

The Electron is a Charged Photon with the de Broglie Wavelength

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A charged photon and its light-speed helical trajectory form a surprising new solution to the relativistic electron's energy-momentum equation. This charged photon model is a new model for the electron, and quantitatively resembles the light-speed electron described by Dirac. The charged photon model of the electron is found to quantitatively predict the relativistic de Broglie wavelength of the free electron. This suggests a new interpretation of quantum mechanics, where the electron is seen as a charged photon, and the quantum mechanical wave equations for the electron are generated by the waves produced by the circulating charged photon that composes the electron.

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1. Introduction

In his Nobel Prize lecture Paul Dirac¹ said: "It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment."

One surprising result of the Dirac equation was that the electron moves at the speed of light and has a characteristic associated length $R_o = \hbar/2mc = 1.93 \times 10^{-13} \text{ m}$, where $\hbar = h/2\pi$ and h/mc is the Compton wavelength $\lambda_{Compton} = 2.426 \times 10^{-12} \text{ m}$. A second and related surprise was that the electron has a "trembling motion" or zitterbewegung in addition to its normal linear motion. Dirac's finding of light-speed for the electron is particularly problematical because electrons are experimentally measured to travel at less than the speed of light. Dirac did not offer a spatially extended model of the electron to correspond to these results, though his results did contain the characteristic length R_o .

Various researchers such as Hestenes², Williamson and van der Mark³, Rivas⁴, Hu⁵, and Gauthier^{6,7} have suggested spatially-extended electron models where the internal energy of the electron circulates at light speed with the characteristic Dirac radius $R_o = \hbar/2mc$. Both the Dirac equation's light-speed motion and its zitterbewegung are reflected in these models.

Hestenes and Rivas independently analyzed the Dirac equation for spatial and dynamical characteristics of the electron's motion. Based on these analyses, they independently proposed that the trajectory of a moving free electron is a helix along which the electron's charge moves at light-speed. When the linear speed and momentum of the electron are zero, the helix becomes a circle of radius $R_o = \hbar/2mc$. Neither author associates this circulating light-speed electric charge with a photon.

2. Modeling the Moving Electron as a Charged Photon

Hestenes' and Rivas' light-speed helical trajectory of an electron's charge can also be described as the helical trajectory of a charged photon. This charged photon has the energy, momentum, speed, wavelength and frequency relationships of a normal photon, but carries the point-like electron's charge with it at the speed of light along the charged photon's helical trajectory.

There is an infinite set of relativistic equations for the helical motion of a charged photon with energy $E = \gamma mc^2 = hv$ and momentum $p = \gamma mc = hv/c$ that

satisfy the electron's relativistic energy-momentum equation $E^2 = p^2c^2 + m^2c^4$ and for which the charged photon's velocity component v in the helix's longitudinal direction equals the velocity of the electron. But only one of this infinite set of helical equations is consistent with the charged-photon model of the electron, which at non-relativistic electron speeds has a helical radius R_o , a z -component of electron spin equal to $s_z = \pm\hbar/2$, and a double-looping helical motion that gives rise to the Dirac equation's *zitterbewegung* electron frequency $\nu_{zitt} = 2mc^2/h$, and the electron's de Broglie wavelength $\lambda_{deBroglie} = h/\gamma mv$.

The parametric equations for the full set of these relativistic helical light-speed trajectories are:

$$\begin{aligned}x(t) &= (\lambda_{Compton} / 2\pi n \gamma^2) \cos(2\pi n \gamma ct / \lambda_{Compton}) \\y(t) &= \pm (\lambda_{Compton} / 2\pi n \gamma^2) \sin(2\pi n \gamma ct / \lambda_{Compton}) \\z(t) &= vt\end{aligned}$$

where $n = 1, 2, 3, \dots$ are the positive integers. The “ \pm ” corresponds to a right and left-turning helix respectively. The meaning of n is the number of complete turns of the helical trajectory around the z -axis corresponding to a length along the helical trajectory of exactly one wavelength $\lambda = \lambda_{Compton} / \gamma$.

The one formula for the charged photon's relativistic trajectory that corresponds to the electron's relativistic energy/momentum equation and Hestenes' and Rivas' helically circulating charge models has $n = 2$. The light-speed along all of the above helical trajectories can be seen by differentiating $x(t)$, $y(t)$, and $z(t)$ with respect to time, and combining the resulting velocities $v_x(t)$, $v_y(t)$ and $v_z(t)$ to give $v_{total} = c$:

$$\begin{aligned}v_x(t) &= -c / \gamma \sin(2\pi n \gamma ct / \lambda_{Compton}) \\v_y(t) &= \pm c / \gamma \cos(2\pi n \gamma ct / \lambda_{Compton}) \\v_z(t) &= v\end{aligned}$$

This gives v_{total} as

$$\begin{aligned}v_{total} &= \sqrt{v_x(t)^2 + v_y(t)^2 + v_z(t)^2} \\&= \sqrt{(c^2 / \gamma^2)[1] + v^2} \\&= \sqrt{c^2(1 - v^2/c^2) + v^2} \\&= \sqrt{c^2 - v^2 + v^2} \\&= \sqrt{c^2}\end{aligned}$$

$$v_{total} = c$$

For $n = 2$ the above parametric equation reduces to

$$\begin{aligned}x(t) &= (\lambda_{Compton} / 4\pi \gamma^2) \cos(4\pi \gamma ct / \lambda_{Compton}) \\y(t) &= \pm (\lambda_{Compton} / 4\pi \gamma^2) \sin(4\pi \gamma ct / \lambda_{Compton}) \\z(t) &= vt\end{aligned}$$

which, since $R_o = \lambda_{Compton} / 4\pi$, becomes

$$\begin{aligned}x(t) &= (R_o / \gamma^2) \cos(\gamma ct / R_o) \\y(t) &= \pm (R_o / \gamma^2) \sin(\gamma ct / R_o) \\z(t) &= vt\end{aligned}$$

The radius of the light-speed helix is $R = R_o / \gamma^2$ and its angular frequency is $\omega = \gamma c / R_o$ which for $\gamma \rightarrow 1$ becomes $\omega_{zitt} = 2\pi \nu_{zitt} = c / R_o$ where the frequency $\nu_{zitt} = c / 2\pi R_o = 2mc^2/h$ is the well-known *zitterbewegung* frequency that characterizes the Dirac electron. These results for the helical radius R and helical frequency ν_{zitt} for the helically circulating charged photon model of the electron are derived below.

3. Dynamical Properties of the Charged Photon Model of the Electron

Let us look closely at the energy and momentum characteristics of the charged photon model of a moving electron. The charged photon carries the electron's charge and moves helically at the speed c with the normal energy-frequency relationship $E = hv$ of a photon. Further, the total relativistic energy $E = \gamma mc^2$ of a moving free electron with longitudinal speed v and longitudinal momentum $p = \gamma mv$ is proposed to equal the helically circulating charged photon's energy $E = \gamma mc^2 = hv$. When the longitudinal component of the charged photon's velocity is identified with the electron's velocity, the electron's momentum $p = \gamma mv$ will then be found to equal the longitudinal component of the helically moving charged photon's momentum $p_{total} = \gamma mc$. In summary, a helically-moving charged photon with energy $E = hv = \gamma mc^2$ and helically-directed momentum $p_{total} = E/c = \gamma mc$ is the model for a moving electron that has energy E and momentum p that are related by $E^2 = p^2c^2 + m^2c^4$.

4. Relations of the Charged photon's Dynamics to the Moving Electron's Dynamics

Let us assume that both the z -axis of the helix of the circulating electron charge above and therefore the

electron's forward linear velocity $v_z = v$ are directed to the right. The helical trajectory given above makes an angle θ with the electron's forward velocity v . The light speed velocity c has a component velocity v in the forward direction, such that $v = c \cos \theta$ or $\cos \theta = v/c$. That forward velocity component v of the charged photon is proposed to be identical to the forward velocity v of the electron. The ratio of the electron's momentum $p = \gamma mv$ to the charged photon's total momentum $p_{total} = \gamma mc$ is seen to be

$$\begin{aligned} p / p_{total} &= \gamma mv / \gamma mc \\ &= v/c \\ &= \cos \theta \end{aligned}$$

So the momentum p of the electron is seen to be equal to the longitudinal or z -component of the total momentum of the circulating charged photon.

Summarizing: $p = p_{total} \cos \theta$, $v = c \cos \theta$, and $\theta = \cos^{-1}(v/c)$. For example, if $v = c/2$, then $\theta = \cos^{-1}(1/2) = 60^\circ$. θ decreases as the electron's speed v increases towards c . But $v < c$ always, as is found experimentally for an electron moving in a vacuum.

5. The Charged Photon's Transverse Momentum Component and Electron Spin

The helically directed total momentum p_{total} of the charged photon makes an angle $\theta < 90^\circ$ with the electron's momentum p , which is the longitudinal momentum component of p_{total} . There is therefore also a transverse momentum component p_{trans} of the charged photon's p_{total} that is perpendicular to the longitudinal momentum p of the electron. This p_{trans} completes a right triangle having p_{total} as the third side.

$$\begin{aligned} p_{trans}^2 + p^2 &= p_{total}^2 \\ p_{trans}^2 &= p_{total}^2 - p^2 \\ &= (\gamma mc)^2 - (\gamma mv)^2 \\ &= (\gamma mc)^2 (1 - v^2/c^2) \\ &= (\gamma mc)^2 / \gamma^2 \\ p_{trans} &= \sqrt{(mc)^2} \\ &= mc \end{aligned}$$

So there is a circulating transverse component $p_{trans} = mc$ of the helically-circulating charged photon's total momentum p_{total} , perpendicular to the momentum $p = \gamma mv$ of the electron. (see Figure 1)

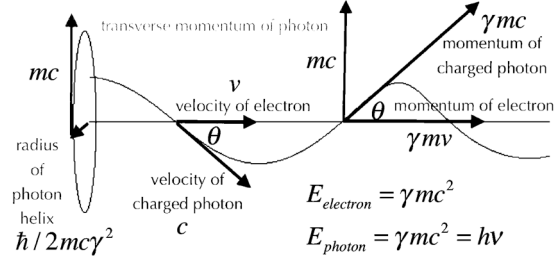


Figure 1. Energy and momentum relationships in the charged photon model of the electron.

Since the charged photon's total momentum circulates along the helix, the value of the transverse momentum $p_{trans} = mc$ can be associated with an internally circulating momentum of the charged photon, or the electron that is modeled by the charged photon. This internally circulating momentum rotates at the same frequency $\nu = \gamma mc^2 / h$ as the charged photon along its helical trajectory. If the electron has no longitudinal momentum, the direction of the transverse momentum will not be clearly defined, and there will be no clearly defined circle of rotation of the transverse momentum $p_{trans} = mc$. When the electron has a small external longitudinal momentum along the z -axis, the internal transverse momentum $p_{trans} = mc$ moves in the xy plane along a circle of radius R_o that is also transverse to the z -axis. Depending on whether the rotation of this momentum mc is positive or negative, this gives the charged photon model a z -component of its angular momentum or spin of

$$s_z = R_o \times p_{trans} = \pm (\hbar / 2mc)(mc) = \pm \hbar / 2$$

which is the experimental z -component of the spin of an electron. The "+" corresponds to a spin-up electron while the "-" corresponds to a spin-down electron. The charged-photon only has these two states for its spin. This corresponds to the quantum description of an electron's spin states, and could be the origin of these spin states of the electron.

Because the radius R_o of the charged photon's open helix corresponds for small electron speeds to a circumference of $2\pi R_o = 0.5 \lambda_{Compton}$, the circulating frequency of the charged photon's internal transverse momentum $p_{trans} = mc$ and its light-speed electric charge would actually be

$$\begin{aligned}
v_{zitter} &= c / \text{circumference} \\
&= c / 0.5 \lambda_{\text{Compton}} \\
&= c / (0.5 h / mc) \\
&= 2mc^2 / h \\
&= 2\nu_0
\end{aligned}$$

This v_{zitter} is the *zitterbewegung* frequency described for the Dirac electron.

6. The Charged Photon's Magnetic Moment

When the internal momentum $p_{\text{trans}} = mc$ with the resting energy $E = mc^2$ of the circling, charged photon are combined with the radius (when the electron's speed $v=0$) of the Hestenes and Rivas helix of $R_0 = \hbar / 2mc$ (Dirac's characteristic electron distance), the result is a model of the electron as a double-looping charged photon whose closed double-looped length is $1\lambda_{\text{Compton}} = h / mc$. When this length λ_{Compton} is double-looped to correspond to the charged photon making a double loop in a resting electron, this double-looped circle has a circumference of $\lambda_{\text{Compton}} / 2 = h / 2mc$, and a radius $R_0 = (\lambda_{\text{Compton}} / 2) / 2\pi = \hbar / 2mc$.

The z -component of the magnetic moment of the charged photon is calculated using the classical method of $M_z = IA$, where I is the effective electric current $I = -e/T$ produced by the circulating electric charge $-e$ in a time T at light-speed in a circular loop of radius R_0 , and A is the loop area.

$$\begin{aligned}
M_z &= IA \\
&= [-e/T] \times (\pi R_0^2) \\
&= [-e / (2\pi R_0 / c)] \times (\pi R_0^2) \\
&= (-ec/2) \times R_0 \\
&= -0.5e\hbar / 2m \\
&= -0.5\mu_{\text{Bohr}}
\end{aligned}$$

where $\mu_{\text{Bohr}} = e\hbar / 2m$ is the Bohr magneton. The Dirac equation predicts the magnitude of the electron's magnetic moment's z -component to be $M_z = -1\mu_{\text{Bohr}}$.

7. How the Pitch of the Charged Photon's Helix Depends on the Electron's Speed

We have seen that $\cos\theta = v/c$ where θ is the angle the circulating charged photon's total momentum $p_{\text{total}} = \gamma mc$ along its helical trajectory makes with the longitudinal direction. A simple calculation starting with $\cos\theta = v/c$ and using the Pythagorean theorem

yields $\tan\theta = (\sqrt{c^2 - v^2}) / v = (c/v)\sqrt{1 - v^2/c^2} = c/\gamma v$. Similarly, $\sin\theta = \cos\theta \tan\theta = (c/\gamma v)(v/c) = 1/\gamma$.

For an electron moving with speed v , the wavelength λ of the charged photon moving along its helix is found from $E = \gamma mc^2 = hv = hc/\lambda$. That is, $\lambda = hc/\gamma mc^2 = h/\gamma mc = \lambda_{\text{Compton}}/\gamma$. This length λ along the helical trajectory corresponds to two turns of the helix or a longitudinal distance $2P$ where the helical pitch P is the longitudinal distance for one full helix rotation.

$$\begin{aligned}
2P &= \lambda \cos\theta \\
2P &= (\lambda_{\text{Compton}}/\gamma)(v/c) \\
P &= (v/2c)(\lambda_{\text{Compton}}/\gamma) \\
P &= (v/2c)(4\pi R_0/\gamma) \\
P &= (2\pi v/c\gamma)R_0
\end{aligned}$$

For example: if $v = 0.01c$, $P = 0.063 R_0$. If $v = 0.5c$, $P = 2.72 R_0$. If $v = 0.999c$, $P = 0.28 R_0$.

The pitch has a maximum value P_{max} for one particular value of the electron's speed v . Calculation of this maximum pitch gives $P_{\text{max}} = \pi R_0$ when $v = (\sqrt{2}/2)c$. At this value of v , the value of θ is $\theta = \cos^{-1}(v/c) = \cos^{-1}(\sqrt{2}/2) = 45^\circ$ exactly. The value of γ for $v = (\sqrt{2}/2)c$ is $\gamma = \sqrt{2}$. The value of the charged photon's wavelength λ for this γ is $\lambda = \lambda_{\text{Compton}}/\gamma = \lambda_{\text{Compton}}/\sqrt{2} = (\sqrt{2}/2)\lambda_{\text{Compton}}$.

8. How the Radius of the Charged Photon's Helix Depends on the Electron's Speed

The radius R of the charged photon's helix varies with the electron's speed v . When two turns or a longitudinal length $2P$ of the helix are "unrolled" flat, a right triangle is formed having one angle θ as previously seen, where the charged photon's wavelength $\lambda = \lambda_{\text{Compton}}/\gamma$ is the hypotenuse. The vertical side of the triangle has length two helical circumferences or $4\pi R$. The horizontal side of the triangle has length $2P = \lambda \cos\theta = \lambda(v/c)$. Using $\sin\theta = 1/\gamma$, the value of R can be calculated:

$$\begin{aligned}
\text{vertical side} &= \text{hypotenuse} \times \sin\theta \\
4\pi R &= (\lambda_{\text{Compton}}/\gamma) \times (1/\gamma) \\
R &= R_0/\gamma^2
\end{aligned}$$

where $R_o = \lambda_{Compton} / 4\pi = \hbar / 2mc$, the characteristic Dirac distance. This $R = R_o / \gamma^2$ is the radius of the helical trajectory for the charged photon given in the three parametric equations earlier in this article.

For example, when the electron's speed is $v = 0.1c$, then $R = 0.99R_o$. For $v = 0.5c$, $R = 0.75R_o$. For $v = 0.9c$, $R = 0.19R_o$, and for $v = 0.999c$, $R = 0.002R_o$. The corresponding values of θ , where $\theta = \cos^{-1}(v/c)$, are 84° , 60° , 26° and 2.6° .

9. How the Frequency of Rotation of the Charged Photon Depends on the Electron's Speed

In the parametric equation for the helix of the circulating charged photon, the argument of the sine and cosine functions is $\gamma ct / R_o$. Where does this come from? For an electron of speed v , the energy of the charged photon is $E = \gamma mc^2 = h\nu$, where ν is the frequency of the charged photon. In the time that the charged photon has traveled one wavelength $\lambda = \lambda_{Compton} / \gamma$ along its helical trajectory, the trajectory has looped twice around its axis. This double-looping of the trajectory in each cycle of the helical photon produces a cycling frequency around the helical axis of

$$\begin{aligned} \nu_{zitter} &= 2\nu \\ &= 2\gamma mc^2 / h \\ &= 2\gamma c / \lambda_{Compton} \\ &= 2\gamma c / (4\pi R_o) \\ &= \gamma c / 2\pi R_o \end{aligned}$$

The corresponding angular frequency for the charged photon around its longitudinal axis is

$$\begin{aligned} \omega_{zitter} &= 2\pi \nu_{zitter} \\ &= 2\pi (\gamma c / 2\pi R_o) \\ &= \gamma c / R_o \end{aligned}$$

and the argument of the sine and cosine functions in the parametric equation for the charged helix is

$$\omega_{zitter} t = \gamma ct / R_o.$$

10. The Apparent Upper-Limit Size of the Electron as Found from High-Energy Electron Scattering Experiments

Bender et al⁸ (1984) found from high-energy electron scattering experiments at 29GeV (one GeV equals

one thousand million electron volts) that the upper limit of the size of the electron is around 10^{-18}m . This result has been difficult to reconcile with spatially extended models of the electron having a radius approximating $R_o = 1.93 \times 10^{-13}\text{m}$. But for an electron scattering experiment with electrons having energies of $E = \gamma mc^2 = 29\text{GeV}$, with $mc^2 = .511\text{MeV}$ for an electron, this gives $\gamma = E / (mc^2) = 5.68 \times 10^4$, the value of the radius R for the charged photon's helix would be

$$\begin{aligned} R &= R_o / \gamma^2 \\ &= 1.93 \times 10^{-13}\text{m} / (5.68 \times 10^4)^2 \\ &= 6.0 \times 10^{-23}\text{m} \end{aligned}$$

This is well below the upper limit of the electron's size found in such high-energy scattering experiments. When a model of the photon is incorporated into the charged photon model, the size of the charged photon a high energies is expected to be larger than the above result, but the photon size is still expected to decrease with increasing γ .

11. Testing the Charged Photon Model of the Electron—the Electron-Clock Experiment

The way to test the charged photon hypothesis is to see what new predictions the hypothesis leads to and to test these predictions experimentally to see if they are supported or not. Currently there is ongoing experimental research by Gouanère et al⁹ (2005) to test the idea that an electron behaves as if it has an internal "clock" based on the frequency ν_o found in the Einstein-de Broglie equation for the electron $h\nu_o = mc^2$ for a resting electron. If there were such a clock in the electron, special relativity predicts that the rate of ticking of that clock would be slower in a moving electron, as measured in a stationary laboratory. According to Gouanère et al, the slowing down of this "clock" with electron speed might be detectable by observing the variable scattering or absorption of electrons when passing with a range of energies through certain crystals at specific angles. The *zitterbewegung* frequency $\nu_{zitter} = 2\nu_o$ might also be observed by using this "clock model".

But according to the charged photon model of the electron, the electron does not behave like a clock whose frequency of ticking would slow down as the electron moved faster. Rather, the frequency ν of the charged photon modeling the electron increases with speed as $\nu = \gamma \nu_o$ since the energy of the circulating charged photon increases as $E = \gamma mc^2 = \gamma h\nu_o = h\nu$. A

moving electron-frequency “clock”, measured by a stationary observer, would beat faster rather than slower with increasing electron speed. So the charged photon model predicts a negative result for Gouanère-type electron-clock experiments. This is not to deny that relativistic time dilation takes place for a moving electron or other particles.

A continuing negative result of Gouanère-type electron-clock experiments would of course not be direct support for the charged photon hypothesis, since there could also be technical reasons for a negative result even if the electron-clock hypothesis were correct. But a confirmed positive result for these electron-clock experiments, which assume a decrease of the internal electron frequency with increasing speed of the electron, would disprove the charged photon hypothesis of the electron, which, like de Broglie’s derivation of a moving electron’s wavelength, claims that the electron’s internal frequency increase as the speed and total energy of the electron increase. So the charged photon model of the electron is falsifiable, an important property of a scientific hypothesis.

Dirac claimed in his Nobel lecture that the speed-of-light velocity of the electron could not be directly verified by experiment because of the high frequency of the oscillatory motion of the electron and its small amplitude of motion. But what was not feasible in 1933 might be feasible in 2015 or later, particularly since conceiving of the Dirac speed-of-light electron as a circulating charged photon may introduce new thinking about testing this hypothesis.

12. The Charged Photon Model and the Origin of the Electron’s Spin

The electron (and positron) with a z -component of spin $\pm\hbar/2$ corresponds to the solution of the light-speed circulating charged photon for the value $n=2$ in the general parametric equation for solutions to the well-known relativistic energy-momentum equation $E^2 = p^2 c^2 + m^2 c^4$ presented above. The calculated spin $\pm\hbar/2$ is only for a slowly moving electron. The solutions to the energy-momentum equation are quantized in the sense that only integer values of n are permitted in the charged photon model. The model predicts that spin could be quantized differently, depending on the value of n , which is the number of times that a single wavelength $\lambda = \lambda_{Compton} / \gamma$ of a charged photon corresponding to an electron with speed v turns around its helical axis during one period $T = 1/v = h/\gamma mc^2$ of a charged photon. Mathematically, there are also solutions to the energy-momentum equation corresponding to $n=1,3,4,5,6\dots$. The value of the transverse component of momentum $p_{trans} = mc$ for a hypothetical particle with mass m

composed of a circulating charged photon does not depend on the value of n , but on the radius of the helix formed, which is inversely proportional to n . This leads to other predictions of possible particles with spin $s_z = \pm\hbar, \pm\hbar/3, \pm\hbar/4$ and in general $\pm\hbar/n$. The value $n=1$ corresponds to a charged boson, possibly the W^+ and W^- particles of the weak interaction. $n=2$ corresponds to the electron, which is a fermion, to the muon and tau particles of the electron family which are also fermions, to the six types of quark, which also are fermions, and to all their fermion antiparticles. Just as the electron is modeled as a circulating charged photon, the quarks might be modeled as circulating charged gluons. Higher values of n would correspond to smaller spin but also do not correspond to any known physical particles. All of these charged-photon particles are mathematical solutions of the relativistic energy-momentum equation. It is interesting that the Dirac equation only gives solutions to the energy-momentum equation for $n=2$, describing the electron and positron and the spin-up and spin-down values of the z -component of their spins. The charged photon model predicts that there should be equations similar to the Dirac equation that provide mathematical solutions for charged photon particles corresponding to $n=1,3,4,5\dots$ of the energy-momentum equation, with corresponding $s_z = \hbar, s_z = \hbar/3, s_z = \hbar/4, s_z = \hbar/5$ and so on.

13. The Charged Photon Model Predicts the de Broglie Wavelength of the Electron

The derivation of the de Broglie wavelength from the charged photon model goes as follows. In the circulating charged photon model of the electron, the photon and the electron have energy $E = \gamma mc^2$. This equals $E = h\nu$ for the charged photon. By equating these two energy terms, the frequency of the charged photon is then found to be $\nu = \gamma mc^2 / h$. There is a charged photon wavelength λ corresponding to this charged photon frequency ν . Its wavelength is $\lambda = c/\nu = c/(\gamma mc^2 / h) = h/\gamma mc$. The charged photon has a wavenumber $k_{total} = 2\pi/\lambda$, which corresponds to a wave vector \vec{k}_{total} pointing along the charged photon’s helical trajectory. For the circulating charged photon

$$\begin{aligned} k_{total} &= 2\pi/\lambda \\ &= 2\pi/(h/\gamma mc) \\ &= 2\pi\gamma mc/h \end{aligned}$$

Like the charged photon's total circulating momentum $\vec{p}_{total} = \hbar \vec{k}_{total}$, \vec{k}_{total} makes an angle θ with the longitudinal direction of the circulating charged photon, where $\cos\theta = v/c$, and v is the velocity of the electron along the helical axis, as well as the longitudinal velocity of the circulating charged photon that is modeling the moving electron. The charged photon's wave vector \vec{k}_{total} has a longitudinal component

$$\begin{aligned} k &= k_{total} \cos\theta \\ &= (2\pi\gamma mc / h)(v/c) \\ &= 2\pi\gamma mv / h \end{aligned}$$

The wavenumber component k has a longitudinal wavelength

$$\begin{aligned} \lambda_{longitudinal} &= 2\pi / k \\ &= 2\pi / (2\pi\gamma mv / h) \\ &= h / \gamma mv \\ &= \lambda_{deBroglie} \end{aligned}$$

Summarizing this result, the longitudinal component of the wave vector \vec{k}_{total} of the helically circulating charged photon that is a model for the moving electron, yields the relativistic de Broglie wavelength $\lambda_{deBroglie} = h / \gamma mv$. The de Broglie wavelength was not explicitly designed into the charged photon model of the electron, which as described above is based on the relativistic energy-momentum formula $E^2 = p^2 c^2 = m^2 c^4$ for the electron and the light-speed helical path of the charged photon described above.

14. Derivation of the Quantum Wave Function for a Free Electron

There is a related way to show how the charged photon model of the electron moving with velocity v generates the de Broglie wavelength as well as the electron's phase velocity $v_{phase} = c^2 / v$ along the direction that the electron is moving. Consider an electron moving to the right along the $+x$ -axis with velocity v . The charged photon's helix winds around the electron's trajectory making an angle θ with the electron's forward direction given by $\cos\theta = v/c$ in the charged photon model. A photon of wavelength λ , wave number $k_{total} = 2\pi / \lambda$, frequency $\nu = c / \lambda$ and angular frequency $\omega = 2\pi\nu$ can be mathematically described as a plane wave $\Phi(\vec{r}, t) = Ae^{i(\vec{k}_{total}\vec{r} - \omega t)}$ where \vec{k}_{total} is the photon's wave vector pointing in the

direction the photon is moving and ω is the photon's angular frequency. The vector \vec{r} indicates the distance and direction in which the magnitude of the plane wave is to be calculated. The phase velocity of the plane wave of the photon is $v_{phase} = \omega / k_{total} = 2\pi\nu / (2\pi / \lambda) = v\lambda = c$.

In the charged photon model, the circulating charged photon is proposed to correspond to a plane wave $\Phi(\vec{r}, t) = Ae^{i(\vec{k}_{total}\vec{r} - \omega t)}$ which however is changing its direction because the charged photon's wave vector \vec{k}_{total} is changing direction as the charged photon circulates helically around the axis along which the electron is moving, in this case the $+x$ -axis. Although the direction of the wave vector \vec{k}_{total} keeps changing with time, \vec{k}_{total} maintains its angle θ with the x -axis, where $\cos\theta = v/c$. So the value of $\Phi(x, t)$ along the x -axis of the circulating plane wave of the charged photon model becomes

$$\begin{aligned} \Phi(x, t) &= Ae^{i(k_{total}x \cos\theta - \omega t)} \\ &= Ae^{i(k_{total}xv/c - \omega t)} \end{aligned}$$

since in this case, $\vec{k}_{total} \cdot \vec{r} = k_{total}x \cos\theta$. But in the charged photon model, the value of the wave number k_{total} of the circulating photon of energy $E = \gamma mc^2 = h\nu$ and wavelength

$$\begin{aligned} \lambda &= c / \nu \\ &= c / (\gamma mc^2 / h) \\ &= h / \gamma mc \end{aligned}$$

is calculated to be

$$\begin{aligned} k_{total} &= 2\pi / \lambda \\ &= 2\pi / (h / \gamma mv) \\ &= 2\pi\gamma mc / h \\ &= \gamma mc / \hbar \end{aligned}$$

Substituting the photon's value of k_{total} into $\Phi(x) = Ae^{i(k_{total}xv/c - \omega t)}$ gives

$$\begin{aligned} \Phi(x, t) &= Ae^{i(\gamma mc/h)xv/c - \omega t} \\ &= Ae^{i(\gamma mv/h)x - \omega t} \\ &= Ae^{i(kx - \omega t)} \end{aligned}$$

where $k = \gamma mv / \hbar$ is the longitudinal or x -component of the photon's wave vector \vec{k}_{total} . Since $k = 2\pi / \lambda_{longitudinal}$ where $\lambda_{longitudinal}$ is the wavelength along the x -axis that corresponds to k , this gives

$$\begin{aligned}
\lambda_{\text{longitudinal}} &= 2\pi / k \\
&= 2\pi / (\gamma mv / \hbar) \\
&= h / \gamma mv \\
&= \lambda_{\text{deBroglie}}
\end{aligned}$$

which is the relativistic de Broglie wavelength for a moving electron $\lambda_{\text{deBroglie}} = h / \gamma mv$.

The expression for $\Phi(\vec{r}, t)$ has become $\Phi(x, t) = Ae^{i(kx - \omega t)}$ along the longitudinal or x -axis of the circulating charged photon, where $k = \gamma mv / \hbar$. The value for the phase velocity of the charged photon model of the electron along the x -axis becomes

$$\begin{aligned}
v_{\text{phase}} &= \omega / k \\
&= (\gamma mc^2 / \hbar) / (\gamma mv / \hbar) \\
&= c^2 / v
\end{aligned}$$

where v is the electron's velocity and $\hbar\omega = hv = \gamma mc^2$ is the charged photon's energy E .

So in the charged photon model of the electron, the charged photon's energy is $E = \gamma mc^2 = hv = \hbar\omega$ and its momentum in the electron's direction is $p = \gamma mv = h / \lambda_{\text{deBroglie}} = \hbar k$. The charged photon has the properties of the electron that formed the basis of the quantum mechanical Schrodinger equation for an electron.

The expression $\Phi(x, t)$ above corresponds to the one-dimensional quantum mechanical wave function $\Psi(x, t) = Ae^{i(kx - \omega t)}$ for a free electron having the de Broglie wavelength and moving in the x -direction. The wave function's wave number $k = 2\pi / \lambda_{\text{deBroglie}}$ equals the longitudinal component of the charged photon's wave vector. The angular frequency ω is that of the charged photon corresponding to the charged photon's total energy $E = \gamma mc^2 = \hbar\omega$. The phase velocity is $v_{\text{phase}} = c^2 / v$ where v is the velocity of the electron.

15. Geometric Relation of the de Broglie Wavelength to the Charged-Photon Model

The relationship of the de Broglie wavelength of a moving electron to the wavelength of the charged photon in the charged photon model of the electron can also be shown geometrically in a simple diagram (see Figure 2 below). Consider an electron as moving horizontally to the right (along the $+x$ -axis) with velocity v , momentum $p = \gamma mv$ and total energy $E = \gamma mc^2$. The charged photon, having momentum $p_{\text{photon}} = \gamma mc$ and energy $E = \gamma mc^2 = hv$, moves in a

helix around the electron's trajectory, making an angle θ (taken from the charged photon model, where $\cos\theta = v/c$) with the x -axis. (In a two-dimensional view this helix looks like two complete cycles of a sine wave that has a maximum angle θ with the x -axis.) The two cycles (in the charged photon model these two cycles are due to the *zitterbewegung* frequency $\nu_{\text{zitter}} = 2mc^2/h$ of the Dirac electron) of the helix corresponds to a distance along the helical path of one charged photon wavelength $\lambda_{\text{photon}} = h / \gamma mc$ (from the charged photon model.)

Now (see Figure 2) unroll the helical trajectory for these two full cycles of the helix so that the two-cycle helical trajectory of length $\lambda_{\text{photon}} = h / \gamma mc$ becomes a straight-line segment of the same length $\lambda_{\text{photon}} = h / \gamma mc$ that makes an angle θ with the x -axis. At the top end of this line segment draw a line perpendicular to the line segment in the downward direction to the right until this line meets the x -axis. The total length of the horizontal line segment produced in this way is the de Broglie wavelength of the electron $\lambda_{\text{deBroglie}} = h / \gamma mv$. As the speed v of the electron increases, the value of θ decreases, the wavelength λ_{photon} of the circulating charged photon decreases, and the de Broglie wavelength decreases. The de Broglie phase wave travels to the right with velocity $v_{\text{phase}} = c^2 / v$ as the electron travels to the right with velocity v and the charged photon travels along its helical path with velocity c .

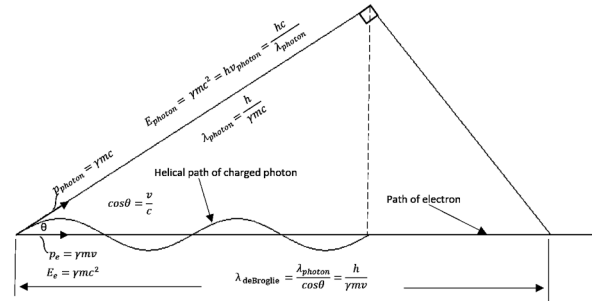


Figure 2. The de Broglie wavelength generated from the charged photon model of electron.

16. Conclusions

The de Broglie equation for the electron forms the basis of quantum mechanics, described by the Schrödinger equation for the electron in non-relativistic quantum mechanics and by the Dirac equation in relativistic quantum mechanics. The charged-photon approach may lead to a somewhat more fundamental and unified description of the quantum world than is provided by the current standard model of physics and

the dominant interpretation of quantum mechanics, the Copenhagen interpretation. Not all fundamental particles lend themselves to the charged-photon approach to matter however. For example, neutrinos and the Higgs boson are fundamental particles that have mass but are neutral. But quarks might be reconceptualized as circulating charged gluons using a similar approach to the charged-photon model of the electron. And uncharged photons or other light-speed quantum particles may be able to helically circulate in ways that form neutrinos and other uncharged fundamental particles having mass.

At the very least the charged-photon model of the electron indicates why it should be no surprise that, in the proper circumstances, the electron shows all the interference, diffraction, and quantum entanglement properties of the photon, such as in the well-known double-slit experiment, since in this view the electron IS a charged photon. Reconceptualizing the electron as a charged photon can also lead to new hypotheses about the charged photon that can then be tested experimentally. De Broglie correctly proposed that matter has a wave-like nature. The charged-photon approach may explain why this is.

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A typo has been corrected that is in the first edition of this article. On page 2, column 1, line 23, the mathematical expression was corrected from $\lambda_{Compton}$ to $\lambda = \lambda_{Compton} / \gamma$. No other content of the article is affected by this change. Updated Feb 12, 2015.