Mass and the Coulomb potential in positronium[©]

A. Meulenberg,[a)] K P Sinha,[b)] W.R. Hudgins,[c)] and R. F. Penland, Jr.[d)]

Abstract

When mass is defined as a function of electrostatic potential energy and the unit charge is defined as a strong resonance of the electro-magnetic field, then the creation and annihilation of electron-positron pairs may be viewed as continuous relativistic processes rather than as 'mystical' quantum transitions between different energy states. Charge and rest mass can no longer be considered as relativistic 'invariants'. They must be redefined to be determined in a potential-free environment. The relativistic 'effective' mass of an electron may not increase with velocity in the expected manner when close to a positron. The fundamental assumption of this paper is that mass, including rest mass, is a measure of this potential energy, i.e., a change in Coulomb potential results in the change in mass of the causative charged particle. In the case of positronium, all of the mass is proposed to be from the Coulomb interaction.

1. Introduction

The energy source of the 1/r Coulomb potential is generally attributed to the charge of particles as if the 'unit' charges themselves, considered relativistically invariant, could contribute energy changes to the system. However, when the potential energy changes, does the unit charge also change? Perhaps not; but the associated electric field changes and therefore the field-energy may also. How are these changes and 'invariants' to be reconciled?

We start with a very 'simple' classical picture for energy transformation in relativity as a baseline and look for these changes. When oppositely charged leptons (e.g., electrons and positrons) 'fall' together, they accelerate, gain kinetic energy, and 'radiate' electromagnetic (EM) energy. Nearly all, or all, of this EM energy remains bound to the leptons' electric-field lines. Being bound to the leptons, this EM energy, as does the leptons' kinetic energy, constitutes a portion of the leptons' total energy.[1, 2] When the leptons are very close, this additional energy (kinetic and EM field) can be significant (MeV range). As their momentum carries them past each other, the leptons slow down and begin to reabsorb much or all of this bound radiant field energy. Since there is almost no chance for the leptons to collide 'head-on', they will circle each other (elliptic orbits) in their common potential well.[i]

[i] This assumes that their kinetic energy is insufficient to carry them out of the potential well.

Over time and many cycles, the lepton dynamics are established as a bound state, eventually in steady state with the EM field. The leptons can radiate photon energy until they reach the atomic ground state, at which point they do not have sufficient angular momentum to create single photons (photon pairs are still a viable option, but with a very low transition probability). The cyclic nature of this steady-state condition comes from the fact that, as the leptons move closer together, on average their 'orbital' frequency increases. The bound EM field responds to that increased velocity (acceleration in a bound state) and thereby increases both its dominant frequency and total energy. Likewise, the EM field can give back energy to the leptons; in this case, both leptons and EM field experience a decrease in frequency. Thus, there is a reversible transfer of energy from the leptons to their EM field.

The ground state is only a local minimum (a mechanical resonance between the electron and the Coulomb potential well); therefore, the leptons are only metastable in this orbit. The photon pair after positron-electron annihilation is the true minimum for positronium, not the bound lepton pair. If the lepton pair is perturbed out of its atomic 'ground' state, it can decay by EM-field generation toward the annihilation point. In free space, the decay mechanism, via double or multiple photon emission, is somewhat different since neither phonons nor single photons are possible.

The question that is seldom addressed is "transfer of energy from the leptons' WHAT to their EM field." The standard response is that the Coulomb potential provides this field energy. Using Feynman's Lectures on Physics[3] as a guide, we will explore that response to see where it, and an alternative, might lead. Electron-positron pair production and annihilation have been studied in detail over the last decades.[4] However, the present authors believe that none of the studies has bothered with this area. The importance of the question is that, at annihilation, the EM field becomes free photons and the WHAT disappears.

At relativistic velocities, the energies for each lepton of an isolated, but widely separated, pair, neglecting any potential energy,[5] are $E_e = \gamma_e m_{eo} c^2$, where $\gamma_e = (1- v_e^2/c^2)^{-1/2}$ and c is the speed of light in free space. The rest mass of the electron, m_{eo} is the same as that of the positron, m_{po}, and v_e is the electron velocity in the laboratory system. Therefore, in the conventional view, the presence of their mutual electrostatic field as they get closer together causes the leptons to gain kinetic energy KE, electromagnetic (EM) field energy, and effective mass, $m_{eff} = \gamma_e m_{eo}$, as they accelerate toward one another. All three variations are incorporated in the single γ_e that extends to at least a factor of two for leptons transiting their near-zone. The increase in effective mass coincides with the increase in EM-field energy.[6] The potential (but not obvious) problem with this picture is that γ_e is always greater than one.

The starting point for this paper's approximation to the problem is that the lepton pair is initially separated and with negligible kinetic energy with a 1/r Coulomb potential. Thus, $E_{e\infty} = m_{e\infty} c^2 =$

$E_{p\infty}$, with $\gamma_e = \gamma_p = 1$, and therefore $E_{total} = E_e + E_p = 2m_{e\infty}c^2$. The distinction between the restmass notations, $m_{e\infty}$ and m_{eo}, will be made clear later.

In the standard view, the restmass, until the instant of annihilation, is invariant and equal to the maximum radiation available from the process. However, as they approach one another in this picture, the sum of each lepton's effective mass and total energy can then become greater than the energy available in the system.[7] At r = 0, the potential energy of the pair is $|V| = -\infty$ and to balance that, the EM-field and/ or the effective mass energies must be infinite. The only observable EM-field energy is $E_\gamma = 2m_{eo}c^2$. Feynman states that it is all right to have infinite energies, as long as the energy differences remain finite.[8] However, the prospect of having an infinite EM field or effective mass being generated in a finite-energy system should still cause most physicists a certain level of angst.

The electrostatic potential energy between opposite charges, $V(r) = (q_1q_2)/4\pi\varepsilon\ r = -e^2/r$, is most naturally taken to be zero at $r = \infty$, despite being a *relative* measure of the capability of doing work. It is the relative nature of the potential energy, expressed as $V(r) = V_o + (-e^2/r)$, and the singularity at r = 0 that will concern us in a later paper. In the present case, we consider V(r) to be the work done in moving a charge from r (>0) to infinity. If either of the leptons is at infinity, then we let $V_o = (-e^2/r)_{r=\infty} = 0$. Therefore, $V(r) = -e^2/r$ is the economical expression for the Coulomb potential.

The combined system of energies for two charges must include its total potential energy, $V_{total} = V_{ep} = V_1 + V_2$. However, since the Coulomb potential between oppositely charged bodies is attractive, the potential as defined above goes more negative as the charges come together. This must balance the increase in positive energies of the charges. This balance may be expressed in Eq. 1 as the constant total energy, $E_t = E_{ep}$, of the lepton pair being a function of the particles' relativistic energy and total potential energy V_t.

$$E_t = \gamma_e m_{eo}c^2 + \gamma_p m_{po}c^2 + V_t\,. \qquad (1)$$

With the particle rest masses generally considered to be fixed values, any variation in potential energy must be balanced by a change in the gammas (γ_e and γ_p). The EM energy, being bound as perturbations primarily to the leptons' field lines,[9] is included in the leptons' kinetic energy terms[1] and is not explicitly expressed unless/until it leaves as a photon. From Eq. 1, it is assumed that a change in the leptons' relativistic energy (and/or their EM and photonic radiation) must compensate any change in $V_t = V_{ep}$.

If one is looking for it, the theoretical work of a few researchers actually hints at the association of potential with the particle mass,[10] or vice versa.[11] However, by mid-20th century, the 'meaning' and actual source of the electromagnetic and potential energy itself and its relation to mass had not been definitively addressed.[6] Feynman identified the velocity-dependent component of relativistic effective mass as being electromagnetic. Boyer, extended this and, using Eq. 1, actually calculated how the proximity of two charges altered their responses to an accelerating or gravitational field.[12,13] So, while he demonstrated the mathematical effect of electrostatic potential on inertial mass and weight (and therefore on the mass of particles), he also showed that its relative effect was too small to be measureable. In the same vein, we will demonstrate a relative effect of 100%; yet the consequences may be too 'fleeting' and occurring at too small a scale to be measureable and uniquely identifiable.

Potential energy is the ability to do work, $W = \mathbf{F}\bullet\Delta\mathbf{x}$, where $\Delta\mathbf{x}$ is the distance an object moves under an applied force. Since, in free space, both leptons move the same distance under the other's influence, both do the same work and have the same rest mass and gamma. Including the leptons' rest mass energy into the constant total energy, $E - 2m_{eo}c^2 = E''$ (and $E'' = 0$), we now have an implicit statement of the energy sources and sinks,

$$E = 2\gamma_e m_{eo}c^2 + V_{ep} = 2m_{eo}c^2 + 2(\gamma_e - 1)m_{eo}c^2 + V_{ep}.\ \text{ This leads to:}$$

$$E'' = E - 2m_{eo}c^2 = 2(\gamma_e - 1)m_{eo}c^2 + V_{ep} = 0. \tag{2a}$$

The $(\gamma_e - 1)$ term is the non-rest-mass energy[ii] of the system, other than the potential energy. This is the kinetic energy of each lepton, $KE_e = (\gamma_e - 1)m_{eo}c^2$, including any relativistic mass (bound EM radiation). However, since $E'' = 0$, if both velocity and potential energy are equal to zero at very large r, then $\gamma_e = 1$ and

$$E'' = 2(\gamma_e - 1)m_{eo}c^2 + V_{ep} = KE_{e+p} + V_{ep} = 0, \quad \text{so that} \quad KE_{e+p} = 2KE_e = -V_{ep}\,. \tag{2b}$$

This is not the statement that the maximum KE is equal to the maximum V as for a simple harmonic oscillator system. It states that the leptons' combined kinetic energy ($KE_{e+p} = 2KE_e$) is <u>always</u> equal in magnitude to their combined potential energy (assuming KE = 0 at $r = \infty$). Since $KE_{e+p} = -V_{ep}$, energy is conserved and the picture is mathematically self consistent. At r = 0, the Coulomb potential is $-\infty$ and the kinetic energy is infinite ($\gamma_e = \infty$).

But, wait a minute! If the conversion is not instantaneous, restmass is not invariant. If rest mass goes to zero non-instantaneously, then effective mass may not go to infinity. What is zero times infinity? This little problem is solved by requiring that the leptons annihilate before r = 0. If the pair annihilates before r = 0, then the KE, mass, and charge are converted into photons and the picture is complete. Nevertheless, we are left with the problem that restmass may be relativistically invariant, but not necessarily invariant in all electric fields.[14] The energy needed to do work in bringing the electron-positron pair together must come from the lepton mass.

[ii] The kinetic energy clearly depends functionally on the particle rest mass. However, since we subtracted the rest mass from this component of the total system energy and called the remainder a 'non-rest-mass' energy for the free particle with no potential applied, we will continue the name here and following for the generalized KE.

How do we quantify a statement of the sources of potential energy, when all we start with is the assumption that $V_t = V_{ep} = V_e + V_p$,[14] where the individual potentials are one-half of the total,[iii] $V_t = -e^2/d$, for d as the distance between the particles? Using the definition of work for a 2-body problem, the distance moved from d = ∞, where V is chosen to be zero, is $\Delta\mathbf{x} = \mathbf{d} - \mathbf{r}_i$. Here, $\mathbf{r_i}$ (equal to $\mathbf{r_e}$ or $\mathbf{r_p}$) are the distances of the respective particles from their common center of mass and $(\mathbf{r_e} + \mathbf{r_p}) = \mathbf{d}$. Nevertheless, since the particles do no work on themselves (there is no $\Delta\mathbf{r}_{ii}$),[iv] the work done is in bringing the other charge closer and, therefore, the potentials are:

$$\Delta V_e = -|e^2/d^2|\Delta r_p, \quad \Delta V_p = -|e^2/d^2|\Delta r_e, \quad \text{and}$$

$$|\Delta V_t| = |\Delta V_p + \Delta V_e| = -|e^2/d^2|(\Delta r_e + \Delta r_p) = -|e^2/d^2|\,\Delta d. \qquad (3)$$

We use the absolute values of potential because keeping track of the relative values of negative potentials is cumbersome and negative numbers do not graph well on log plots. With $V_\infty = 0$, the potential at r is $V(r) = \Delta V(r, \infty)$.

With this background, it is possible to examine (with a simplified standard model in Section 2) the velocities, accelerations, and energies of the electron and positron as they approach to within fermis of each other. The relativistic correction, γ, is then described in an isolated system as a function of Coulomb potential energy, instead of just the particle's velocity. In Sections 3 and 4, γ is further described as a function of potential-energy-dependant mass[15] that allows it to become infinite near the origin, for the electron-positron case. With help from the electron-positron analysis, Section 4, we will show that lepton mass is the limiting value for the Coulomb potential in that case. The self-annihilation of positronium is demonstrated to be a continuous and finite-time event. In our discussions of the relativistic electron-positron pair, we do no more than mention their magnetic interaction and the retardation of the Coulomb attraction, etc., that become comparable to the non-relativistic terms.[16] We wish to focus on a basic assumption that may have been overlooked in many physics problems today.

This paper can be summarized in the observation that, just as the effective 'mass' of a charge increases at relativistic velocities, its rest mass decreases at nuclear distances from an opposite charge of sufficiently low mass. We start with the assumption that potential energy must have an energy source (or some mechanism must exist with which to store energy). In an isolated system, e.g., positronium alone, the Coulomb potential and lepton mass are shown below to be two views of the same energy. As the leptons approach one another close enough in their common potential well to attain relativistic velocities and significant EM fields, their 'residual (not their rest) mass

[iii] This division of the potential energy is not common. However, Feynman devotes a whole section to this topic and it is critical to the conceptual development of this paper.

[iv] The potential field of a charge is symmetric about the particle; therefore, ignoring any internal interactions (the basis for decades of debate at the highest levels of physics) there is no net force on the particle from its own field. In the same sense, the potential and relativistic corrections used in this paper are not rigorous. The conclusions are independent of such details.

and charge must decrease correspondingly. Work is done and energy must be conserved. The term residual mass (where residual mass is less than rest mass) will become clear as the paper progresses.

2. *Gamma as a function of potential energy*

Assume tight circular orbits for a highly relativistic lepton pair (total rest mass = $2m_{eo}$).[17, 18] Then, the relativistic centrifugal force $F_c = \gamma m_{eo}v^2/r$ and the Coulomb force $F_C = -(e)^2/(2r)^2$ for steady state must be equal in magnitude. Thus, letting $r = r_e$,

$$|F_C| = F_c \quad \text{gives} \quad |(e)^2/4r_e^2| = \gamma_e m_{eo} v_e^2/r_e \quad \text{and then} \quad v_e^2 = |e^2|/\,4\gamma_e m_{eo} r_e. \qquad (4a)$$

Using Eq. 3, remembering that $V_t = 2V_e$ and $4r_e = 2d$:

$$(v_e/c)^2 \;=\; e^2/4\gamma_e r_e\, m_{eo}c^2 \;=\; |V_e|/\gamma_e m_{eo}c^2\,. \qquad (4b)$$

Substituting this expression for $(v_e/c)^2$ into the definition for γ_e, leads to a potential-dependent correction to the relativistic mass, $\gamma_e m_{eo}$ for a bound electron in the center-of-mass system:

$$\gamma_e \;=\; 1/\text{sqrt}(1 - v_e^2/c^2) \;=\; 1/\text{sqrt}(1 - |V_e|/\gamma_e m_{eo}c^2) = \; 1/\text{sqrt}(1 - |V_{ep}|/2\gamma_e m_{eo}c^2). \qquad (5)$$

From Eq. 2a and for $V_e < 0$:

$$2\gamma_e m_{eo}c^2 = E - V_{ep} \;=\; E + |V_{ep}| \quad \text{gives} \quad \gamma_e = (E + |V_{ep}|)\,/\,2m_{eo}c^2. \qquad (6)$$

Thus, the values of γ_e, from Eqs. 5 and 6, and effective mass, $\gamma_e m_{eo}$ in Eq. 6, can be determined to good approximation as a function of potential energy V_e (and therefore of r_e alone) and of the constant total energy of the electron, E_e, positron, E_p, or the e-p pair, E_{e+p}:

$$\gamma_e \;=\; 1/\text{sqrt}(1 - |V_e|/(E_e + |V_e|) \;=\; 1/\text{sqrt}(1 - |V_{ep}|/(E_{e+p} + |V_{ep}|))\,. \qquad (7)$$

Eq. 7 is valid for $V_p < 0$, since we are only considering a bound pair. It does not hold for a repulsive potential, $V_{pp} > 0$. The two expressions in Eq. 7 are simply to indicate that γ_e can be defined either in terms of the individual- or of the paired-lepton energies.

3. The electron-positron pair

It is commonly taught[19] that in real, as distinct from virtual, e-p pair creation the photon energy is immediately converted entirely into mass energy. How this happens is not mentioned and no equations describe the process. Energy balance says that the minimum photon energy for e-p formation is the rest-mass energy of the separated lepton pair ($E_{\gamma min} = h\nu_{min} = (m_{eo}c^2 + m_{po}c^2)$. The actual conversion process between photon and separated lepton pair is generally treated as a 'black box' and otherwise ignored. The effort of the thousands of papers on e-p creation and annihilation is to provide accurate answers for the output of the black box in terms of the input.

The binding energy E_B necessary to separate the pair must be subtracted from the input photon energy. This deficit in the photon energy available to form lepton mass implies that the lepton rest masses, m_{eo} & m_{po}, are not the predicted rest masses at the moment of creation. There is also the problem of how the creation-photon energy equals the lepton mass energy. If the lepton mass changes, do their charges also change from the moment of conversion to the point of separation? Present laws of physics include conservation of charge and the relativistic invariance of charge. These laws are often interpreted to mean that the electron charge does not vary as a function of velocity, or for any other reason;[20] and, by extension, its charge does not change as it approaches an opposite charge and attains relativistic velocities. However, there is an option that is almost always ignored. Since only total charge is conserved, when two equal-mass opposite charges approach one another, their charges can both change equally. Without this latter concept, the physical absurdity of a singularity at r = 0 in the 1/r Coulomb potential has become incorporated into mathematical physics. With the concept that these lepton charges can simultaneously go to zero at r = 0 (or some other small separation), even the view that electrons are point charges will not result in a singularity.[v]

Feynman, in his Lectures on Physics, spent a whole chapter (II-28) on the electromagnetic mass of the electron and its implications. So, instead of thinking about the conversion of photon energy into mass energy during the e-p pair creation, we should be thinking about the 'separation' of the alternating (AC) fields of the photon into the apparently steady-state (DC) fields of separated electrons and positrons. Thus, we have a similarity between the charge separation of an electron from the nucleus of an atom and the EM field separation of the photon into an e-p pair. The bound radiation of an orbiting electron in a neutral atom (an oscillating dipole) corresponds to the AC field of a 'charge-free' (neutral) photon. The DC charge fields of the separated proton and electron correspond to the separated DC charges of the e-p pair. As physicists, we would never think that the dipole moment of a charge pair is fixed at specific values. Nevertheless, we seem ready to accept that individual charges of a pair are fixed at either 0 or 1. This fits with our concepts of resonants or the quantum. There are no measurements that are presently interpreted in a manner that would prove the charge-invariance concept wrong. The proposed charge 1/3 or 2/3 of quarks is the only hint of another view in modern physics and the inability of quarks to be separated supports the 0 or 1 values for summed charges. In this respect, pair production might fit into this quark-like category (but as a transient rather than a resonant condition) and even contribute to its understanding.

In the details of e-p pair production from an energetic photon that are never mentioned, the first step is field separation. The interaction of a photon with the intense **E**-field gradient of a nucleus or electron catalyses this process. In the black-box model, the massive charged-body needed for pair production from a photon is only used to balance the energy and momentum conservation requirements. Field separation creates the potential energy of the charge pair. Only if there is enough energy to stabilize the separated fields can full pair production occur. Both the photon and the lepton are stable resonances, just as are the different electron orbitals in an atom. If there

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[v] Recognition that the proton is not a point charge should have long ago eliminated the mathematical arguments against the anomalous solution of the Dirac equations for the hydrogen atom.

is insufficient photon energy to form the leptons, the separating-field system collapses back into the now-scattering photon. If there is sufficient energy to form the leptons, but not to separate them, then they immediately (not instantly) recombine and reform a photon (or two?). Both of these cases are examples of Compton scattering. Only if there is sufficient energy (and/or external field) available to separate the photon fields, to form the stable leptons, to separate them sufficiently, and to provide any resultant recoil energy of the scattering nucleus or electron will the threshold for this creation process be reached. At higher photon energies, just as in ionization, any excess energy is converted into kinetic energy of the leptons. In our development below of the potential-dependent mass, we will assume no excess energy.

If a remote electron and positron come within ‘range’ of their mutual Coulomb potential, then the process may reverse. In this next section, we seek to understand the details of this relativistic conversion (annihilation) process and thereby to better understand the creation process as well. We will do this in terms of the central-potential process of Section 2.

4. Gamma as a function of particle mass for the electron-positron pair

In the hydrogen atom, the source of potential energy V is seldom important. The average value $\langle V \rangle$ for even a relativistic electron is a very small percentage of the energy available in the system, predominantly in the proton rest mass. Nevertheless, for a very brief time ($\sim 10^{-22}$s), the proton loses mass energy (< 0.1%) as a bound electron gains in relativistic effective mass when it transits the nuclear region. In the case of electron-positron pair interaction, the limited resource of V is crucial. There is no large energy resource in the system from which to derive this potential energy without producing a noticeable impact.

We know that lepton mass and charge disappear in the pair-annihilation transition. Recognizing, from relativity, that effective mass is equal to or greater than the rest mass, we might assume that lepton mass, as a source of energy, is not a constant in this process. If we further assume that the total effective mass energy ME is a function of Coulomb potential energy V and the lepton masses are equal, we define a residual mass so that $\gamma m_p(V)c^2 = \gamma m_e(V)c^2 = ME(V)/2$.

We can include the system potential in the potential-dependent mass terms and we will use the asterisk on these mass-energy terms to indicate that they may also include more than just the rest mass. We therefore define a potential-dependent mass energy,

$$ME(V)^* = ME_\infty + f(V) , \tag{8}$$

in place of a far-field rest mass, ME_∞ (now defined at infinity, where the potential is presumed to be zero). Thus, Eq. 1’ below, for the electron-positron case, may be defined without an explicit Coulomb potential term. The potential is imbedded in the mass and, therefore, in the kinetic energy terms. However, recognizing that the total energy is still a constant value at $E_{e+p} = 2m_{e\infty}c^2$:

$$E_{e+p} = \gamma_p m_p(V)^*c^2 + \gamma_e m_e(V)^*c^2 = 2m_{e\infty}c^2 = ME(V)^* + (\gamma_e - 1)ME(V)^* = ME(V)^* + KE(V)^*. \quad (1')$$

We can proceed to a form of Eq. 2 by subtracting the lepton far-field rest masses ($m_{e\infty}$ and $m_{p\infty}$) from both sides (to get $E_{e+p}'' = 0$), by remembering that the far-field mass is the largest non-relativistic mass, by expanding and equating the lepton terms, and by recombining them.

$$E_{e+p}'' = E_{e+p} - (m_{e\infty} + m_{p\infty})c^2 = [KE(V)_{e+p}^* + ME(V)_{e+p}^*] - (m_{e\infty} + m_{p\infty})c^2$$

$$= [(\gamma_p - 1)m_p(V)^* + (\gamma_e - 1)m_e(V)^*)c^2 + (m_p(V)^* + m_e(V)^*)c^2] - (m_{e\infty} + m_{p\infty})c^2, \text{ so that}$$

$$E_{e+p}'' = 2(\gamma_e - 1)m_e(V)^*c^2 + 2[m_e(V)^* - m_{e\infty}]c^2 = 2(\gamma_e - 1)m_e(V)^*c^2 - \underline{2|\Delta m_e(V,r)|c^2} = 0. \quad (2a')$$

This last line is identical in form to Eq. 2a; but, the rest mass of Eqs.2 has become the potential-dependent mass of the lepton pair, $2m_e(V)^*$, and the potential is replaced by the change in lepton mass energy. Thus, V_{ep} is replaced by $-2|\Delta m_e(V,r)|c^2$, which is the difference in non-relativistic (but potential-dependent) lepton-mass energy between infinity and r. That is to say, Eq. 2a' becomes Eq. 2b'.

$$2(\gamma_e - 1)m_e(V)^*c^2 = 2|\Delta m_e(V,r)|c^2 \text{ implies that } 2KE_e = 2|V_e| \text{ and } KE_{e+p} = |V_{ep}|. \quad (2b')$$

Some of the later equations for positronium are also slightly different. There are a few complications in the relative values of d and r introduced by relativistic velocities at small distances. While these and other details are important, they do not alter the conclusions of this paper and will not be discussed here since they only dilute the many main points being made.

For the present, we will assume that Eq. 3 remains valid. However, with the potential-dependent mass, a new Eq. 4' is derived by balancing Coulomb and centrifugal forces and solving for v^2:

$$|F_C| = F_c \text{ leads to } |(e)^2/(2r)^2| = \gamma_e m_e(V_e)^* v^2/r \text{ and } v^2 = e^2/4r\gamma_e m_e(V_e)^*, \quad (4a')$$

so that, from 3 and 4a':

$$(v_e/c)^2 = e^2/4r\gamma_e c^2 m_e(V_e)^* = (|V_{ep}|)/2\gamma_e c^2 m_e(V_e)^* = (|V_e|)/\gamma_e c^2 m_e(V_e)^*. \quad (4b')$$

Note that the residual mass, $m_e(V_e)^*$, accelerating in the presence of the Coulomb attraction, is potential dependent and, at small d from an opposite charge, $m_e(V_e)^*$ will be less than $m_{e\infty}$. On the other hand, its relativistic velocity will increase the effective mass of the leptons. Does one effect dominate the other? Furthermore, the accelerating force may no longer be that predicted by the Coulomb potential. When the residual masses decrease, so do the lepton charges (the DC component of the leptons' EM fields, which goes to zero as the leptons become massless, chargeless, photons).

Extending the development of Eq. 5 to 5', to include the potential-dependent mass in the value for relativistic correction γ_e, has major implications.

$$\gamma_e = 1/\text{sqrt}(1 - v_e^2/c^2) = 1/\text{sqrt}(1 - |V_e|/\gamma_e c^2\, m_e(V_e)^*) = 1/\text{sqrt}(1 - |V_{ep}|/2\gamma_e c^2\, m_e(V_e)^*). \quad (5')$$

Both Eqs. 5 and 5' have values of γ_e that approach infinity, v => c. The difference is where this occurs. In Eq. 5, the limiting value is at r = 0 and $|V_{ep}| = \infty$. Eq. 5' is not yet sufficiently defined to be able to predict exactly where $\gamma_e = \infty$; but, it may not be at $|V_{ep}| = \infty$.

As in the development of Eqs. 5, 6, and 7, Eqs. 1', 4b', and 5' lead to Eqs. 6' and 7':

$$2\gamma_e m_e(V)^* c^2 = E_{e+p} \text{ and } E_{e+p} = 2m_{e\infty}c^2 \text{ gives } \gamma_e = m_{e\infty} / m_e(V)^*. \quad (6')$$

The differences between equations 4b' and 6' and 4b and 6 are only in the masses, $m_e(V_e)$ vs. m_{eo}, and in where and how the potential is expressed. In Eq. 6, assuming an isolated system where total energy can never change from that of the initial photon, $E_\gamma = h\nu$, the potential-<u>independent</u> mass in Eq. 6, $V_{ep} = 2(1 - \gamma_e)m_{eo}c^2$, implies $\gamma_e = (1 + |V_{ep}|/2m_{eo}c^2)$. This means that the effective mass energy, $\gamma_e\, m_{eo}c^2 = (m_{eo}c^2 + |V_{ep}|/2)$, is always greater than or equal to the rest mass energy. This statement leads to a misconception that elementary-particle mass can never drop below that of its rest mass. Does introduction of the potential-dependent mass energy ME(V)* concept in Eq. 6' change anything?

Remembering that all primed equations in this paper are unique for positronium and the potential-dependant mass interpretation, Eq. 6', indicates that $m_e(V) = 0$ at the point that $\gamma_e = \infty$. This statement has a certain beauty about it. Since mass cannot reach the speed of light (for $\gamma_e = \infty$), it makes sense for mass to be gone at that point. It does not take a singularity (infinite energy) to violate this speed limit. Eq. 7' now puts specific limits on Eq. 5'.

$$\gamma_e = 1/\text{sqrt}(1 - v_e^2/c^2) = 1/\text{sqrt}(1 - |V_{ep}|/2\gamma_e m_e(V_e)^* c^2) = 1/\text{sqrt}(1 - |V_{ep}|/E_{e+p})$$
$$\gamma_e = 1/\text{sqrt}(1 - |V_{ep}|/2m_{e\infty}c^2)\,. \quad (7')$$

In Eq. 6', $\gamma_e = m_{e\infty} / m_e(V)^*$ allows γ_e to have the same range as before, $1 < \gamma_e < \infty$, since $m_e(V)^*$ can vary from $m_{e\infty}$ to 0. However, solving Eqs. 6' and 7' together, for $|V_{ep}|$ (equal to $2c^2 (m_{e\infty} - m_e(V)^{*2}/m_{e\infty})$), limits $|V_{ep}|$ to a range of 0 to $2m_{e\infty}c^2$, not 0 to ∞. This violates the 1/r Coulomb potential; but, it also eliminates the non-physical singularity at r = 0 that limits acceptance of some solutions to the Dirac equation.[21,22] There is no 'real' solution of Eq. 7' for r smaller than the value that provides for $|V_{ep}| = 2m_{e\infty}c^2$. This value ($r = e^2/m_{e\infty}c^2$, assuming no change in lepton charge) is equivalent to the classical radius of an electron. Is this a coincidence?

For positronium, Eq. 6' gives $\gamma_e m_e(V)^* = m_{e\infty}$, which, as will be seen, means that the effective mass of the leptons is constant during the annihilation process. As the leptons accelerate toward one another, they gain no net mass (where would the energy come from?). Does this result of a process, which does away with a singularity at r = 0, actually violate relativity? Or, does it provide important information about the relationship between mass, charge, and potential energies. It does allow a common example of matter approaching the speed of light without the aid of a large accelerator. Since the relativistic lepton mass comes from the loss of rest mass in the Coulomb potential and the EM field mass grows as the lepton velocity and acceleration increases, we can assume that the relativistic mass is electromagnetic. By logical extension, all relativistic mass is probably EM (even for net neutral particles).

5. Discussion

There are two important differences between Eqs. 6 and 7 and Eqs. 6' and 7'. First, just as the relativistic effective mass increases with velocity, and therefore is a function of potential, our potential-dependent mass for an e-p pair has the potential V as a modifier. In this case, the mass decreases as the potential increases. At a certain point, the mass disappears. The difference between positronium and the hydrogen atom (H, discussed in our next paper) is that the relativistic mass gain of a proton-bound electron generally comes from the large mass of a proton. Only when the energy comes from the electron mass itself does its potential-dependent mass decrease.

From Eq. 6', the only means for the velocity to become relativistic is for $m_e(V)^*$ to become smaller than $m_{e\infty}$, the electron mass in a zero-potential region. For the lepton velocity to approach that of light (i.e., $\gamma_e \Rightarrow \infty$), $m_e(V)^*$ must go to zero. This is appropriate for two reasons. First, if the potential-dependent mass decreases, then the attraction between the charged leptons will cause greater acceleration and greater velocity. Second, as the velocity approaches c, the actual mass goes to zero and the transition from mass and charge to EM radiation is continuous, complete, and natural. This particular interaction between velocity and mass is a feedback mechanism. If there were no negative feedback, the transition would be too abrupt to provide the 'clean' (mono-energetic) energies and emergence angles indicated by the annihilation radiation data.

The necessary negative feedback for a smooth and reproducible transition from lepton pairs to photons comes from the reduction in the charge field of the individual leptons as they get close enough together. This reduction in elementary charge lowers the attractive force between the leptons relative to that from a full-charge Coulomb potential. Details of the change in this relativistic invariant, mentioned above, will be the topic for another paper.

Figure 1 represents the three different levels of lepton mass that must be considered in the e-p interaction. With zero relative velocity at infinity, all three are identical to $m_{e\infty}$.

1. The effective mass is the rest mass multiplied by the relativistic correction factor, gamma. It is the mass expected for an atomic electron at the given radius from a proton.
2. The residual, or potential-dependent, mass, m(V), diminishes as the lepton pair comes together.

3. The lepton's net mass is the potential-dependent mass, when corrected for its relativistic velocity.

The constant value for the net mass is both expected and unexpected. It is expected based on energy conservation. Any energy going into the relativistic mass (a bound EM radiation field) must come from the lepton masses (internal EM field). It was not expected that the velocity-dependent relativistic correction to, and the potential-dependence of, the residual mass would exactly cancel (as predicted by Eq. 6'). A demonstration of why this is so requires an examination of the relativistic virial theorem and the relativistic nature of the kinetic and EM energies of the leptons (our next paper).

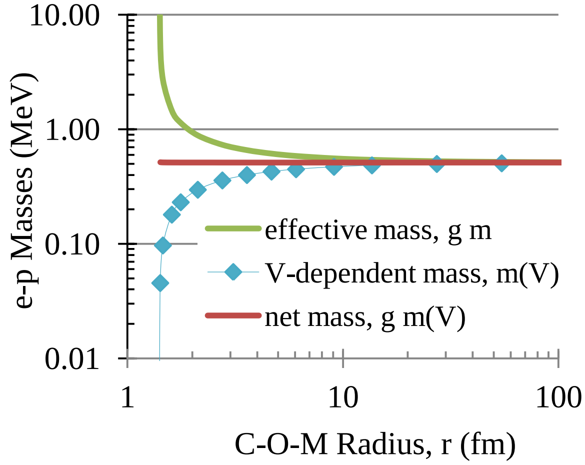

Fig. 1 Electron and positron masses in the region of near approach.

What are the consequences of this electron-positron annihilation model? Nothing has changed as far as measurements are concerned. It introduces what may become controversial concepts. Why should we bother to look at something that happens nearly instantaneously over near-nuclear ranges and has no measurable consequences? Can any of the conjectures be proven? Does it change or improve the experimentally confirmed predictions that have been made assuming instantaneous conversion of kinetic energy, mass, and charge to electromagnetic energy? It may shed some light over presently mysterious processes. Nevertheless, if no one looks at the process, there is no mystery, it is just a 'fact'.

What are the advantages of looking at creation and annihilation in this manner, rather than just in terms of a black-box model and quantum-mechanical operators? First, it may lead to an understanding of some of the mysteries of classical, relativistic, and quantum mechanisms at nuclear dimensions. Second, it may relate directly to the interpretation of, and models for, experiments going on today at these levels. Third, it can be extended to the hydrogen atom, where the effects are much more subtle, and from there to understanding some nuclear reactions. This understanding may lead to reinterpretation of old and new data and to some new physics. Modern quantum mechanical models can assume that nuclear interactions may take place 'off-mass-shell' and make use[23] of changes in nucleon and electron masses. Often, no explanation of how or why this occurs is provided other than statements of the Heisenberg Uncertainty Principle and relativistic effects. Does this new model provide some greater insight?

4. Summary and Conclusions

We have asked, "from whence comes the energy for doing work via Coulomb interactions?" Starting with the simplest case of a single energetic photon becoming an electron-positron pair, the near-inverse interaction is examined and compared. In the examination of this electron-positron pair annihilation, several non-standard conclusions have been reached. A few of them have been explored and modelled in this paper:

a. Conversion of a photon to a pair of leptons is described by analogy with ionization of an atom.
    i. the photon is a structure composed of alternating (AC) EM fields; it is separated into a pair of objects described by DC fields (electron and positron).
    ii. the atom is a structure that includes AC EM fields (an oscillating electric dipole); it is separated into a pair of objects described by DC fields (a nucleus and an electron).
    iii. both conversion processes are continuous; they depend on the 'smooth' transition from one stable state (a resonance) to another.

b. Electron-positron annihilation provides more details to understanding their interaction than does their creation (at least at this level of understanding). Comparison between e-p annihilation and atomic-electron decay is instructive:
    i. the early annihilation stage processes are similar to atomic physics and its models
    ii. radiation from bound charges is well known and characterized
    iii. effective mass increase with relativistic velocity has been quantifiably determined, but not for the self-generated velocity of e-p pairs
    iv. the null-particle state is the deepest energy level for the lepton pair; in e-p annihilation, it can be reached only by double- or multi-photon emission

c. For positronium, all energies and energy sources are balanced between the electron and positron, e.g., masses contribute equally to the Coulomb potential and work done.

d. In some situations, it is necessary to consider three types of mass:
    i. rest mass ($m_{e\infty}$, a constant at zero potential, e.g., at $r = \infty$),
    ii. actual (or residual) mass, $m(V)$, that is potential dependent and, when near opposite charges, is less than or equal to the rest mass, and
    iii. relativistic (effective) mass, $\gamma m_{e\infty}$, $\gamma m_{p\infty}$, or $\gamma m_{\infty}(V)$, that is equal to or greater than the rest masses (unless very close to opposite charges, in which case, $\gamma m_{\infty}(V)$ must be constant if the charges have equal mass)

e. Understanding the e-p transition at its deepest energy level requires an extension to the understanding of atomic physics and beyond.
    i. charge is conserved, but individual charges are not always relativistically invariant
    ii. potential energy may have a specific source in a Coulomb potential
    iii. physically, the Coulomb potential cannot be singular
    iv. electron-positron pair creation and annihilation are not instantaneous
    v. 'matter' can reach the speed of light without an infinite-energy source, etc.

f. Just as relativistic mass increases with particle velocity, there must be a potential-dependent mass that decreases as two oppositely charged leptons approach one another.
    i. The standard expression of the 1/r Coulomb potential energy between two equal-mass charges is extended to show how, in positronium, leptons provide the interaction energy.
    ii. The interaction energy must come from the lepton masses.
    iii. This energy extraction process is modelled as a potential-dependent mass that, during the annihilation process, goes to zero as the mass energy is converted first into kinetic energy, then into EM energy, and finally into radiant energy.
    iv. A fundamental tenet of relativity, the velocity-dependent effective mass increase, appears violated during the e-p annihilation process. However, with a slight change in the definition of effective mass, relativity is validated down to, and including, the point of e-p annihilation.

g. A potential-dependent mass concept is modelled. It provides:
    i. zero mass and zero DC **E**-field at annihilation,
    ii. at annihilation, an AC EM-field energy equivalent to $2m_{e\infty}c^2$,
    iii. a maximum magnitude Coulomb potential energy equivalent to $2m_{e\infty}c^2$,
    iv. no singularity (the charge pair never reaches r = 0 or, if it does, the DC charge and mass field of the leptons at that point = zero),
    v. a ‘smooth’ transition from e-p pair to photons at annihilation.

Other concepts that have come out of this model, but not yet detailed are:

a. time-continuity of charge-field separation of a photon into opposite unit charges;
b. the relativistic effects on the bound EM-radiant energy and kinetic energy of the leptons;
c. reduction of lepton charge as its mass goes to zero (this implies that, as charge grows during the lepton separation process, then their mass and potential energy do also);
d. implications for the hydrogen atom and nuclear reactions; and
e. implications for the concept and non-unit charge of quarks.

Several more papers will be necessary to complete the picture begun here.

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[a] CEO, Science for Humanity Trust, Inc. Tucker, GA, USA, mules333@gmail.com
[b] INSA Honorary Scientist, Department of Physics, Indian Institute of Science, Bangalore, 560012, India, Kritpsinha@gmail.com
[c] Chief Scientist, Science for Humanity Trust, Inc. Tucker, GA, USA, hudginswr@msn.com
[d] Science for Humanity Trust, Bangalore, India, rpenland@gmail.com

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