version of April 9, 2015

Notes on "A Greater ElectroMagnetism"

M.B. van der Mark[a] and J.G. Williamson[b]

[a]Philips Research Laboratories, HTC 34, WB-21,
Prof. Holstlaan 4, 5656 AA Eindhoven, The Netherlands
[b]Glasgow University, Department of Electronics & Electrical Engineering,
Glasgow G12 8QQ, Scotland

Contents

Appendix XV: Multiplication of multivectors using matrices

The multiplication of two general multivectors involves 256 components, and together with its non-commuting nature, this makes it useful to be implemented in a computerprogram as a matrix multiplication.

$$
\begin{aligned}
\Psi\Phi &= (M(\Phi)\vec{\Psi})\cdot\vec{e} \\
&= (M(\Phi)_{XY}\Psi_Y)\cdot e_Y \qquad (1) \\
\Psi &= \Psi_Y\cdot e_Y \quad Y\in\{\ ,\nu,\mu\nu,\kappa\mu\nu,0123\} \qquad (2)
\end{aligned}
$$

If Ψ is of the form given by Eq. (**??**), so that $s_0=\phi$, $v_\nu=\phi_\nu$, $b_i=\phi_{i0}$, etc., the matrix M looks like this:

$$
\begin{pmatrix}
\phi & \phi_0 & -\phi_1 & -\phi_2 & -\phi_3 & \phi_{10} & \phi_{20} & \phi_{30} & -\phi_{23} & -\phi_{31} & -\phi_{12} & -\phi_{023} & -\phi_{031} & -\phi_{012} & \phi_{123} & -\phi_{0123} \\
\phi_0 & \phi & -\phi_{10} & -\phi_{20} & -\phi_{30} & \phi_1 & \phi_2 & \phi_3 & -\phi_{023} & -\phi_{031} & -\phi_{012} & -\phi_{23} & -\phi_{31} & -\phi_{12} & -\phi_{0123} & \phi_{123} \\
\phi_1 & -\phi_{10} & \phi & \phi_{12} & -\phi_{31} & \phi_0 & \phi_{012} & -\phi_{031} & -\phi_{123} & \phi_3 & -\phi_2 & -\phi_{0123} & -\phi_{30} & \phi_{20} & -\phi_{23} & \phi_{023} \\
\phi_2 & -\phi_{20} & -\phi_{12} & \phi & \phi_{23} & -\phi_{012} & \phi_0 & \phi_{023} & -\phi_3 & -\phi_{123} & \phi_1 & \phi_{30} & -\phi_{0123} & -\phi_{10} & -\phi_{31} & \phi_{031} \\
\phi_3 & -\phi_{30} & \phi_{31} & -\phi_{23} & \phi & \phi_{031} & -\phi_{023} & \phi_0 & \phi_2 & -\phi_1 & -\phi_{123} & -\phi_{20} & \phi_{10} & -\phi_{0123} & -\phi_{12} & \phi_{012} \\
\phi_{10} & -\phi_1 & \phi_0 & \phi_{012} & -\phi_{031} & \phi & \phi_{12} & -\phi_{31} & \phi_{0123} & \phi_{30} & -\phi_{20} & \phi_{123} & -\phi_3 & \phi_2 & -\phi_{023} & \phi_{23} \\
\phi_{20} & -\phi_2 & -\phi_{012} & \phi_0 & \phi_{023} & -\phi_{12} & \phi & \phi_{23} & -\phi_{30} & \phi_{0123} & \phi_{10} & \phi_3 & \phi_{123} & -\phi_1 & -\phi_{031} & \phi_{31} \\
\phi_{30} & -\phi_3 & \phi_{031} & -\phi_{023} & \phi_0 & \phi_{31} & -\phi_{23} & \phi & \phi_{20} & -\phi_{10} & \phi_{0123} & -\phi_2 & \phi_1 & \phi_{123} & -\phi_{012} & \phi_{12} \\
\phi_{23} & \phi_{023} & -\phi_{123} & \phi_3 & -\phi_2 & -\phi_{0123} & -\phi_{30} & \phi_{20} & \phi & \phi_{12} & -\phi_{31} & \phi_0 & \phi_{012} & -\phi_{031} & -\phi_1 & -\phi_{10} \\
\phi_{31} & \phi_{031} & -\phi_3 & -\phi_{123} & \phi_1 & \phi_{30} & -\phi_{0123} & -\phi_{10} & -\phi_{12} & \phi & \phi_{23} & -\phi_{012} & \phi_0 & \phi_{023} & -\phi_2 & -\phi_{20} \\
\phi_{12} & \phi_{012} & \phi_2 & -\phi_1 & -\phi_{123} & -\phi_{20} & \phi_{10} & -\phi_{0123} & \phi_{31} & -\phi_{23} & \phi & \phi_{031} & -\phi_{023} & \phi_0 & -\phi_3 & -\phi_{30} \\
\phi_{023} & \phi_{23} & \phi_{0123} & \phi_{30} & -\phi_{20} & \phi_{123} & -\phi_3 & \phi_2 & \phi_0 & \phi_{012} & -\phi_{031} & \phi & \phi_{12} & -\phi_{31} & -\phi_{10} & -\phi_1 \\
\phi_{031} & \phi_{31} & -\phi_{30} & \phi_{0123} & \phi_{10} & \phi_3 & \phi_{123} & -\phi_1 & -\phi_{012} & \phi_0 & \phi_{023} & -\phi_{12} & \phi & \phi_{23} & -\phi_{20} & -\phi_2 \\
\phi_{012} & \phi_{12} & \phi_{20} & -\phi_{10} & \phi_{0123} & -\phi_2 & \phi_1 & \phi_{123} & \phi_{031} & -\phi_{023} & \phi_0 & \phi_{31} & -\phi_{23} & \phi & -\phi_{30} & -\phi_3 \\
\phi_{123} & \phi_{0123} & \phi_{23} & \phi_{31} & \phi_{12} & \phi_{023} & \phi_{031} & \phi_{012} & \phi_1 & \phi_2 & \phi_3 & -\phi_{10} & -\phi_{20} & -\phi_{30} & \phi & -\phi_0 \\
\phi_{0123} & \phi_{123} & -\phi_{023} & -\phi_{031} & -\phi_{012} & -\phi_{23} & -\phi_{31} & -\phi_{12} & -\phi_{10} & -\phi_{20} & -\phi_{30} & \phi_1 & \phi_2 & \phi_3 & -\phi_0 & \phi
\end{pmatrix}
$$

In case ϕ_Y are the unit vectors, this matrix is the sum of 16 matrices that form a representation of the algebra of 16×16 matrices $\{e_Y\}$ with real entries: $M_{16}(I\!R)$.

The multiplication of two arbitrary multivectors $\Psi = s+v+b+r+t+q$ yields

$$
\Psi_\alpha\Psi_\beta = \Psi_\alpha\cdot\Psi_\beta + \Psi_\alpha\wedge\Psi_\beta = \frac{1}{2}(\Psi_\alpha\Psi_\beta+\Psi_\beta\Psi_\alpha)+\frac{1}{2}(\Psi_\alpha\Psi_\beta-\Psi_\beta\Psi_\alpha) \qquad (3)
$$

with

$$
\Psi_\alpha\cdot\Psi_\beta = s_{0\alpha}s_{0\beta}+v_{0\alpha}v_{0\beta}-\vec{v}_\alpha\cdot\vec{v}_\beta+\vec{b}_\alpha\cdot\vec{b}_\beta-\vec{r}_\alpha\cdot\vec{r}_\beta-\vec{t}_\alpha\cdot\vec{t}_\beta+t_{0\alpha}t_{0\beta}-q_{0\alpha}q_{0\beta}
$$

$$
\begin{aligned}
&+ \quad e_0(s_{0\alpha}v_{0\beta} + s_{0\beta}v_{0\alpha} - \vec{r}_\alpha \cdot \vec{t}_\beta - \vec{r}_\beta \cdot \vec{t}_\alpha) \\
&+ \quad \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} (s_{0\alpha}\vec{v}_\beta + s_{0\beta}\vec{v}_\alpha - t_{0\alpha}\vec{r}_\beta - t_{0\beta}\vec{r}_\alpha + \vec{b}_\alpha \times \vec{t}_\beta + \vec{b}_\beta \times \vec{t}_\alpha) \\
&+ \quad \begin{pmatrix} e_{10} \\ e_{20} \\ e_{30} \end{pmatrix} (s_{0\alpha}\vec{b}_\beta + s_{0\beta}\vec{b}_\alpha + q_{0\alpha}\vec{r}_\beta + q_{0\beta}\vec{r}_\alpha + \vec{v}_\alpha \times \vec{t}_\beta + \vec{v}_\beta \times \vec{t}_\alpha) \\
&+ \quad \begin{pmatrix} e_{23} \\ e_{31} \\ e_{12} \end{pmatrix} (s_{0\alpha}\vec{r}_\beta + s_{0\beta}\vec{r}_\alpha + v_{0\alpha}\vec{t}_\beta + v_{0\beta}\vec{t}_\alpha - t_{0\alpha}\vec{v}_\beta - t_{0\beta}\vec{v}_\alpha - q_{0\alpha}\vec{b}_\beta - q_{0\beta}\vec{b}_\alpha) \\
&+ \quad \begin{pmatrix} e_{023} \\ e_{031} \\ e_{012} \end{pmatrix} (s_{0\alpha}\vec{t}_\beta + s_{0\beta}\vec{t}_\alpha + v_{0\alpha}\vec{r}_\beta + v_{0\beta}\vec{r}_\alpha + \vec{v}_\alpha \times \vec{b}_\beta + \vec{v}_\beta \times \vec{b}_\alpha) \\
&+ \quad e_{123}(s_{0\alpha}t_{0\beta} + s_{0\beta}t_{0\alpha} + \vec{v}_\alpha \cdot \vec{r}_\beta + \vec{v}_\beta \cdot \vec{r}_\alpha) \\
&+ \quad e_{0123}(s_{0\alpha}q_{0\beta} + s_{0\beta}q_{0\alpha} - \vec{b}_\alpha \cdot \vec{r}_\beta - \vec{b}_\beta \cdot \vec{r}_\alpha) \qquad (4)
\end{aligned}
$$

$$
\Psi_\alpha \wedge \Psi_\beta = \qquad (5)
$$