Energy Density of Spacetime Calculated from General Relativity: Previously we showed that it was possible to deduce the impedance of spacetime $Z_s = c^3/G$ from quantum mechanical considerations, zero point energy and an equation from acoustics. However, now we will show that it is possible to calculate the energy density of the spacetime field using just equations from general relativity and acoustics. Since general relativity and quantum mechanics are often considered to be incompatible, it might seem unlikely that we would turn to general relativity to analyze the quantum mechanical energy density of spacetime. The reason for suspecting that this might be a fruitful approach is that gravitational waves are like sound waves propagating in the medium of spacetime. It is well known that analyzing the acoustic properties of a material can reveal some of its physical properties of the medium including its density. Gravitational waves are like sheer acoustic waves propagating in the medium of the spacetime field. Therefore, we will make analogies to acoustics and attempt to calculate the energy density of the spacetime field. The following equation from acoustics relates the density of the medium ρ to intensity $\mathcal{I}$, particle displacement Δx, acoustic speed of sound c_a, and angular frequency ω.

$\mathcal{I} = k\,\rho\omega^2 c_a \Delta x$.

The spacetime field does not have rest mass like fermions, but gravitational waves do possess momentum. As previously explained, if we could confine gravitational waves in a hypothetical 100% reflecting box, then the gravitational waves would exhibit rest mess. The box is merely turning traveling waves into standing waves. The waves themselves possess characteristics that can be associated with not only energy density but also mass density under specialized conditions. If we can calculate the energy density of the spacetime field using equations from acoustics and gravitational waves, then this will be important not only for establishing the quantum mechanical properties of spacetime, but also for making a connection between general relativity and quantum mechanics.

Earlier in this chapter, an equation was referenced which connects the intensity $\mathcal{I}$ of gravitational waves with the frequency υ and the strain amplitude A of the gravitational waves. This equation assumes the weak field limit where nonlinearities are eliminated and also assumes plane waves. That equation is repeated below. The amplitude A of the gravitational wave is given as the dimensionless strain amplitude (maximum slope) of $A = \Delta L/\lambda\!\!\!^{-}$ where ΔL is the maximum displacement of spacetime and the reduced wavelength is: $\lambda\!\!\!^{-} = \lambda/2\pi = c/\omega$.

$$\mathcal{I} = \left(\frac{\pi c^3}{4G}\right)\upsilon^2 A^2 = kA^2\omega^2\left(\frac{c^3}{G}\right) = \mathrm{k}\left(\frac{\Delta L}{\lambda\!\!\!^{-}}\right)^2 \omega^2 \frac{c^3}{G}$$

We will set the intensity of the above equation equal to the intensity of the acoustic equation $\mathcal{I} = k\,\rho\omega^2 c_a \Delta x$ and solve for density ρ. To achieve this we will set the acoustic displacement Δx equal to the gravitational wave spatial displacement ΔL and set acoustic speed $c_a = c$.

$k\rho c_a \Delta x = k\left(\frac{\Delta L}{\lambda\!\!\!^{-}}\right)^2 \omega^2 \frac{c^3}{G}$ set $\Delta x = \Delta L$, $c_a = c$, $\lambda\!\!\!^{-} = c/\omega$, solve for ρ and U

$\rho_i = k\frac{\omega^2}{G} = k\frac{c^2}{\lambda\!\!\!^{-} G}$

$U_i = k\frac{c^2\omega^2}{G} = k\frac{F_p}{\lambda\!\!\!^{-2}} = k\frac{\omega^2}{\omega_p^2}U_p = k\frac{L_p^2}{\lambda\!\!\!^{-2}}U_p$

Where: ρ_i is the interactive density of spacetime

U_i is the interactive energy density of spacetime

$U_p = c^7/\hbar G^2 \approx 10^{113}$ J/m^3 = Planck energy density

$\omega_p = \sqrt{c^5/\hbar G} \approx 1.85\times10^{43}$ s^{-1} = Planck angular frequency

$L_p = \sqrt{\hbar G/c^3} \approx 1.6\times10^{-35}$ m = Planck length

$F_p = c^4/G \approx 1.2\times10^{44}$ N = Planck force

The terms "interactive density" and "interactive energy density" are necessary because the spacetime field does not have density and energy density in the conventional use of the terms. When we think of the density of an acoustic medium such as water, this has the same density even if the acoustic frequency is equal to zero. The spacetime field only exhibits an "interactive density" when there is a wave in spacetime with a finite frequency. If the frequency is 0, then $\rho_i = 0$ and $U_i = 0$.

I want to briefly point out that the above equations derive the energy density of spacetime that must be there in order for gravitational waves to propagate. The presence of this energy density and the frequency dependence was obtained from a gravitational wave equation and an acoustic equation with no assumptions from quantum mechanics. Proceeding with the spacetime field interpretation of these equations, a gravitational wave is oscillating a part of the sea of dipole waves that forms the spacetime field. These dipole waves are slightly compressed and expanded by the gravitational wave, so they reveal the energy density that is actually interacting with the gravitational wave. The dipole waves in the spacetime field are primarily at Planck frequency $\omega_p \approx 2\times10^{43}$ s^{-1}.

If there was such a thing as a Planck frequency gravitational wave filling a specific volume, then this Planck frequency gravitational wave could efficiently interact with all the energy density in that specific volume of the spacetime. No known particles could generate this frequency, but this represents the theoretical limits of the properties of spacetime. For example, suppose we imagine two hypothetical Planck mass particles forming a rotating binary system. They would both be black holes with radius equal to Planck length L_p. As they rotated around their common center of mass, they would generate gravitational waves. If they were close to merging, then the frequency would be close to Planck frequency. To explore this limiting condition, we will assume a gravitational wave with Planck angular frequency and substitute $\omega = \omega_p = \sqrt{c^5/\hbar G}$ into $U_i = c^2\omega^2/G$. This gives Planck energy density $U_p = c^7/\hbar G^2 \approx 4.63\times10^{113}$ J/m^3.

Before proceeding, we should pause a moment and realize that this simple calculation has just proven that general relativity requires that spacetime must have Planck energy density for spacetime to be able to propagate gravitational waves at Planck frequency. General relativity also specifies how waves less than Planck frequency interact with the energy density of the spacetime field. We normally think of general relativity as being incompatible with quantum mechanics. However, general relativity actually supports and helps to quantify the proposed quantum mechanical model of the spacetime field.

At frequencies lower than Planck frequency, a gravitational wave experiences a mismatch with the spacetime field that primarily has waves at Planck frequency. There is only a partial coupling to the energy density of the spacetime field. The scaling of the lower frequencies is given by the equation $U_i = (\omega^2/\omega_p^2)U_p$. A numerical example will be given which assumes a gravitational wave with an angular frequency of 1 s^{-1} and reduced wavelength of 3×10^8 m. For this wave, the frequency mismatch factor is $(\omega^2/\omega_p^2) \approx 2.9\times10^{-87}$. Therefore, according to $U_i = (\omega^2/\omega_p^2)U_p$ the interactive energy density encountered by this frequency is: $U_i = 1.35\times10^{27}$ J/m^3 or $\rho_i = 1.5\times10^{10}$ kg/m^3. If a gravitational wave with angular frequency of 1 s^{-1} is assumed to have intensity $\mathcal{I} = 1$ w/m^2, then using the previously stated gravitational wave equation, the oscillating spatial displacement produced over a distance equal to the reduced wavelength is: $\Delta L = 4.7\times10^{-10}$ m. I will not go through the entire numerical example, but a $\lambda\!\!\!^{-3}$ volume has an interactive mass of 4×10^{35} kg. Ignoring numerical constants, the energy deposited by the gravitational wave in this volume is $E = \mathcal{I}\lambda\!\!\!^{-2}/\omega = 9\text{x}10^{18}$ J. If you calculate the distance that this energy will move a $4\text{x}10^{35}$ kg mass in time $1/\omega$, it turns out to also be $4.7\text{x}10^{-10}$ m (ignoring numerical constants near 1). Therefore, the displacement of spacetime Δx obtained from general relativity corresponds to the distance ($4.7\text{x}10^{-10}$ m) that the interactive mass (or interactive energy) can be moved in a time of $1/\omega$.

The dipole waves in spacetime contained in the gravitational wave volume cannot be physically moved because they are already propagating at the speed of light. Instead, the gravitational wave is causing a slight change in frequency which produces a shift in energy equivalent to imparting kinetic energy to a mass equal to the interactive mass discussed. Now we can conceptually understand why gravitational waves are so hard to detect. They are interacting with the tremendously large energy density of the spacetime field. Even with a large frequency mismatch, the gravitational waves are still changing the frequency of a very large energy of dipole waves in spacetime.

I have also derived U_i and ρ_i using different approaches. One of these incorporated the impedance of spacetime $Z_s = c^3/G$ and an acoustic impedance equation: $z_o = \rho c_a$. This approach gives the same answers but is not shown here because it is more complex.

Connection to Black Holes: The previous equations were given using the gravitational wave's angular frequency ω and reduced wavelength $\lambda\!\!\!^{-}$. However, the energy density characteristics of the spacetime field should really be expressed using r; the radius of a spherical volume of space. For example, later it will be proposed that gravity and electric fields both are the result of a distortion of the spacetime field. Even though the spacetime field has Planck energy density, this implies a Planck length interaction volume. A larger radius volume interacts in such a way that there is a reduction in the coupling efficiency similar to the effect described for gravitational waves. Therefore the equations for U_i and ρ_i can be rewritten using radius r rather than using ω and λ. Therefore, we have:

$$U_i = k\frac{F_p}{r^2} = k\frac{L_p^2}{r^2}U_p \quad \text{and} \quad \rho_i = k\frac{c^2}{r\,G}$$

These equations should be compared to the equations for a black hole with classical Schwarzschild radius R_s. The energy density is U_{bh} and the density of a black hole is ρ_{bh}.

$$U_{bh} = k\frac{F_p}{R_s^2} = k\frac{L_p^2}{R_s^2}U_p \quad \text{and} \quad \rho_{bh} = k\frac{c^2}{R_s\,G}$$

Therefore, it can be seen that we have the same equations. This is another case of general relativity confirming the energy density characteristics of the spacetime field. The picture that will emerge is that black holes occur when the energy within a spherical volume of radius r from fermions and bosons equals the interactive energy of dipole waves (when $U_{bh} = U_i$)

Another insight into black holes can be gained by imaging two reflecting hemispherical shells confining photons at energy density of about 3 J/m^3. This photon energy density striking a reflecting surface generates pressure of $\mathbb{P} = 2$ newton/m^2. To hold together the two hemispherical shells would take two opposing forces of 2 newton times the cross sectional area of the hemispheres. Next we will imagine increasing the photon energy density to the point that it meets the energy density of a black hole with a radius equal to the radius of the hemispherical shells. Ignoring gravity, the force required to hold the black hole size spherical shells together can be easily calculated. For energy propagating at the speed of light, energy density equals pressure ($U = k\mathbb{P}$). The equation $U_i = F_p/\lambda^2$ becomes $\mathbb{P} = F_p/R_s^2$ or $\mathbb{P}R_s^2 = F_p$. Ignoring constants near 1, Planck force must be supplied by the spacetime field to contain the internal pressure of any size black hole. The smallest possible black hole consisting of photons would be a single photon with Planck energy in a volume Planck length in radius. A confined photon of this energy density would generate Planck pressure $= F_p/L_p^2 \approx 10^{113}$ N/m^2 but since the area is only L_p^2, the total force required to hold the two hemispheres together is Planck force $\approx 10^{44}$ N. A super massive black hole such as found at the center of galaxies has much lower energy density but also requires the same amount of force (Planck force) to hold the shells together.

Normally physicists merely accept that gravity can generate this force and they do not try to rationalize the physics that causes the various "laws" of physics. In the case of gravity, the spacetime field will be shown to apply a repulsive force (pressure) which we interpret as the force of gravity. The maximum force which the spacetime field can generate is Planck force, therefore all black holes, regardless of size, require this force to confine the internal energy.

Why Does the Energy Density of the Spacetime Field Not Collapse into a Black Hole? The energy in the spacetime field does not collapse and become black holes because this form of energy is the essence of spacetime (vacuum) itself. These waves form the background energetic "noise" of the universe. Some quantum mechanical calculations require "renormalization" which assumes that only differences in energy can be measured. Therefore the background energetic fluctuations which only modulate distance by $\pm L_p$ and the rate of time by $\pm T_p$ can usually be ignored. However, when we are working on the scale which characterizes vacuum energy, then these small amplitude waves must be acknowledged and quantified. These small amplitude waves are the building blocks of everything in the universe. They are ultimately responsible for the uncertainty principle and they give spacetime its properties of c, G, $\hbar$, and ε_o.

The standard model has 17 named particles with a total of 61 particle variations (color charge, antimatter, etc.). Each of the fundamental particles is described as an "excitation" of its associated field. Therefore, according to the standard model there are at least 17 overlapping fields, each with its associated energy density. For example, the Higgs field has been estimated to have energy density of 10^{46} J/m^3. Therefore, even the standard model has energetic "fields" which do not collapse into black holes. The spacetime-based model merely replaces the 17^+ separate fields with unknown structure with one "spacetime field" with quantifiable structural properties. Gravitational wave equations have been shown to imply the existence of this vacuum energy density. Zero point energy has long characterized the vacuum as being filled with "harmonic oscillators" with energy of $E = ½\, \hbar\omega$ and energy density of $U = k\hbar\omega/\lambda^3$. The spacetime-based model of the universe characterizes the vacuum energy as dipole waves in spacetime which lack angular momentum. This homogeneous energy density is responsible for the properties of the quantum mechanical vacuum.

Curvature of the spacetime field occurs when energy possessing quantized angular momentum (fermions and bosons) is added to this homogeneous energy density. Black holes with radius r are formed when the energy density of fermions and bosons (quantized angular momentum) equals the interactive energy density of a spherical volume with radius r (when $U_i = U_{bh}$). In this case $r = R_s$ and we need to measure the radius by the circumferential radius method previously explained. Stated another way, the energy density of the spacetime field does not cause black holes, it forms the homogeneous vacuum with no curvature. Introducing fermions and bosons into this homogeneous field distorts this uniform background energy density. When this distorting energy equals the interactive energy density of the spacetime field, then this is the limiting condition. This "contamination" distorts the spacetime field to the extent that it forms a black hole.

Insights into the Speed of Light: The key concept of spacial relativity is that the speed of light is constant in all frames of reference. In the 19th century physicists postulated that there was a fluid-like medium that propagated light waves called the "luminiferous aether". This concept was largely abandoned after the Michelson Morley experiment showed that the speed of light was constant in all frames of reference and Einstein's special relativity generated equations which did not require the aether. However, Einstein himself continued to refer to the aether or "physical space" until his death.[1] The description of the aether was merely changed to correspond to the physical properties of space.

Is there any reason to believe that the spacetime field possesses the property that would allow waves propagating in the spacetime field to propagate at the speed of light in all frames of reference? Gravitational waves propagate in the medium of the spacetime field and they propagate at the speed of light. In any frame of reference, an observer would see a gravitational wave propagating at the speed of light. If it was possible to do a Michelson Morley experiment using gravitational waves, no motion could be detected relative to the medium of the spacetime field. If spacetime is visualized as a medium consisting of interacting dipole waves propagating at the speed of light, then it is reasonable that it would be impossible to detect any motion relative to this medium. It can be shown that waves of any kind that propagate in the medium with impedance $Z_s = c^3/G$ and strain amplitude $A_s = L_p/\lambda\!\!\!-$ will propagate at c, the speed of light.

[1] L. Kostro, *Einstein and the Ether*, (2,000) Apeiron, Montreal, Canada