# Absolute relativity in classical electromagnetism: the quantisation of light

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## ABSTRACT

A rigorous introduction of the underlying nature of space and time, through a sharpening of the principle of relativity, forces qualitatively new kinds of solutions in the classical theory of electromagnetism. A class of wavefunctions are derived which are solutions to the first order, free-space Maxwell equation, governed by a single parameter. That parameter is the exchange frequency, related to the exchange energy through Plank's constant. Though the theory remains that of classical, continuous electromagnetism, allowed travelling-wave solutions are quantised in that they come in "lumps" and are associated with a fixed angular momentum.

**Keywords:** Relativistic photon wavefunction

## 1. INTRODUCTION

Since the early twentieth century theoretical effort has focused largely on the understanding of quantum mechanics and the development of gauge theories following on from the hugely successful theory of quantum electrodynamics. This paper picks up on an older path, that of classical electromagnetism. It develops a relativistic mathematics designed to parallel experiment as closely as possible. This is related to certain Dirac[1] and Clifford algebras,[2] but is more restrictive than either. The point of departure is represented by Maxwell's classic text-book,[3] rather than more recent formulations of electromagnetism with a more complex superstructure.[4]

The Maxwell theory has been re-cast in a minimal mathematics forced to parallel the experimental properties of space and time. A warning is in order: the resulting system, while logically self-consistent is, in many ways, not very "well behaved" mathematically.[5] In particular there are regions where such long-familiar concepts as division (and even subtraction) are not defined. No excuse will be made for this: experiment is not required to follow any mathematics, even if the mathematics may eventually prove an efficient description of certain aspects of experiment.

## 2. OUTLINE OF THE THEORETICAL BASIS

Often, it is argued that a more general mathematics is more powerful than a simpler one. If one wishes to make an attempt to properly parallel reality, as in a solution of Hilbert's sixth problem for example, then one needs to find the simplest mathematics that achieves this, just and no more. Here, an attempt will be made to keep the mathematics as restrictive as possible. This is not merely a philosophical choice here: it is some of these restrictions which will prove to lead the necessity of travelling wave solutions of continuous classical electromagnetism being quantised in the following.

A priori four (and only four) frame-independent unit elements are introduced. These represent unit lines in one dimension of time and three (orthonormal) dimensions of space. Magnitudes or extents are represented by positive-definite quantities with the appropriate units. These are used to express an amount of "stuff" (mass, energy or charge, for example) , or the apparent magnitude of (4- or multi-) vector elements in a particular frame of reference. So far this is five degrees of freedom, of which only one, the positive definite real quantities, represents a magnitude, the others being strictly unit quantities. Additional inner complexity arises rapidly under a proper, relativistic definition of "multiplication" or "division" of the unit vector elements amongst themselves, leading to a further 12 linearly-independent unit elements, corresponding to a unit point and to unit planes, unit

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volumes and a unit hyper-volume, as discussed in the following. The definitions are chosen such that these parallel (special) relativity precisely. The unit point is always a Lorentz invariant and is the"direction" of such things as the invariant mass. The four base vectors always transform as the components of a 4-vector for any quantity with this form. The 6 distinct unit areas transform, for example, as the six components of the electromagnetic field. The four tri-vectors transform as an angular momentum density. The quadri-vector is Lorentz invariant, but not invariant under inversion. In addition to these sixteen linearly independent"directions" a set of signs are required for each taking, potentially, the values + and − only. In particular cases, however, they not be required (or allowed) at all. For example, clearly, one needs to distinguish "forwards" and "backwards" in Cartesian space. It is at least debatable,however, whether a minimal description of reality will require both "forwards" and "backwards" in time. In the definition of the positive direction of the unit plane formed from the product of two perpendicular lines, one should distinguish the left-handed and right-handed choices with different signs. Note in particular that it has no meaning to addition or subtraction of the unit elements themselves, but only the magnitudes which condition them: $1\alpha_0 + 1\alpha_0 = (1 + 1)\alpha_0 = 2\alpha_0$ (seconds, for example). The sign appearing in the addition or subtraction of real numbers is, again, different conceptually from the signs of the unit elements themselves. It is most important, to avoid confusion, to be mindful of the nature of the sign at hand. Also an extension of the simple new basis here into the standard model may require more signs for various quantum numbers to distinguish such aspects as positive and negative charge, spin, lepton number and so on. It should be clear that the potential number of different "algebras" which may be defined in this way is rather large. Which, if any, is necessary as an element of the eventual solution of Hilbert's sixth problem is left to future work. The approach followed here has been to choose a system which works at the level of the Maxwell equations and which, further, corresponds as closely as possible with the conventions adopted in the standard textbooks.[4]

In this formalism, a four vector is written ($\mathbf{v} = a_0\alpha_0 + a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3$). The $a_\mu$ are quantities expressing a magnitude (e.g. 3 Amps $m^{-2}$) or an extent (e.g. 42 metres) and the $\alpha_\mu$ are unit lines which may take at most the two values $\pm\alpha_\mu$. Here and in the sequel, Greek indices run from 0 to 3 and Roman from 1 to 3, with 0 representing the time "direction".

A product of these base unit elements with themselves is defined such that the unit time vector, $\alpha_0$ squares to the positive invariant scalar unity (the unit point) $\alpha_0^2 = \alpha_P$ and the three spatial vectors $\alpha_1$,$\alpha_2$ and $\alpha_3$ square to the negative scalar unity $\alpha_i^2 = -\alpha_P$. This is the point at which special relativity is introduced and is all that is required such that all derived quantities transform correctly under all products and quotients The quantity $\alpha_P$ represents a physical point, not in size but rather as opposed to a line or a plane or a volume, in the new algebra. Note that, for neither product, is the value assigned to the real number 1 (as in a Clifford algebra, for example). The quantity $\alpha_P$ is distinguished, here, from the real or natural number unity, 1, in that it is invariant under a Lorentz transformation and may take only the two values $\pm\alpha_P$. The positive value is idempotent such that $+\alpha_P^2 = +\alpha_P$. Here, the negative value is also taken to square to the positive unit scalar, $-\alpha_P^2 = +\alpha_P$. It is also perfectly possible, and more general, to work with the definition $-\alpha_P^2 = -\alpha_P$, but the definition adopted is that appropriate for real invariant masses which are always positive definite. It is worth noting that, properly, the multiplication (or division) of unit vectors, of magnitudes and of numbers are, in principle, three different kinds of operations. The first results in an object of a different form, the second in quantity with a different dimension and the third in merely a different magnitude. A consequence of this definition is that the square of a four vector is $(a_0^2 - a_1^2 - a_2^2 - a_3^2)\alpha_P$, a manifestly Lorentz invariant quantity, as it is experimentally. For the $\alpha_\mu$ taking the dimensions of a 4-momentum, for example, this is the positive-definite invariant mass.

The ordered product or quotient of one spatial unit element with another, for example $\alpha_1\alpha_2$ leads to a unit right-handed ordered spatial plane (bivector) element. This spatial plane is denoted $\alpha_1\alpha_2 = \alpha_{12}$. The reverse ordering gives a plane in the opposite (left-handed) direction, that is $\alpha_{12} = -\alpha_{21}$. There are three such right-handed objects: $\alpha_{12}, \alpha_{23}, \alpha_{31}$. Because this is a four-dimensional basis there are three further space-time planes, represented by products such as $\alpha_1\alpha_0 = \alpha_{10}$. Because of the properties of the base elements introduced above and the nature of the product, these elements transform relativistically as the magnetic ($\alpha_{ij}$) and electric ($\alpha_{i0}$) field elements which take this form in the following. This is a general and defining feature of the algebra being developed: anything with a particular unit element form inherits the relativistic transformation properties of that form. There are 4 tri-vectors representing unit volume elements ($\alpha_{123}$,$\alpha_{012}$, $\alpha_{023}$, $\alpha_{031}$). The latter three are a momentum density multiplied by a perpendicular unit vector, and therefore transform as the components of an angular momentum density. Finally, there is a quadri-vector ($\alpha_{0123}$) which, just as the scalar, is invariant under a Lorentz transformation but may change sign under other operations such as Hermitian conjugation.[6]

Several considerations should be noted. The system is non-commutative, hence the implicit ordering of quantities is important. In the sequel a system has been chosen which works, at least up to the derivation of the Maxwell equations. In principle, the elements derived from ordered multiplication or ordered division may be different. In particular, quantities of this form scale differently under a Lorentz transformation, as discussed below. The ordering of division (whether one divides by or divides into a quantity) introduces a sign change. Further, there are several choices to be made about the handedness and ordering of the operations between the various unit elements. In particular, the time element may be taken to come first or last (implying a change of sign and of handedness of the base elements in which it appears). Importantly, both choices give a same-handed set of products amongst each other ($\alpha_1\alpha_0 \times \alpha_2\alpha_0 = \alpha_0\alpha_1 \times \alpha_0\alpha_1 = \alpha_1\alpha_2$). This would imply that there was an intrinsic sign of and an intrinsic handedness between certain elements. The conventions adopted here work with the standard left-to-right ordering of products, the standard (right-) handedness of co-ordinate systems and the standard signs chosen for the directions of the electric and magnetic fields. This can equally be made to work with a left-handed basis. A comment is in order here: nature is intrinsically handed. The feeling of the author is that the left-handed choice is very likely to be more correct, though the right-handed choice has the advantage that it agrees with convention and with all the results derived in the literature and in textbooks. Taking the convention that the base elements $\alpha_1, \alpha_2, \alpha_3$ are right-handed, this ordering, with space first then time, forms a right-handed set for angular-momentum products (such as $r \times p$), the reverse ordering a left-handed one. The conventional signs in the Maxwell equations then arise if one adopts the convention that the multiplication of a unit vector in the 1 direction into an inverse unit vector in the 2 direction has the reverse sign to the simple product. That is $\alpha_1/\alpha_2 = -\alpha_{12}$. It should be immediately apparent that, with these degrees of freedom, there is more than one way of choosing a consistent system at the level of the Maxwell equations. Further, conventionally, the scaling and sign properties are taken up by a real number factor (introducing positive and negative reals then) and, rather than introducing many more base elements than the 16 and obfuscating the simple development to follow, that approach will be followed here. Provided one is not working with addition or subtraction, but just with multiplication and division this is not an issue. Where it becomes an issue (in the addition of energies and fields), rather than being a problem it becomes a solution, as will be seen.

In the following, the proper form of quantities will be represented by a unit token with ordered lettering, thus $\alpha_{\mu\nu}$ represents a general bivector and, $\alpha_{0ij}$,$\alpha_{ij}$ and $\alpha_{i0}$ are right-handed tri-vectors, space-space bi-vectors and space-time bi-vectors respectively. Given this, the "direction" of a hypercomplex element is assigned to the way it transforms under a planar rotation.[6] For example, the unit volume $\alpha_{012}$ rotates in the same way as $\alpha_3$ hence, in cartesian co-ordinates it represents the "z" component of the angular momentum density. Here and in the sequel, Greek indices run from 0 to 3 and Roman from 1 to 3.

The multiplication and division of unit vector elements has been defined. The division of 4-vectors within the algebra is now discussed. Note firstly that the algebra developed is not a division algebra. There are many regions, apart from zero, where division is not defined. Primary amongst these is the vector itself. Consider the 4-vector case:

$$\mathbf{v} = \alpha_0 v_0 + \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 = \alpha_0 v_0 + \alpha_i \vec{v} \tag{1}$$

$$\mathbf{v}^{-1} = \mathbf{v}/\mathbf{v}^2 = v/(v_0^2 - v_1^2 - v_2^2 - v_3^2) = \frac{\mathbf{v}}{(v_0^2 - v_1^2 - v_2^2 - v_3^2)} \tag{2}$$

The inverse is in the same direction as the original vector, but with a different (real) scale factor, corresponding to the usual relativistic scaling. The over-arrow is used to denote the components corresponding to a conventional 3-vector. Here these are just three real numbers, with the proper (4-dimensional) unit elements being given by the $\alpha_i$ factors. Note, for the case of the space-time coordinates, the divisor corresponds to the invariant interval squared and that all inverses are scaled relativistically, by construction, according to this quantity. The underlying unit elements, when squared, give quantities of opposite sign, as they must relativistically. At the same time, if the real number factors for the magnitude of the spatial and temporal parts are equal, as they are everywhere on the lightcone for example, the interval goes to zero. Hence there is no inverse, not only at zero, but also in the crucial case of anywhere on the lightcone such that $(v_0^2 - v_1^2 - v_2^2 - v_3^2) = 0$. That is, the plane where division is undefined corresponds precisely to the physical limitations imposed by the speed of light. There are other combinations as well (such as that corresponding to the photon energy and momentum, for example) where division is undefined as well. Further discussion of where division is and is not defined is of great interest in itself, but not relevant to the simple cases discussed here. It is reserved for future work. Note that, for the case of the definition of the 4-vector derivative below, division is always well-defined. In that case the scaling is precisely unity in the frame of the derivative.

There is a feeling in some quarters that algebras which are not division algebras are somehow not well behaved.[5] This is true in the narrow sense of the properties of a "nice" mathematics. Here, it is precisely the proper scaling properties of 4-vectors in special relativity, perfectly paralleled in the present algebra, that leads to the new results. Far from being ill-behaved, this kind of behaviour is essential to precisely parallel the proper relativistic transformations of space and time, as observed in experiment, and to force the quantisation of allowed solutions.

Given that the algebra is designed to parallel relativity, it should come as no surprise that it is closely related to certain Dirac algebras.[1] It is most closely related to one of the Dirac gamma matrix algebras, but differs in that it is more restrictive as outlined above. In particular, neither the unit imaginary nor the popular factor $\alpha_5$ is defined for the treatment here. It is also related to one of the Clifford algebras,[2] sometimes referred to as the space-time algebra, or the Dirac-Clifford algebra.[5] Those familiar with such algebras will recognise some of the properties summarised above. It differs in detail from both Dirac and Clifford algebras in the definitions of addition, subtraction multiplication and division and the treatment, transformation properties and restrictions of the scalar unit element $\alpha_P$, discussed above.

For Cartesian co-ordinates a 4-vector 4-differential is defined within this framework as:

$$\begin{aligned} d &= \frac{\partial}{\alpha_\mu \partial x_\mu} = \partial_\mu / \alpha_\mu \\ &= \alpha_0 \partial_0 - \alpha_1 \partial_1 - \alpha_2 \partial_2 - \alpha_3 \partial_3 = \alpha_0 \partial_0 - \alpha_i \vec{\nabla} \end{aligned} \tag{3}$$

Note the change of sign of the space components due to the implicit quotient of the unit vectors. Note also that, though the differential is a special case of a division, the scale properties discussed above are simply unity, since the differential is taken with respect to each base unit vector element separately for each observer. If one were to define an differential with respect to an interval, this would be different. For any interval lying on the light-cone, for example, as the interval approached zero the result would tend towards infinity. The resultant does not tend to a finite quantity, but may be made "as large as you like" as one approached precise lightspeed . It may be speculated that this kind of behaviour may be part of the reason that zero-interval events dominate exchanges where the cross-section is vanishingly small, such as photon exchanges over intergalactic distances.

The 4-differential of a 4-vector potential yields field components. Writing a vector potential as:

$$A = \alpha_\mu A_\mu = \alpha_0 A_0 + \alpha_1 A_1 + \alpha_2 A_2 + \alpha_3 A_3 = \alpha_0 A_0 + \alpha_i \vec{A} \tag{4}$$

The 16 ($= 1 + 3 + 3 \cdot 2 + 3 \cdot 2$) terms of the 4-derivative of the 4-potential $dA$ may be gathered together and written as:

$$dA = \alpha_P(\partial_0 A_0 \quad + \quad \vec{\nabla} \cdot \vec{A}) - \alpha_{i0}(\partial_0 \vec{A} + \vec{\nabla} A_0) - \alpha_{ij} \vec{\nabla} \times \vec{A} = P\alpha_P + F\alpha_{\mu\nu} \tag{5}$$

which is the sum of a scalar (pivot) part $P\alpha_P$ and a bivector (field) part $F\alpha_{\mu\nu}$.

In Eq. (5) the term in $\alpha_{i0}$ is usually identified with the electric field $\vec{E} = -\partial_0 \vec{A} - \vec{\nabla} A_0$ and that in $\alpha_{ij}$ with the magnetic field $\vec{B} = \vec{\nabla} \times \vec{A}$. Some care is required with the signs in relating these quantities to the conventional electric and magnetic fields. Taking the convention in Jackson[4] $F\alpha_{\mu\nu} = E_i \alpha_{i0} - B_i \alpha_{jk}$). The standard electric field then maps to the set of three ordered right-handed space-time unit elements $\alpha_{10}, \alpha_{20}, \alpha_{30}$ and the magnetic field to the terms $\alpha_{23}, \alpha_{31}, \alpha_{12}$ respectively.

Over each of the sixteen multivector-quantities defined above, a general dynamical multi-vector field $G$ is defined over a scalar term $P$, a vector potential $A$ a field term $F\alpha_{\mu\nu} = E_i \alpha_{i0} - B_i \alpha_{jk}$, a trivector potential term

$T$ and an eventual quadri-vector potential $Q$ such that: $G = P\alpha_P + A_0\alpha_0 + A_i\alpha_i + E_i\alpha_{i0} - B_i\alpha_{jk} + T_k\alpha_{0ij} + T_0\alpha_{123} + Q\alpha_{0123}$. In an obvious notation, the constant terms are defined as $C = C_P\alpha_P + C_0\alpha_0 + C_i\alpha_i + C_{i0}\alpha_{i0} - C_{jk}\alpha_{jk} + C_{0ij}\alpha_{0ij} + C_{123}\alpha_{123} + C_Q\alpha_{0123}$.

Writing, by analogy with the form of the Maxwell equation $dF = J$, $dG = C$, and again using the conventional 3-space patterns for reference, one obtains from the odd terms a generalisation of the Maxwell equations as:

$$\alpha_0(\vec{\nabla}\cdot\vec{E} + \partial_0 P) = C_0\alpha_0 \tag{6}$$

$$\alpha_{123}(\vec{\nabla}\cdot\vec{B} + \partial_0 Q) = C_{123}\alpha_{123} \tag{7}$$

$$\alpha_i\left(\vec{\nabla}\times\vec{B} - \partial_0\vec{E} - \vec{\nabla}P\right) = C_i\alpha_i \tag{8}$$

$$\alpha_{0ij}(\vec{\nabla}\times\vec{E} + \partial_0\vec{B} + \vec{\nabla}Q) = C_{0ij}\alpha_{0ij} \tag{9}$$

and four further equations in the even terms:

$$\alpha_P(\vec{\nabla}\cdot\vec{A} + \partial_0 A_0) = C_P\alpha_P \tag{10}$$

$$\alpha_{0123}(\vec{\nabla}\cdot\vec{T} + \partial_0 T_0) = C_Q\alpha_{0123} \tag{11}$$

$$\alpha_{i0}\left(\partial_0\vec{A} + \vec{\nabla}A_0 + \vec{\nabla}\times\vec{T}\right) = C_{i0}\alpha_{i0} \tag{12}$$

$$\alpha_{jk}\left(\partial_0\vec{T} + \vec{\nabla}T_0 - \vec{\nabla}\times\vec{A}\right) = C_{jk}\alpha_{jk} \tag{13}$$

In the first set of four equations corresponding to the Maxwell equations, the main difference is the introduction of two new dynamical terms $P$ and $Q$ transforming like an invariant mass. The constant terms on the right correspond to the electric and magnetic charge and current. Setting $P$ and $Q$ zero, but setting $C_0$ to the charge and $C_i$ to the current density these reduce to the standard, inhomogenous Maxwell equations. Note that, in this limit, all four Maxwell equations are present at once in the present formalism, with all the correct signs, in contrast to the usual derivation not using the principle of absolute relativity.[4] Further, within the new formalism, setting $P$ non-zero leads to the possibilities of new kinds of self-confined solutions with rest-mass[9,12] and underpinning the underlying quantised nature of charge.[10] The possibility that this may lead to a new, linear theory of light and matter, treating leptons and photons on the same footing, will be explored further in a companion paper.[13] Here, all teh constant terms on the left will be set to zero. This corresponds to the free-space (Lorenz gauge) condition appropriate for photons.

In the second set of four the main additions are the presence of the tri-vector terms $T$. These represent the possibility of introducing a 4-tri-vector potential as well as the conventional 4-vector potential into an extended theory of electromagnetism. This may be expected to be of value in understanding the underlying nature of angular momentum in particles. Further these equations express (potential) gauge conditions. With $C_P = 0$, equation (10) is just the Lorenz gauge condition and one obtains other conventional gauges by setting this constant to other quantities. In other words, $C_P$ non-zero expresses a gauge degree of freedom. Here, there are other constants which, if expressed, would introduce new physics. In particular, the dual term $C_Q$ expresses a further gauge degree of freedom. In principle, the odd set and the even set should both constrain the physics, but the even set will not be used in deriving the main results of this paper. Indeed, the main results here will be obtained by demanding that solutions satisfy the standard set of free-space Maxwell equations alone ($dF = 0$). The full set will be used in developing the linear, first order wave-functions used to construct the solutions. Note that, with $T = 0$ the final two equations are just the standard expression for the electric and magnetic field in terms of the vector potential.

## 3. A NEW PHOTON WAVE-FUNCTION

Using the new algebra new kinds of wave function may be generated with properties more strongly constrained than is possible conventionally. There are a plethora of such solutions. Complexity has arisen rapidly from the simplicity of the four-dimensional basis due to the properties ascribed to multiplication and division. As a prequel, it is worth exploring these new solutions in general.

Conventionally, one often writes wave functions introducing a complex scalar $i$ and exploiting the property that:

$$e^{i\theta} = e^{\theta i} = (e^{i})^{\theta} = (e^{\theta})^{i} = cos(\theta) + isin(\theta) \tag{14}$$

The ordering and nesting of the exponents in the equation above is unimportant for complex numbers, as all factors commute, but will prove crucial in the more complex discussion to follow. As is well-known, a non-relativistic wave function propagating in the z direction may be written:

$$F_{NR} = Ae^{i(kz-\omega t)} = Ae^{i\theta} \tag{15}$$

Where $k$ is the spatial frequency (wavenumber in z) and $\omega$ is the temporal angular frequency. Such forms have wide practical application. They may be further modified by well-known generalisations of the harmonic functions, Bessel functions, spherical harmonics, half-integral Legendre polynomials and so on to describe exponential-like solutions in cylindrical, spherical and toroidal systems. Despite their power and elegance, they have one major flaw if one wishes to use them in a relativistic theory: space and time appear in the combination $(kz-\omega t)$ as a (Lorentz) scalar factor. Since space and time, however, transform differently under a general Lorentz transformation, such a wave function does not reflect the differences between space and time properly. An alternative route inserting the proper relativistic transformations at all levels, even in the exponent, leads to qualitatively different kinds of solutions. It is the rigour of forcing all instances of space or time to have their proper form, wherever they occur, that is the sharpening of the principle relativity referred to here as "absolute relativity".

Describing the proper relative difference of space and time within all elements of a wave-function will prove to lead to a raft of side-benefits. It will prove necessary that such solutions to the Maxwell equation correspond more closely to the physical photon, in that light must come in "lumps" that scale in total energy with the frequency. This section will develop the general case, the next- the photon.

In the present formalism, there is no single simple complex scalar $i$, but several quantities which may play the same role in describing travelling wave solutions. Three of the base unit elements and seven of the unit elements derived from these square to negative unity $(\alpha_1, \alpha_2, \alpha_3, \alpha_{12}, \alpha_{23}, \alpha_{31}, \alpha_{012}, \alpha_{023}, \alpha_{031}, \alpha_{0123})$. Any of these may be used to describe travelling waves. For example, by analogy with complex numbers one may expand an exponential with $\alpha_{12}$, corresponding to a rotation of angle $\theta$ in the 12 plane as:

$$e^{\alpha_{12}\theta} = \alpha_P cos(\theta) + \alpha_{12} sin(\theta) \tag{16}$$

In the physical association made above, this would describe an oscillation back and forth between a rest mass component $(\alpha_P)$ and a magnetic field component$(\alpha_{12})$. Such a formalism is descriptive in a similar way to complex numbers. Using the scalar $\alpha_P$ and $\alpha_{0123}$ alone provides an even more precise parallel, since the sub-algebra containing this pair alone is isomorphic to complex numbers. Such exponents will be denoted in general as hypercomplex exponents. Though this may sound like some progress, eq. (16) retains the problem of general covariance alluded to above and such solutions are not necessarily proposed as representing a physical process (governed by a 4-vector derivative) as the proper elements corresponding to neither space nor time are present. To describe physics, they would require a bi-vector or quadri-vector derivative in an angular measure to operate. This would retain the essential feature of the deficiency sketched above, that space and time are treated identically. They serve merely to point the way to further progress.

Note that $\alpha_P$ has been used in the expansion above. This is because energy conservation considerations require that any physical wave should transform between elements of substance constituting, at the very least, an equal integrated energy. It would be preferable, of course, to require a local microscopic conservation of energy (and momentum), if this could be defined. In either event, this means that both terms should square to an energy density rather than to a simple number. This is one reason why at least two distinct kinds of scalar unity must be considered. The unity $\alpha_P$ appearing in hypercomplex exponentials describing unitary transformations in nature is necessarily different to the unit real number 1. Moreover, it will appear that consideration of the non-commutativity of elements may lead to a kind of black-body quantisation, as will be discussed in the following.

To make proper progress, a second extension is required such that elements may be nested with each other leading to a combined motion observed as a wave. This leads to a far richer structure than is available in a merely complex algebra. The new axiom requires the inclusion of the proper (in the Lorentz sense) relative transformation properties of space and time directly into the exponential. This may be achieved by associating the proper unit element directly with the appropriate propagation direction in the hypercomplex exponent, as is now shown. For example, for propagation in the 3 (z ) direction, solutions are sought for some appropriate unit element, denoted $\alpha_?$, of the form:

$$F_{SR} = Ae^{(\alpha_3 kz - \alpha_0 \omega t)\alpha_?} \tag{17}$$

The different unit elements $\alpha_3$ and $\alpha_0$ ensure the proper relative transformation of space and time. By analogy with the expansion of real exponentials to complex exponentials, the element $\alpha_?$ converts this to a travelling wave-function. The pre-factor $A$ may be viewed as that quantity transported by the exponential factor. Here it will not be a simple magnitude. To satisfy the Maxwell equations it will prove to be, necessarily, a field with a strongly constrained form. That form will prove to be precisely that observed for the physical photon. Elements of the wave may act in concert with other elements to produce a self-sustaining oscillation, provided they are mutually in harmony[7,8] and represent an overall force-free and energy conserving whole. Note that though equation (17) is a conceptual extension, it also embodies a physical restriction in that the factors corresponding to space and time are forced to have their proper relative form.

For a wave-like overall solution $\alpha_?$ is required to be some unit element which ensures that both the spatial and the temporal element of the development is governed by a unit element squaring to negative unity. Within the principle of absolute relativity it is axiomatic that space and time, and any other quantities such as angular momenta, should appear everywhere with their proper form.

By inspection, it is apparent that substituting neither $\alpha_P$, nor the real number 1 for $\alpha_?$ leads to a travelling wave solution. In both cases the temporal development will square to positive unity, leading to falling exponential-like solutions rather than waves. The question is: by analogy with the extension from ordinary to complex exponentiation outlined in equation. (14), is it possible to pre- or -post multiply the exponent by some other unit element, corresponding to exponentiation by or exponentiation into the whole expression? For a given propagation direction there are six unit elements $\alpha_?$ which achieve this, each leading to new kinds of wave-particle solutions. For the particular case of the 3 direction in eq. (17), these are explicitly: $(\alpha_{012}, \alpha_{23}, \alpha_{31}, \alpha_{123}, \alpha_{10}, \alpha_{20})$. The first three themselves square to negative unity; the second three to positive unity. This means the first three afford the possibility of inserting a scalar phase factor, in harmony with the multi-vector component, into the hypercomplex exponent. Of these, $\alpha_{012}$ (corresponding to the introduction of a unit vector in the direction of the angular momentum) leads to the possibility of substituting for $A$ in equation (17) above, a pure field solution. The crucial difference to more conventional solutions, is that this is then a first order solution of the Maxwell equations - describing all six components of the electromagnetic field in any proper frame.

The exponential factor alone in equation (17), though it represents a wave, is not itself of solution of the free-space Maxwell equations $dF = 0$ as it contains terms transforming as a rest-mass as well as field terms. It is, however, a solution of the more general set of dynamical equations with the constant terms zero, $dG = 0$ introduced above, however, as is readily verified by substitution. It is tempting to associate this part of the wave-function directly with a massive source particle such as an electron, but this form is still too simple to fully encompass the complexity of such particles.[10,12] Further developments along these lines will be dealt with in a companion paper.

It is worth noting in passing that, of the other five, the elements $\alpha_{23}$ and $\alpha_{31}$ also lead to solutions of the general equation dG=0, and these may correspond more closely to elements of electron-like and positron-like solutions. The remaining three possibilities may also be associated with light-like and particle-like solutions and may indeed be the primary- initial or lightest- solutions. Curiously, the dual bivector pair ($\alpha_{10}$ and $\alpha_{20}$) do not lead by themselves to a magnetic monopole-like but also to an electric monopole-like fields, as can readily be verified by substitution and expansion. Magnetic monopoles may be described, however, by introducing more complicated terms involving a product with the pre-factor $A$. The $\alpha_{123}$ case may be associated with a precursor to the electron-positron pair in the creation process as it resembles a twisted-mode solution, the solution obtained by overlapping counter-propagating right-right or left-left circularly polarised light. This is the configuration for the creation of a particle-antiparticle state at spin zero at sufficiently high energy. It is also possible (by choosing an appropriate pre-factor and/or the relative propagation direction of space and time) to associate these with the primary photon-like and electron-like solutions. This opens up the possibility that the other set, which may introduce mass through the scalar term, may be involved in the description of the weak interaction. Again, the development of these speculations will be left to future work.

In popular expositions of relativity one often talks of rulers and clocks, these often being held by idealised "observers". In reality, there are no observers, only the emitting and absorbing particles or systems and the intermediating photons themselves. Confusion is often introduced in arguments by ascribing "knowledge' to n external observer that it could not possible have.

Consider an idealised system in three frames: an emitter, an absorber and an intermediating photon. Each frame has its own scale of space, time and frequency. Its own "rulers" and "clocks" and its own scale of energy or, equivalently, frequency. In a general "event" where a photon is exchanged between two particles, the particles may be in very different Lorentz frames. These frames, and hence their scales, will also change due to the effect of the exchange. Let the scale-change be denoted by $R$. For example, a photon in one frame may have a particular energy, frequency and wavelength. In another (blue shifted) frame where the energy (and hence the frequency) increases by $R$, the wavelength decreases by $1/R$. It is enlightening to write this scale-factor $R$ in terms of the usual relativistic $\beta$ and $\gamma$ factors:

$$R = \sqrt{\frac{1+\beta}{1-\beta}} = \gamma(1+\beta), \quad 1/R = \sqrt{\frac{1-\beta}{1+\beta}} = \gamma(1-\beta), \quad \omega' = \omega R, \qquad \lambda' = \lambda/R \tag{18}$$

With:

$$\beta = v/c = \frac{R^2-1}{R^2+1}, \qquad \gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{2}(R+\frac{1}{R}) \tag{19}$$

Note that the last relation above means that the gamma factor is the average of the increase in energy of the light travelling against the motion, with that travelling with the motion. One may conclude that the Lorentz scaling of the mass of material particles is just that of the energy of light in a box. Further, provided the magnitudes of the electric and magnetic field components are equal (as they are for propagating free-space electromagnetic waves in general and for photons in particular), so that $|E| = |B|$ , they transform relativistically as:

$$E' = \gamma(E+\beta B) = RE(=RB), B' = \gamma(B+\beta E) = RB(=RE) \tag{20}$$

That is, for light, the fields transform in the same way as does the frequency and energy: linearly with $R$.

The formalism to write down a new, fully relativistic solution to the first-order Maxwell equation in free space ($dF = 0$) is now complete. For the simple case of a propagating free-space electromagnetic wave, forcing $z$ and $t$ to take the proper form $\alpha_3$ and $\alpha_0$ respectively, a solution of a left circularly polarised electromagnetic wave, travelling in the the $+z$-direction and transmitting a quantum of energy $\mathcal{E}$ in the centre of mass frame may be written:

$$F_L = H_0 U_F R\mathcal{E}(\alpha_{10}+\alpha_{31})e^{\frac{\mathcal{E}}{\hbar}R(\alpha_3\frac{z}{c}-\alpha_0 t)\alpha_{012}} = F_0 R(\alpha_{10}+\alpha_{31})e^{R(k\alpha_3 z-\omega\alpha_0 t)\alpha_{012}} \tag{21}$$

Physics is inserted nine times in the proper application of the principle of absolute relativity in this equation. Space is inserted with its proper form in the direction of propagation direction as $\alpha_3$. Time appears associated with $\alpha_0$. The proper "direction" of the angular momentum around the propagation direction is $\alpha_{012}$. The electromagnetic field is inserted not only with its proper form ($\alpha_{10}$ and $\alpha_{31}$), but also with two further implicit constraints in order for equation. (21) to be a solution: the starting fields should be perpendicular ($E_x$ and $B_y$ here) and of equal magnitude. The (scalar) energy appears twice: once in the pre-factor (expressing the linearity of field with $R$) and once in the exponent (expressing the linearity of energy).The real-number constants $c$ are the (scalar) speed of light and $\mathcal{E}(= \hbar ck = \hbar\omega)$ the (scalar) quantum of energy transmitted in the centre-of-momentum frame respectively. $U_F$ is a universal constant converting to field units. This is a universal factor (containing $\hbar$) which takes the same value for all photons. $H_0$ is a distribution function representing the spread of field or energy over phase, whose square integrates to unity, as the number of cycles in phase in the same in all frames. This distribution is the same for all frames, right up to the limit of light-speed where the energy goes to zero. The single parameter R is that factor which determines the scales of energy, frequency, length and time in the three frames discussed above. Taking this to be unity in the centre of momentum frame (the photon frame - determining the proper magnitude of the energy-momentum transmitted), the factor $\mathcal{E}R$ is then the energy in each relevant frame. The factor of $R$ in the exponent pertains to the relativistic transformation of "rulers" and "clocks". The proper reference "ruler" and "clock" for any given photon exchange event scales with the centre of momentum energy. This is just the wavelength and frequency of the photon in the proper (centre of momentum) frame, for which $R = 1$. The factor or $R$ in the pre-factor corresponds to the relativistic transformation of fields and frequencies in the emitter and absorber frames. In the photon frame the proper frequency is the energy divided by Plank's constant $\omega_0 = \frac{\mathcal{E}}{\hbar}$. Note that the energy density in the field is proportional to field squared. Explicitly, $D(\mathcal{E}) = \frac{1}{2}\epsilon_0(E^2 + c^2B^2)$. This means the wave-function in equation (21) may be converted to square-root energy density units by the simple expedient of defining another universal factor $U_{\mathcal{E}}$ in place of $U_F$. This raises the question of why the factor for energy appearing in the prefactor should be $\mathcal{E}$ and not $\sqrt{\mathcal{E}}$. The reason is that, experimentally, both energy and field must add linearly. If one accepts this experimental fact, then it is space and time themselves which much deform, relativistically, to accommodate this. That is exactly what the Lorentz transformation achieves, as sketched above. If the scaling factor is 2, for example, the relativistic transformation is such that the field doubles, the energy density quadruples, but the length of the wave train in the new frame halves, giving a linear increase in the total energy with frequency overall, as is observed experimentally. It is the number of cycles of phase which remains the same in all frames. It is the stringent constraints of linearity of energy and of field, together with this, that forces allowed solutions to be only those where the frequency in the exponent corresponds to the proper magnitude of the energy in the pre-factor. One can fill in any value of energy from radio-waves to high energy gamma photons - but this must affect exponent and pre-factor proportionately. Equally one can , as a photon emitter or absorber, be in any arbitrarily blue or red-shifted frame, the wave-function takes the same form, differing only by the change in the relative scale of frequency, energy and length. In every frame however, these properly relativistic wave-functions are necessarily quantised with an energy given by $\mathcal{E} = h\nu$. The new, fully relativistic, quantised wave-function is the main result of this paper.

Though the exponential factor represents elements other than fields, for (and only for) the experimentally allowed field configuration of the the pre-factor such that electric and magnetic fields are mutually perpendicular and of equal magnitude, these cancel, leaving a pure field solution of the free-space Maxwell equations alone. In other words, only for the cases observed experimentally are expressions encompassing the principle of absolute relativity as in equation. (21) also solutions to the free-space Maxwell equations.

Usually one looks for solutions of the second order equations eliminating one of the fields in favour of the other. Equation. (21) is a solution of the first order equation $dF = 0$ directly. It describes both electric and magnetic fields simultaneously. The development of these in time and space parallel the field transformations expressed by the Maxwell equations. Under a Lorentz transformation the whole solution scales in energy proportional to the frequency alone, as is observed in experiment. Taking the frequency to zero, the energy also goes to zero. Because of the construction, all elements, in any frame and to any order of expansion, scale properly relativistically. This is a result of the rigorous implementation of the principle of absolute relativity, particularly in that the proper relative form of space, time and angular momentum appears in the exponent and that the energy appears both in the exponent and in the pre-factor.

It is also worth noting that, in contrast to conventional wave-functions of the form of equation (15), the development of the wave-function in space and time is different. In space one has a wave, in time one has a rotation. As one moves in space there is an alternation between electric and magnetic field - just as described by the first order Maxwell equations. Sitting at one point in space, and allowing time to pass, one has a rotation of the field vectors. The new wave-functions, in and of themselves, parallel the physical development of the field components in the Maxwell equations far more closely than do conventional solutions.

Equation (21) may be readily be expanded in any particular frame. For the conditions corresponding to experimentally observed photons, the non field (scalar and quadri-vector) terms in the exponential part cancel. Setting $F_1 = H_0 U R \mathcal{E}$ and $ck = \omega = \frac{\mathcal{E}}{\hbar}$ one obtains:

$$F_L = F_1[(\alpha_{10} + \alpha_{31})\cos(kz - \omega t) + (\alpha_{23} - \alpha_{20})\sin(kz - \omega t)] \tag{22}$$

This describes electric ($\alpha_{i0}$) and magnetic ($\alpha_{ij}$) fields rotating in time in a plane perpendicular to the direction of momentum transport and transforming in space from magnetic to electric and vice-versa. The resultant field configuration is that shown in Fig. 1. It appears identical that found in any elementary textbook on electromagnetism for a left-handed circularly polarised wave. This is comforting: despite its apparent complexity the new wave-function reduces to that form measured in experiments and familiar from elementary text-books.

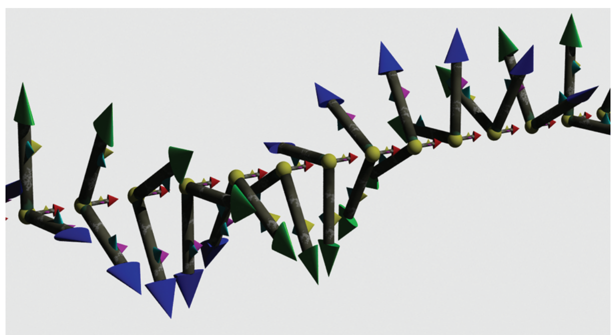

Figure 1. Representation of a single wavelength of a circularly polarised photon. The electric field direction is represented using green arrowheads, the magnetic field blue and the momentum density red.

## 4. DISCUSSION

The new wave-function is consistent with the experimentally-observed field pattern for a photon. The extent that it truly describes a light quantum, a photon, is now discussed.

Firstly, consider that if equation (21) is a solution, then the linearity of field addition and the condition of energy conservation require that light comes in"lumps". Proof: consider an emission-absorption event in the frame of the photon. Now consider whether or not it is possible to superimpose a second such solution, where both overlap precisely in phase and wave-train length, such that twice the energy is transmitted at the same frequency in a single overlapping event. In this case the fields add everywhere. Since the energy density goes as the field squared, this would give four times the energy density everywhere and hence four times the energy transmitted. This violates energy conservation and hence such a process is"not allowed". This is similar to the argument proposed in earlier work to explain the origin of the exclusion principle.[9] It is not a problem to have a second photon, arriving at an infinitesimally different point in space-time, following"behind" or"ahead" of it. The point is that, for an individual photon, the phases of the emitter and absorber are always in harmony at the same point in space time, for the photon - and hence in harmony for any other frame in general and for the frames of emitter and absorber in particular. Any two distinct photons are always outside one anothers light-cone. If they are not outside one another light-cone they are the same photon, and of a different colour. Viewing the same photon from a different frame scales energy density by $R^2$, but length by $R^{-1}$ leading to a linear increase in energy overall. The only way to increase the energy in a single event, self-consistent with both the linearity of energy and field and with relativity, is to increase the frequency. This is tantamount to varying the factor "$R$" in equation (21), affecting both the frequency and the overall energy by the same factor. This process, and this process only, gives a linear increase in both energy and field, as is required by the relativistic transformation of the solution and by experiment. This is the primary reason why the new first order, relativistic expression of equation (21) is necessarily physically quantised. Note that, conversely, relativity itself may be viewed as that transformation required to ensure the linearity of both energy and field as expressed by equation (19) and equation (20) above. Manifestly, for such wave-functions, the energy scales with frequency with some universal constant $h$ such that $E = h\nu$. Clearly, this universal constant $h$ must be identified with the constant of Plank. In other words, the Plank constant defines the scale of length for any given photon wave-function of the form of equation (21) with characteristic energy $\mathcal{E}$ . For a given proper energy, it sets the scale of rulers and clocks for that event. One may have different wave-lengths, but then one must also have correspondingly different energies - just as is observed. This is the key result of this paper: the quantisation of allowed solutions of the continuous theory is a consequence of the experimentally-observed conservation of energy and the linearity of field.

Secondly, note that though the fact of quantisation of the kind of solution represented by equation (21) has not, here, required the introduction of a differential operator, a calculation of the value of the constant of proportionality between energy and frequency (Plank's constant) does. Charge appears at the level of the vector, and the field at its differential, bringing in a factor of R such as that in the pre-factor of equation (21). The question is then: can an expression be found relating the value of the elementary charge and that of Plank's constant in the present formalism? Such an estimate requires a study of the internal dynamics of the emitting and absorbing particles, at its simplest an electron, and this is beyond the scope of the present paper. In earlier work, however, a consideration of a simple semi-classical model of the electron as a localised photon did lead to such a relation.[10] This gives an estimate for Plank's constant in terms of the elementary charge in that model of $\hbar = 1.27 \times 10^{-34} Js$ which is, at least, of the right order of magnitude.

Thirdly, note that the field development and transformation parallel the Maxwell equations more closely than do more conventional solutions. The microscopic development of the field components is not merely a rotation. In equation (21), as one progresses forwards in space the field elements in the solution transform back and forth between electric and magnetic field components, just as in the case of equation (8) and equation (9). Although equation (22) looks just like a conventional simple electromagnetic wave-function, the underlying origin of the elements of electric and magnetic field, as described by eq. (21) is back and forth between each other, just as described by the Maxwell equations. Nonetheless, for a fixed position in some frame, as time progresses the field components appear to rotate (or oscillate), just as is the case for physical sources such as incandescent lights or a transmitter. In these respects, the new solutions match not only what is observed, but also parallel more closely the underlying field transformations of the Maxwell equations.

Fourthly, consider the relativistic transformations of space, time and field. By construction, the internal elements of space and time retain their proper relative form. This leads to the correct transformation properties of all components under a general Lorentz transformation.[6] In any other frame the proper relative transformation of the spatial and temporal field components is ensured in that they are constructed in such a way as to differ by the proper unit element from each other. In these solutions space and time transform properly with respect to each other. The result is that the transformed solution remains a solution in any proper Lorentz frame, right up to the limit of lightspeed. Put simply: all elements in the solution, both exponent (energy) and pre-factor (field) must scale linearly with R. Conversely, the Lorentz transformation is that transformation which ensures the linearity of both energy and field addition. Demanding that both energy and field should add linearly requires the introduction of the principle of absolute relativity. Athough the new principle was taken here as an ansatz, it appears that the sharpening of the principle of relativity of space and time is required to be consistent with the deeper principles of the conservation of energy and the linearity of the field.

Experimentally, the wave train of a photon may have any length, the energy is characterised only by the frequency. It does not matter, either experimentally or within the current theoretical framework, what the absolute value of field is in any frame, merely that it scales properly relativistically and, in particular, linearly with R. In equation (21), $H_0$ is a real normalisation distribution over phase. It is the frequency alone that determines the integrated energy, not the length of the wave train, as is observed in experiment. A functional dependence of the conventional wave-number factors $\omega$ (and $k$, for non-massless objects) appearing in the exponent is required, such that the length scale (the ruler) and the time scale (the clock) transform correctly relativistically. The case for massless, lightspeed objects such as a photon simplifies further in that $|\omega| = |k|$. The simplest functional form consistent with the relativistic transformations of field and the relativistic Doppler formula is then simply linear, $\omega = f\mathcal{E}$, where $f$ is some universal (real) factor converting energy to frequency in appropriate units. That factor is defined by the experimental value of the constant of Plank.

Fifthly, in any extension where $\omega$ and $k$ are not precisely equal, the separation between the space and time oscillations allows an identification with the two-phase harmony of de Broglie which lies at the root of quantum mechanics.[7,8] This corresponds to an extension to lightspeed quantum particles with rest mass, as has been discussed elsewhere.[9–11] The expansion to a full 4-dimensional wave-function introduces, necessarily, a limited extent perpendicular to the propagation direction. In an obvious extension, replacing the exponent with $(\alpha_1 kx + \alpha_2 ky + \alpha_3 kz - \alpha_0 \omega t)\alpha_{012}$ leads to the perpendicular $x$ and $y$ components having and expansion in terms of cosh and sinh instead of cos and sin. Explicitly, these are $F_0(\alpha_P \cosh(kx) - \alpha_{20} \sinh(kx))$ and $F_0(\alpha_P \cosh(ky) + \alpha_{10} \sinh(ky))$ respectively. These terms do not describe propagating solutions. Propagation is supported only along a line joining emitter and absorber and not transverse to the photon path. Further, both sinh and cosh functions increase exponentially in magnitude for larger lateral values, a clearly unphysical condition. $dF = 0$ only if the expansion in the transverse direction is zero or constant. This confines the lateral, non-propagating dimension of the wave-function to lie close to the axis. In particular, some longitudinal components of field may be completely suppressed, since the sinh function is zero on axis. This may help to explain why the field of physical photons is primarily transverse. The scalar component, however, has a finite minimum on axis, and may supply a constant term. This term may prove to express the scalar mass-energy transferred by the photon from emitter to absorber.

Sixthly, if we demand that the exponent should act as a scalar at least at the points of emission and absorption such that it matches to their internal wave-functions (of the form of equation (17), for example), then to achieve this, the factors in the exponent $(\alpha_3 kz - \alpha_0 \omega t)$ and $\alpha_{012}$ should commute. In particular, this requires that $\alpha_{012}$ should commute with the factor for the wavenumber $k$. This is only the case if that wavenumber corresponds to an integral number of half-wavelengths. This extra condition corresponds then to black-body quantisation.

Seventhly, consider simple transformations of the solution proposed in equation (21). Changing the sign of one component of the prefactor alone, for example $(\alpha_{10} + \alpha_{31})$ to $(\alpha_{10} - \alpha_{31})$ has an interesting effect. This is no longer a left-handed solution for a wave propagating in the positive z direction, but a solution for a right-handed photon travelling in the negative z direction. That is, such a transformation matches precisely the physical process of reflection and the handedness of the field with respect to one another matches the direction of momentum transport. A change in the relative handedness of the electric and magnetic field components reverses the propagation direction (and flips the helicity). In other words, just as observed in experiment, the relative handedness of the electric and magnetic field components determines the direction of propagation. Changing the order of the unit angular momentum factor in the exponent from $(\alpha_3 kz - \alpha_0 \omega t)\alpha_{012}$ to $\alpha_{012}(\alpha_3 kz - \alpha_0 \omega t)$ is not a solution to the Maxwell equations. Indeed, for $k = \omega$ it is a frequency doubled oscillation. Though this is not a solution for the Maxwell equation, in the case of the electron-positron annihilation it does correspond precisely to the internal zitterbewegung frequency of the fermions as described by the Dirac equation.[1] It is tempting, then, to identify this solution with the electron. This is not so, the solution remains too simple. For a description of a massive particle the solution must, at the very least, follow periodic boundary conditions such as those described in the simple semi-classical model considered in earlier work.[10] If this is done, this may give a description of purely electromagnetic charged particles with half-integral spin.[9] Changing the sign of the exponent remains a solution, but has the physical effect of transforming from left-handed to right-handed or vice-versa. Thus a linearly polarised photon may be represented as a sum or difference of such solutions. For the particular solution proposed, these are x polarised and y polarised respectively. Elliptical polarisations may be obtained from a linear combination in the usual way. These properties, taken together, mean that the new construction does not describe many non-physical combinations of field. This form is only a solution if a strongly-constrained set of physical conditions are met. Crucial is that *both* the rate of change of phase *and* the field magnitude in both space and time scale with $R$. The magnitudes of the electric and magnetic field components must be equal, but are otherwise arbitrary. To be a solution of the free-space Maxwell equations, $\omega$ must equal $k$. The signs of $\omega$ and $k$ must match the handedness of the field components and their scale must match the scale in the pre-factor.

Further, a factor corresponding to a unit angular momentum is required in the exponent to transform the proper relative form of the spatial and temporal elements to a travelling wave solution with pure fields alone. In the logical extension of the 2D case to the 4D case the lateral components do not propagate. Once again, the required physical conditions match those of the physical photon observed in experiment.

Finally, the wave-function in equation (21) describes a temporal rotation in real space. This means the lateral extent in the photon frame should not exceed a rotation horizon imposed by the speed of light. This imposes conditions on the angular momentum of allowed solutions. The concept was used in previous work to lay bare the physical origin of the anomalous magnetic moment of the electron as a localised photon.[10] For a given frequency the limit imposed by the speed of light on rotation about the photon axis, the rotation horizon, is just $r_h = c\omega$. Introducing the photon momentum observed in experiment, $\vec{p} = \hbar\omega/c$, gives a limit on the integral allowed angular momentum of the solutions of $r_h \times \vec{p} = \hbar$. This sets the intrinsic scale of unit angular momentum for solutions such as that described by equation (21). The form demanded by equation (17) and manifested in equation (21) is not merely descriptive, it is strongly proscriptive. Demanding the principle of absolute relativity, manifested in the form of equation (17), places strong restrictions on allowed solutions, over and above those required by the Maxwell equations alone. In summary: to be a travelling wave solution, the proper form must have electric fields perpendicular and of equal magnitude, must be associated with a unit angular momentum and energy must scale with frequency, just as is observed for the physical photon.

## 5. EXPERIMENTAL TESTS

The theory is already consistent with a great number of classic experiments. The present theory, as discussed above, explains the underlying physical origin of effects such as the quantisation of light itself which must otherwise be taken simply from experiment. It has been argued that the present theory is more closely consistent with the whole existing body of experiment than are any alternatives. This does not, however, mean that it does does not suggest new avenues for experimental investigation.

The fact that the limits on the angular momentum of the photon may arise from the elementary charge opens up an interesting set of experimental possibilities. It may be that photons emitted by collective states of matter, such as in the superconducting or fractional quantum Hall regimes at low energy, or in regimes where fractional charges such as quarks may be present at high energy, may have different constraints on the limits of the photon angular momentum. It may therefore be possible to produce or detect photons with angular momentum a fraction or multiple of $\hbar$, and this should be subject to experiment.

## 6. CONCLUSIONS

In conclusion, a stringent application of the principle of absolute relativity, demanding solutions of the form of equation (21), leads to solutions of the continuous first-order Maxwell equations with many of the properties of the physical photon. Such solutions propagate equal and perpendicular fields along an axis perpendicular to both. Perpendicular to the propagation axis they are strongly confined. The transformation of the fields require that the energy transmitted should come in "lumps" and that this energy is proportional to the frequency. All such solutions have the same angular momentum, meaning that this appears quantised. An extra boundary condition at the point of emission and absorption requires a black-body quantisation. Taken together, it may be argued that these features mean that the new wave-function better represents the physical photon than do more conventional solutions.

### 6.1 Acknowledgments

This work flows from a long and fruitful collaboration with my friend and co-pirate Martin van der Mark. At least half of anything creditworthy in this article is due to him. I am grateful to Phil Butler for his education in algebra, to Stephen Leary and Tim Drysdale for their valuable contributions, to John Duffield for useful suggestions and to Nick Green, Nick Bailey and John Weaver for their constant encouragement. The figure was drawn using Povray.

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