[4]J. A. DeShong, Jr., R. H. Hildebrand, and P. Meyer, Phys. Rev. Letters 12, 3 (1964).
[5]N. F. Ness, C. S. Scearce, and J. B. Seek, Proc. Intern. Space Sci. Symp., 5, Florence, 1964.
[6]H. S. Bridge, A. Egidi, A. Lazarus, E. F. Lyon, and L. Jacobson, Proc. Intern. Space Sci. Symp., 5, Florence, 1964.
[7]D. A. Bryant, G. H. Ludwig, and F. B. McDonald, IRE, Trans. Nucl. Sci. 9, 376 (1962).
[8]A. Maxwell, private communication.
[9]D. A. Bryant, T. L. Cline, U. D. Desai, and F. B. McDonald, Phys. Rev. Letters 11, 144 (1963).
[10]C. Y. Fan, G. Gloeckler, and J. A. Simpson, Goddard IMP symposium, National Aeronautics and Space Administration Goddard Space Flight Center, Greenbelt, Maryland, March 1964 (unpublished).
[11]E. N. Parker, J. Geophys. Res. 69, 1755 (1964).
[12]H. Dodson Prince and R. Hedeman, private communication.

FOURTH TEST OF GENERAL RELATIVITY

Irwin I. Shapiro
Lincoln Laboratory,* Massachusetts Institute of Technology, Lexington, Massachusetts
(Received 13 November 1964)

Recent advances in radar astronomy have made possible a fourth test of Einstein's theory of general relativity. The test involves measuring the time delays between transmission of radar pulses towards either of the inner planets (Venus or Mercury) and detection of the echoes. Because, according to the general theory, the speed of a light wave depends on the strength of the gravitational potential along its path, these time delays should thereby be increased by almost 2×10^{-4} sec when the radar pulses pass near the sun.[1] Such a change, equivalent to 60 km in distance, could now be measured over the required path length to within about 5 to 10% with presently obtainable equipment.[2]

An analytical representation of this predicted increase in delay, useful for discussion, can be obtained by calculating the difference Δt_r between the proper-time delay predicted in general relativity and the corresponding flat-space value. Using the usual form of the Schwarzschild solution to represent the gravitational field of the sun[3] and neglecting the motion of the earth between pulse transmission and echo reception, we find

$$\Delta t_r \approx \frac{4r_0}{c}\left\{\ln\left[\frac{x_p+(x_p^2+d^2)^{1/2}}{-x_e+(x_e^2+d^2)^{1/2}}\right] - \frac{1}{2}\left[\frac{x_p}{(x_p^2+d^2)^{1/2}}+\frac{2x_e+x_p}{(x_e^2+d^2)^{1/2}}\right]\right\} + O\left(\frac{r_0^2}{c^2}\right), \tag{1}$$

where, in this coordinate system, d is the distance of closest approach of the radar wave to the center of the sun, x_e is the distance along the line of flight from the earth-based antenna to the point of closest approach to the sun, and x_p represents the distance along the path from this point to the planet. Both x_e and x_p are measured positively in a direction away from the earth. The gravitational radius r_0 for the sun is $GM_S/c^2\approx1.5$ km, where G is the gravitational constant, M_S the mass of the sun, and c the speed of light. The right-hand side of Eq. (1) is due primarily to the variable speed of the light ray; the contribution from the change in path, being of second order in (r_0/c), is negligible. (This type of result is a general one for refraction phenomena in which the change in index is small.)

At superior conjunction, when the target planet is by definition on the opposite side of the sun from the earth, Eq. (1) reduces to

$$\Delta t_r \approx \frac{4r_0}{c}\left\{\ln\left(\frac{4x_ex_p}{d^2}\right)-\left(\frac{3x_e+x_p}{2x_e}\right)\right\}; \tag{2}$$

$$d \ll x_e,\ x_p,$$

and, at inferior conjunction, when the planet is between the earth and sun, to

$$\Delta t_r \approx \frac{4r_0}{c}\left\{\ln\left|\frac{x_e}{x_p}\right|-\left(\frac{x_e-|x_p|}{2x_e}\right)\right\}; \tag{3}$$

$$d \ll x_e,\ |x_p|.$$

At elongation, when the planet is furthest east or west from the sun, as viewed from the earth, we find

$$\Delta t_r \approx \frac{4r_0}{c}\left\{\ln\left(\frac{2x_e}{d}\right)-1\right\}; \qquad (4)$$

$$x_p \approx 0;\ d^2 \ll x_e^2.$$

This last form is only valid for Mercury, since for Venus $x_e \approx d$ at elongation. As an illustration, we note that for Mercury when d is twice the radius R_S of the sun, Eq. (2) yields $\Delta t_r \approx 1.6\times10^{-4}$ sec, whereas for their respective conditions of applicability, Eqs. (3) and (4) both yield about 0.1×10^{-4} sec. Thus, despite the logarithmic behavior of the dominant term in Eq. (1), the difference between the maximum and minimum effects, which is the significant measurable quantity, is almost equal to the maximum value of Δt_r.[4]

Are these effects on interplanetary time delays likely to be obscured by others? The most important candidates in this latter category are the imprecision in the knowledge of planetary orbits and radii, and the presence of the interplanetary medium. Analysis shows that the orbits of the earth and target planet, as well as the latter's radius, can be determined with more than the required precision from time-delay measurements distributed around the orbits of both planets.[5] The sensitivity of the time-delay measurements to changes in Δt_r is different from the corresponding sensitivity to changes in the initial conditions of the orbits and in the planetary masses and radii. Hence the parameters characterizing Δt_r can be estimated from the data simultaneously with the other relevant ones, without incurring any severe accuracy penalty from nonseparability. The topographical variations on the target planets are probably small enough so that even the most accurate measurements will not be significantly degraded.

The effect Δt_m of the interplanetary medium on the time delay can be represented by

$$\Delta t_m \approx \frac{8.2\times10^7}{f^2 c}\int_{-x_e}^{x_p} N(l)dl \text{ sec}, \qquad (5)$$

where N is expressed in electron/cm^3, f in cps, c in cm/sec, and l in cm. Using recently compiled results on the solar corona,[6] we find that during a "quiet-sun" period

$$N(r) = 5\times10^5\left(\frac{R_S}{r}\right)^2 \text{ electron/cm}^3,\quad r^2 = l^2 + d^2, \qquad (6)$$

represents the data reasonably well from about $r = 4R_S$ to $r = 20R_S$. Inside this range, the actual N increases more rapidly with decreasing r, whereas outside it decreases more rapidly with increasing r. For the period of maximum solar activity, N seems to be about a factor of 5 higher in the radial range represented by (6).[6] Substituting Eq. (6) into (5) yields

$$\Delta t_m \approx \frac{6.5\times10^{24}}{f^2 d}\left\{\tan^{-1}\left(\frac{x_p}{d}\right)+\tan^{-1}\left(\frac{x_e}{d}\right)\right\} \text{ sec}, \qquad (7)$$

where d, x_e, and x_p are expressed in cm. For the Arecibo Ionospheric Observatory's frequency of 430 Mc/sec, the lowest at which interplanetary time-delay measurements are currently being made, Eq. (7) yields $\Delta t_m \approx 3.7\times10^{-4}$ sec for observations of Mercury near superior conjunction with $d \approx 4R_S$. [This latter value corresponds to an angular distance from the sun of 1°, the smallest at which Arecibo measurements can be made (see below).] In this case, Δt_r would equal about 1.4×10^{-4} sec and would most likely be masked by the uncertainty in Δt_m. Although Δt_m varies inversely with d, whereas the corresponding dependence in Δt_r is logarithmic, the difference $\Delta t_r - \Delta t_m$ is nowhere large enough and positive for a really reliable result to be obtained solely from Arecibo data. Since Δt_m varies as the inverse square of the radar frequency, this plasma effect will be reduced by a factor of almost 400 (and will therefore be unimportant) for measurements made at the 8350-Mc/sec frequency of the newly constructed, but not yet fully instrumented, Haystack radar of MIT's Lincoln Laboratory.[7] In any event, simultaneous equivalently accurate time-delay measurements at two frequencies will allow the plasma effect to be deduced and subtracted, since Δt_m is frequency dependent and Δt_r is not.[8]

Other possibly relevant effects on the delays are easily disposed of. A previous study[9] has shown that the earth's and planet's atmospheres and ionospheres will not significantly affect time delays, even for $f = 430$ Mc/sec. The effect of the earth's gravity and motion on the laboratory clock is unimportant for this experiment, since the clock rate remains constant over a year to within about one part in 10^{10}. The gravitational effects of the earth, moon, and target planet on the time delays are far smaller than the sun's, but in any case the former can be neglected since their contributions will be almost identical in each measurement and consequently indistinguishable from a small decrease in the planet's radius. Any lack of precision in the determination of c in terms of terrestrial units (such as km/sec) is clearly irrelevant to our experiment since time delays only are of concern.

In making the time-delay measurements, a radar cannot be directed towards the solar limb because of the radio interference that would result. For the Haystack facility, however, the antenna beam width is sufficiently narrow and the near side lobes of sufficiently low gain that the beam can be directed well within a degree of the limb without the solar radio emanations introducing a significant increase in the overall system noise temperature. For Arecibo the closest possible approach is about 1°. Were both inner planets to have the same ratio of radar to geometric cross section, then at superior conjunction Venus would always be more easily detected, the received power being about a factor of two greater. However, at the X-band frequency of Haystack, Venus may very well have a relatively low radar cross section[10] because of absorption in its atmosphere.

Although Mercury passes through superior conjunction about once in four months, some passages are considerably more useful than others from the points of view of ease of detectability and of close angular approach to the sun. The most favorable in the next two years occur on 11 June 1965 and 27 May 1966; in the former the minimum angular distance is about 1° and in the latter about 0.5°. The superior conjunctions of Venus are less variable and usually lead to an angular distance of closest approach of about 1°, as in the next two which will be on 12 April 1965 and 9 November 1966. The second occurs in southern declinations and so will be invisible to the Arecibo radar whose antenna is not steerable.

---

*Operated with support from the U. S. Air Force.

[1]In an interplanetary radar experiment, the Doppler shift of the radar wave is also measured; but although the effect on time delay of the change in c is cumulative, the corresponding general relativistic effect on Doppler cancels out over the round trip.

[2]J. V. Evans and G. H. Pettengill, private communication.

[3]See, for example, P. G. Bergmann, Introduction to the Theory of Relativity (Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1942), p. 203.

[4]In an earth-moon experiment, however, the relativistic contribution to the difference in measured time delay between new and full moon is quite undetectable, being only about 5×10^{-11} sec.

[5]With measurements extended over a several-year period the precession of Mercury's perihelion could also be estimated accurately and independently of the optical data; in addition, reasonably strict limits could be placed on the possible time dependence of G.

[6]W. C. Erickson, Astrophys. J. 139, 1290 (1964).

[7]The only other facility that could now participate significantly in this measurements program is Jet Propulsion Laboratory's Goldstone radar, operated at a frequency of 2388 Mc/sec.

[8]With one of these frequencies at about 400 Mc/sec or lower, the integrated electron density along the path of the radar wave could be determined accurately, thus providing useful information on the solar corona. Faraday rotation effects may also shed light on certain characteristics of the solar magnetic field.

[9]I. I. Shapiro, to be published.

[10]D. Karp, W. E. Morrow, Jr., and W. B. Smith, to be published.

ION TRAPPING IN ROTATING HELIUM II

R. L. Douglass
Physics Department, American University of Beirut, Beirut, Lebanon
(Received 5 November 1964)

Trapping of negative ions in the cores of quantized vortex lines has been suggested by Careri, McCormick, and Scaramuzzi[1] as an explanation for their discovery of rotation-induced attenuation of space-charge limited ion currents in He II. This Letter reports the direct detection of trapped negative ions in rotating He II, and some measurements of their mean trapped time and of their mobility parallel to the rotation axis at temperatures between 1.20 and 1.72°K. No evidence of positive-ion trapping has been found, in agreement with the ion-current measurements,[1,2] and in contrast to the experiment of Rayfield and Reif[3] at much lower temperatures.

The He II is contained in an electrode assem-