Draft of Paper [9570-53], SPIE Optics + Photonic, San Diego, 9-13 August 2015.

On the nature of “stuff” and the hierarchy of forces

Martin B. van der Mark*
Philips Research Europe, WB-21, HTC 34, 5656 AE Eindhoven, The Netherlands

ABSTRACT

From super clusters of galaxies down to the quarks in the proton, at all length scales the structure of matter is the result of a balance of forces. In this paper it is argued that with decreasing size there must necessarily be an increase of the fraction of kinetic and binding energy with respect to the total energy. Smaller sizes require stronger forces which represent more of the energy available. The smallest possible size of granularity is found to be where the internal kinetic energy and total energy become comparable, which occurs at the size of the proton. We infer that the proton is the smallest stable particle, being a light speed circulation of energy.

Keywords: primordial stuff, hierarchy of forces, rest mass, energy balance, proton structure, light speed knot of energy.

1. MOTIVATION

In this paper the question will be addressed of what it is that every material thing is made of[1-4]. Of course modern science already gives some idea of what it is, at least to some level: It is matter and it is made of atoms! Or more precisely, matter is made of protons, neutrons and electrons, or perhaps, quarks and leptons and gluons and vector bosons, and more. This is quite true, but what then is the nature of the substance that constitutes these elementary particles? Are there, for example, even smaller, more elementary-elementary particles? Or perhaps, is there some underlying primordial “stuff”? If so, is this stuff continuous, like a fluid or field, or is it granular of nature?

Whether we look at galaxies, the solar system or the electrons in the atom, at all length scales the basic structure and shape of matter seems to be that of some objects bound together and rotating around one another. The basis for this universal dynamical structure is a balance of forces: a repulsive force (such as the centrifugal force) and an attractive force. At astronomical sizes the attractive force is provided by gravitation, whereas at molecular and atomic sizes it comes from electromagnetism. For nuclear scales it is provided by the weak and strong interaction.

What we find is that the weakest of the forces (gravitation) governs the largest structures and that the smallest structures, the elementary particles, are stabilized by the strongest of the forces: the strong interaction. There appears to be a hierarchy of forces that dominates the structure of matter at successive length scales. That this must necessarily be so, in fact, can be understood quite readily. The stronger force will simply dominate over the weaker ones and be able to pull in more closely the objects it likes.

While stronger forces lead to tighter binding, with the system releasing more energy in the process, at the same time the internal kinetic energy of the system (by the virial theorem) must increase proportionally to it. As will be explained more precisely in the body of this paper, the consequence of this is that in the tightest bound systems much, if not most, of the mass is coming from the system’s internal kinetic energy, not from the bare mass of the constituents. In contrast, in weakly bound systems the mass is entirely dominated by and (very nearly) equal to the sum of the bare masses of the constituents. This is what we judge to be the “normal” situation and a deviation from it implies the occurrence of what is commonly known as the “mass defect”. A well-known example is given by the Helium nucleus, for which the sum of the masses of the two protons and two neutrons is larger than the mass of the atomic nucleus they form.

In what follows some of the steps above will be detailed. This will then lead to conclusions that may only be appreciated if we first prepare ourselves by asking some more philosophical questions about the possible foundations of matter.

*martin.van.der.mark@philips.com; phone +31 40 2747548

2. SOME FUNDAMENTAL QUESTIONS

Before going into more detail it seems appropriate to prepare ourselves by asking a few questions.

1. What is it that everything is made of? A quite reasonable answer to this question seems to be: "It is either matter or light (radiation, photons)". For the purpose of this paper we will only indirectly address the latter possibility.
2. Then, what is matter? What is its ultimate substance?
3. Does there exist something such as primordial "stuff"?
4. Does this "stuff" have mass? And, as a consequence of its possible mass, does it contain or possess energy?
5. Is "stuff" penetrable? Does it exert a force on some things? Note that this question certainly has bearing on the nature of radiation, which is penetrable, contains energy and can exert a force on matter.

Before trying to answer all these questions we make explicit that we will presume (as usual) that:

- Energy is conserved, and it is conserved locally
- One may only consider closed systems so that a proper energy balance can be obtained
- $E = mc^2$ for all energy E thinkable (or unthinkable), i.e. all energy has the same "essence": it is mass m. Note that the meaning of Einstein's famous formula is that *energy is equivalent to mass* and not another form of mass (as if the formula were to describe a transmutation). This point appears to be part of scientific confusion and therefore raised explicitly in Appendix A and explained in more detail in Ref. 5.

Given the above, if "stuff" does not ever exert a force, nothing happens and we cannot know that it exists at all. It then is effectively not part of our world. Interestingly, however, would it have interaction with its own, separate world, then it must carry energy (a force is the gradient of some energy), and consequently it becomes part of our world through gravitation ($m = E/c^2$, the energy represents mass, and that mass gravitates with the masses of our world). So, all that has energy does exists in our world, albeit only felt by gravitation.

This leads to some further realizations and consequences. The previous paragraph implies that for any stuff to be useful it must interact with some world and it must therefore have energy, in other words: stuff cannot be "sterile"! Conversely, it also implies that anything at all that exists in the universe (our world) must have energy. Then, finally, we may conclude that there is only a single world and it contains everything that exists, and that is our universe.

Hence we must conclude that energy is part of stuff or some form of stuff. This then raises more questions:

1. What is the fundamental appearance that goes with stuff?
2. Is there more than one type of stuff?
3. Are charge, spin or quarks some form of stuff?
4. Is stuff quantized or is it continuous?

While only the purpose of these questions may be clear within the context of the first section of this paper, in what follows it will be shown that these questions are indeed also intelligible and addressable. Note however that the very existence of space and time have been assumed implicitly and as being independent from the existence or the nature of matter. Although that assumption may be incorrect (see Mach's principle, Ref. 7), for the purpose of this paper, it will not appear to give any immediate problems.

3. SOME FIRST ANSWERS

From experiment we can find some first answers to the questions raised above. Consider Eq. (1), showing the annihilation of the singlet bound-state electron-positron pair, called para-positronium (anti-parallel spins). It has a mean lifetime of 125×10^{-12} s and decays highly preferentially into two gamma rays. Energy is conserved; the gammas have energy of 0.511 MeV each, corresponding to the rest mass of electron and positron. Of course, total momentum (which is zero) is also conserved: the gammas are radiated in opposite direction, and angular momentum (which is also zero) is conserved by the polarization of the gammas (which are therefore in an entangled polarization state).

$$e^+ + e^- \rightarrow \gamma + \gamma \tag{1}$$

In Eq. (1) there is a total transmutation of matter into radiation, which implies that for the stuff in the electron and positron only the following possibilities seem to be allowed:

1. There is no different stuff other than the electromagnetic fields. The electron and positron are purely electromagnetic to begin with, hence they can radiate purely electromagnetically.
2. There is one kind of stuff. Bringing together twice the amount shouldn't make it disappear, but what is left are just the two gammas. Hence stuff is electromagnetic, so that comes down to possibility 1, we can say that stuff is some, perhaps alternative or extended, form of electromagnetism.
3. Stuff is its own anti-stuff. Then, if stuff is continuous one may have any amount of it, it cannot be its own anti-stuff. If it is quantized, only one quantum may exist in each fundamental particle (since it is its own anti-stuff it will annihilate even amounts), so that all particles have the same amount of stuff. Then, what makes them different in the first place? It must be different stuff for different particles, or equivalently, more properties to stuff, or properties may be imposed by boundary conditions imposed by space-time. Then, the quantization may come from the periodic or topological nature of these boundaries[8]. Anyhow, later in this paper it will become clear that stuff must be continuous.
4. There is stuff and there is anti-stuff and brought together the two annihilate into radiation. Hence there are two kinds of stuff which must couple to electromagnetism. What then is, or carries, the "anti" property? Is it the electric charge? Charge conjugation is the generally accepted particle to anti-particle transformation. Charge is an electromagnetic property, apparently intimately connected to stuff. Another possibility is, by the CPT theorem, that anti-stuff has the opposite spatial handedness running backwards in time or so, but that would require it to be quite more complicated than just a scalar field. Alternatively, space-time itself must have a non-trivial structure.

The more properties are attributed to stuff, the less fundamental stuff becomes. This is not what we were hoping for because then there is something else, "meta stuff", that determines the differences. But a further question comes to mind: How does stuff know where the electric charge of the electron is, or where it is going, if it has no interaction with it that is at least as strong as the electromagnetic interaction? Hence to solve this issue, one way or another, stuff must at least also couple by the weak force or electromagnetism!!! It cannot be as dull as galactic dark matter; gravitation is far too weak to keep up with a single particle!

At this point it should be realized that at least part of the mass of a charged particle comes from its electric charge. This so-called electromagnetic mass comes from the energy carried by the external electric field. In case of the electron, much of the field seems to be external and hence the contribution of the electromagnetic mass to the total mass of the electron may be substantial. It has therefore been proposed[9,10] that the electron may be purely electromagnetic, an idea that is very appealing, but is associated with some serious problems. One problem is known as the self-energy problem of electrical charge, the other is the illusive nature of the binding forces (known as the Poincaré stresses) that keep the electron together. The problems are very well described in the Feynman Lectures[9], but because the structure of matter is the central theme of this paper, a brief account of it is given in Appendix B.

4. THE STRUCTURE OF MATTER AT DIFFERENT LENGTH SCALES

From super clusters of galaxies down to the quarks in the proton, at all length scales the basic structure and shape of matter seems to be that of some objects orbiting one another, forming a bound system. For example, stars circulating the galactic center, the moon orbiting the earth or electrons waving around the atomic nucleus. The basis for this universal dynamical structure is a balance of forces: a repulsive force (such as the centrifugal force) and an attractive force. At astronomical sizes the attractive force is provided by gravitation, whereas at molecular and atomic sizes it comes from electromagnetism. For nuclear sizes it is provided by the weak and strong interaction. When we go to small enough length scales an additional property emerges due to quantum mechanics, namely the wave-like nature of particles. Due to wave interference quantum mechanics provides a further mechanism (on top of the centrifugal force) against the collapse of the smallest objects. Historically, in 1913, the quantization using De Broglie waves was the answer to Bohr's postulated stability of electron orbits in his model of the Hydrogen atom[2].

To gain some insight on the observed scaling of structures in the universe, an overview of the mass and size relationship is given in Fig. 1. What we see first of all is the truly astronomical scale we are dealing with. At the far left we have the smallest thinkable, an object of Planck mass $m_P = \sqrt{\hbar\, c/G}$ and Planck length $l_P = \sqrt{\hbar\, G/c^3}$ = 1.6×10^{-35} meter, to the upper right the largest and heaviest, the universe itself (with a radius of 46.6 Mly), and in between, at the bottom, the lightest material object, the electron. Well, one may ask whether it is the lightest and why its size of 2.4×10^{-12} meter is so large; shouldn't it be as tiny as a point? It is a matter of choice, what is taken here for the size of an elementary particle is its Compton wavelength:

$$\lambda_C = \frac{h}{mc} \tag{2}$$

Clearly, neutrinos may be lighter, and so are all photons that have less than 511 keV of energy (this energy corresponds to the electron's rest mass). Although radiation is not matter, its relation between energy and wavelength $E = hc/\lambda$ would make the photon fall on the same line as all elementary particles; with proportionality $m \propto d^{-1}$ we may name this the "particle branch". The electron, like all other leptons (muon, tauon, the neutrinos), is special in that it shows no internal structure in scattering experiments, just as a point object. This does not mean however that the electron itself is really a point. For example, the electron has spin, magnetic dipole moment, a de Broglie wavelength and a finite mass and charge. This cannot be reconciled with the notion of point symmetry. The implied infinite energy and charge densities require at least some proper dressing over a length scale related to the electron mass and its quantum mechanical wave nature. That length is what the Compton wavelength is. Note that for the proton, which does have internal structure, the charge radius and Compton wavelength are of similar size.

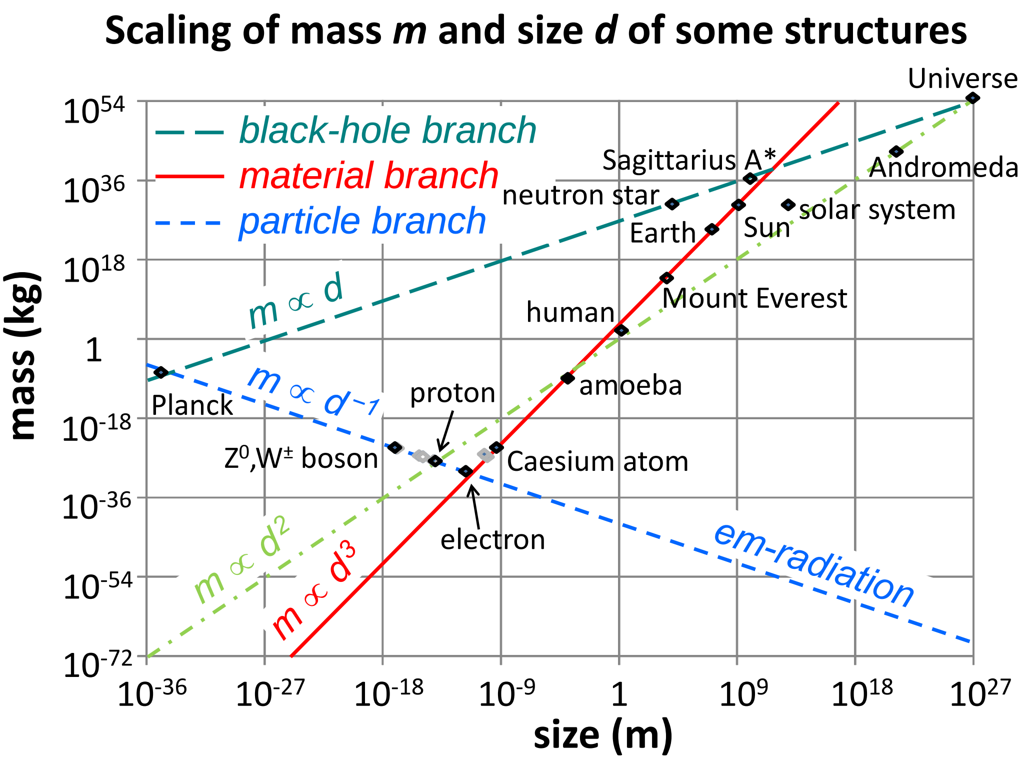

Figure 1. Overview of masses and sizes of particles and structures as found in the universe. Lines indicate scaling laws of mass with size. Three branches stand out: The "material" branch with $m \propto d^3$, the "black hole" or "Schwarzschild" branch with $m \propto d$, and the "particle" or "Compton" branch with $m \propto d^{-1}$. Except for the proton, all elementary particles with more mass than that the electron (in the diagram to the left of the electron) are unstable. The graph shows what is found experimentally: that the electron is the *lightest* stable particle and that the proton is the *smallest* stable particle.

Central to Fig. 1 is what we may call the "material branch" for which mass is proportional to volume: $m \propto d^3$ and it is where we find ordinary matter. The line connects the more familiar material bodies: electron, human, earth and sun. On the material branch, objects get lighter when smaller. On the particle branch, however, objects get heavier when smaller! As a consequence, a minimum weight object exists close to the point where the particle branch and the material branch cross. This appears to be the electron.

Another couple of branches with different scaling laws are indicated as well. The upper one has proportionality $m \propto d$ and is named the "black hole branch". It connects the universe with the Planck scale through some known black holes, the size of which is taken to be the calculated Schwarzschild radius: $r_S = 2Gm/c^2$. There is also line that connects the universe with the proton and it has proportionality $m \propto d^2$. It has been drawn for no better reason than Dirac's Large Numbers Hypothesis (LNH) in which the ratio of sizes of universe and proton are compared to the relative strength of electromagnetic and gravitational interaction between proton and electron (both are 10^{42}), and it appears to coincide with the aspect ratio of the figure signifying the very largest, smallest, heaviest and lightest ever thinkable in our universe.

In this paper we combine two general arguments. First, that with decreasing size (horizontal scale of Fig. 1.) there is a systematic increase of the fraction of kinetic energy (internal dynamics) and binding energy (energy lost at formation) with respect to the total energy (rest mass, vertical scale in Fig. 1.). Second, that the stronger forces are getting balanced at cost of the larger part of the available energy. Consequently, at some small enough length scale the amount of internal energy will approach the total energy in the system, from where no further decrease in size is possible in a stable manner. Experimentally the proton is found to be the smallest stable particle.

4.1 On the strength and hierarchy of forces

Any stable, bound structure must have an internal balance of repulsive forces that prevent collapse and attractive forces that provide binding. The dependence of the strength with distance of either of these forces determines the nature of the structure we see.

Looking at different scales of length, what we find is that the weakest of the forces (gravitation) governs the largest structures and that the smallest structures, the elementary particles, are stabilized by the strongest: the strong interaction. There appears to be a hierarchy of forces that dominates the structure of matter at successive length scales. That this must necessarily be so, can be understood quite readily. The stronger force will simply dominate over the weaker ones and be able to pull in more closely the objects it finds attractive. The weaker bonds, made by the weaker forces, can and will be broken by the stronger force. So the hierarchy becomes obvious: The stronger force wins and pulls in whatever it can. What is pulled in (against any repulsive force) ends up closer to the source of the interaction than the weaker force could do. Hence stronger forces go first and they make smaller, more tightly bound objects.

One may wonder what the influence is of the reach and polarity of the forces on the hierarchy argument as presented above. Whatever initial configuration of particles, we may safely assume that sufficient mixing will occur eventually. Whether unipolar (gravity) or bipolar (electromagnetism) interaction sources are at play, attraction will occur. In the latter case this is true because the universe appears to have no net charge. The same polarity charges will simply be pushed away in favor of the attractive ones. There is another convenient situation in the universe (that may not be accidental) and that is that the strongest forces only work at short range, just where they would "want" to operate. What it means is that the diverse properties of the forces do not at all get in the way of the central argument of this paper.

While stronger forces lead to tighter binding, with the system releasing more energy in the binding process, at the same time the internal kinetic energy of the system must increase proportionally. How much it increases depends on the kind of binding potential and is expressed by the so-called virial theorem, which states that if the potential energy is of the form $E_{pot}(r) = ar^n$ then the bound system has (mean) kinetic energy $\langle E_{kin} \rangle = n \langle E_{pot} \rangle / 2$.

In weakly bound systems the total mass of the system is, for all practical purposes, simply equal to the sum of the bare masses of the constituents. In what follows it is shown that for the tightest bound systems this is no longer the case because the internal kinetic energy and binding energy become so large in comparison to the bare mass of the constituents that it cannot be neglected. A well-known example of this is given by "mass defect" for atomic nuclei.

Now we have arrived at an important point, and a key insight can be obtained if we string together all successive levels of binding. Starting in reverse, from the weakest forces, each successive, deeper level of binding must come from a stronger force at a shorter length scale and hence must have more binding energy associated with it. While this binding energy is released to the environment, a related amount of kinetic energy is kept or built-up inside the system. More kinetic energy is added at each deeper level than all the kinetic energy that was there already. At some point, the kinetic energy may reach a level equal to the bare mass of the system, exhausting the maximum possible amount of energy available, at which point there can no longer be a next level with even more energy and stronger forces to keep even smaller constituents together. This is where further scaling must stop. Due to a pure lack of available energy to keep the integrity of the granules, granularity must be given up in favor of a continuum of energy.

The question is when and where granularity must be given up. At first glance any attempt to give an estimate may seem futile, but it will appear that answers are only needed that are accurate at the order of magnitude level. In what follows, the energy balance and binding process for the hydrogen atom, deuteron and proton will be analyzed.

5. ENERGY BALANCE IN A BOUND SYSTEM

When two particles with an attracting force between them are brought together from infinite separation, the potential energy is reduced and a bound system may be formed if all of the required conservation laws can be fulfilled. Among those are the conservation of energy, momentum and angular momentum. This requires that at least a third particle or (radiation) field is present or is produced to deal with any excess of energy, momentum and angular momentum. In what follows we will first analyze the energy balance in the formation of the hydrogen atom from an electron and a proton. From there we will proceed to analyze the energy balance in the deuteron as well as the proton.

5.1 Formation of the hydrogen atom

In this section the energy balance and internal kinetic energy in the hydrogen atom is compared to that of the free proton and electron at large separation. At time t_0, imagine an electron and proton at very large separation and both at rest so that both the kinetic and potential energy are zero $E_{kin} = 0 = E_{pot}$. If we let the system evolve, see Fig. 2, the electron and proton will gain kinetic energy at the cost of potential energy (times t_1 and t_2) and come together to form a bound state at time t_3, a hydrogen atom, all under emission of electromagnetic radiation with total energy E_{phot}.

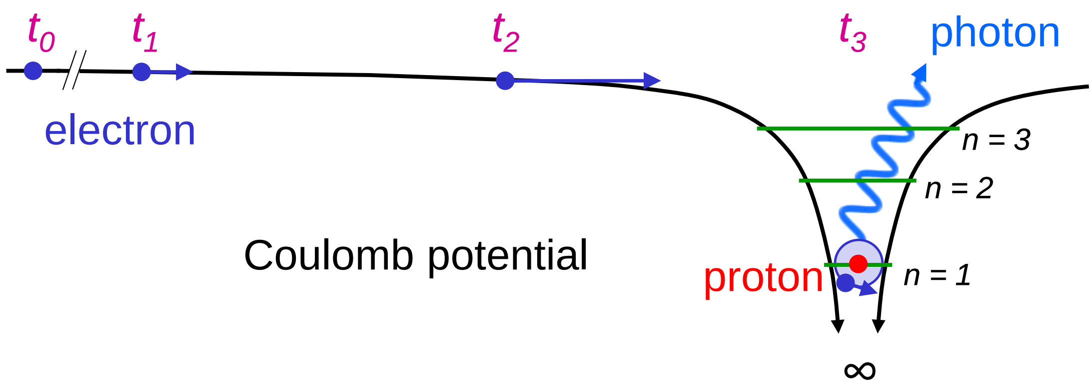

Figure 2. Schematic picture of an electron and proton form a hydrogen atom under emission of electromagnetic radiation.

For the initial state when proton and electron are at rest, the total energy is:

$$E_{tot,i} = E_{pot,i} + E_{kin,i} + E_0 = E_0 \tag{3}$$

Where $E_0 = m_0c^2$ is the amount of (rest) mass of the constituents, the proton and electron. Clearly, by conservation of energy, the total energy in the final state must be equal to the energy in the initial state:

$$E_{tot,f} = E_{pot,f} + E_{kin,f} + E_{phot} + E_0 = E_{tot,bound\ state} + E_{phot} = E_{tot,i} = E_0 \tag{4}$$

Here, $E_{tot,bound\ state}/c^2 = m_H$ is the mass of the hydrogen atom. To find the distribution of energy in the final state, we may apply the viral theorem which states that if a potential is of the form $E_{pot}(r) = ar^n$, then the kinetic and potential energy, averaged over an orbit, are related by $2\langle E_{kin}\rangle = n\langle E_{pot}\rangle$. For the special case of circular orbits, when averaging can be omitted, this reduces to $2E_{kin} = nE_{pot}$. In case of the Coulomb potential $E_{pot,EM}(r) = q_p q_e/4\pi\varepsilon_0 r$ or the gravitational potential $E_{pot,G}(r) = -G\, m_p m_e/r$ we have $n = -1$ and hence $E_{pot} = -2E_{kin}$.

In the Bohr model of the hydrogen atom were the electron and proton are in a circular orbit around each other, the energy balance is given by

$$E_{tot,f} - E_0 = 0 = E_{pot,f} + E_{kin,f} + E_{phot} = E_{phot} - E_{bind} = -E_{kin,f} + E_{phot} \tag{5}$$

from which it is clear that the bound state has a binding energy equal to the energy carried away by the emitted radiation $E_{bind} = E_{phot}$ and also that there remains a kinetic energy inside the atom $E_{kin,f} = E_{bind}$, corresponding to the orbital motion of the proton and electron. The energy has come from the increased overlap and cancellation of the electromagnetic fields of the electron and proton in the final state.

What does this mean in terms of the total mass of the system? In the initial state we only have the mass of electron m_e and proton m_p. The values m_e and m_p can be found in a table of particle properties and represent an invariant property that corresponds to the intrinsic mass m_0 (also called the invariant mass, rest mass or proper mass) of the particle.

Where in the initial state, when everything looks static, we easily could keep the idea that the intrinsic mass is built purely of some static primordial stuff, in the final state we see that this is, at least to some degree an illusion: part of the mass has been radiated away (E_{phot}/c^2) and another part of the mass is present as kinetic energy ($E_{kin,f}/c^2$), and clearly non-static! It is not mysterious; all of the mass has really come from the electromagnetic field between the proton and electron, of course. It is indeed so that the mass of the electron and proton is at least partially of electromagnetic nature to begin with: $m = m_{stuf} + m_{EM}$ (see Appendix B as well as the Feynman Lectures, Ref. 9).

The well of a Coulomb potential is infinitely deep, so in principle the electron and proton could come together as close as they like, making the binding energy arbitrarily high! There is however another restriction, the wave nature of the particles. It appears that there is a smallest average distance r_0 at which the proton and electron want to be; this is the ground state of the hydrogen atom, and the proton and electron are in what we may call a zero-point motion at a length scale imposed by their De Broglie wavelength $\lambda_B = h/\gamma m v$. In the Bohr model, the wave must fit exactly on the circular orbit, $\lambda_B = 2\pi n r_0$ with $n = 1$ for the ground state. In the more advanced solution, that of the Schrödinger equation, the orbital motion is replaced by a radial breathing of the wave, a radical change, but as we all know and remarkable as it is, this goes without immediate consequences for the position of the energy levels. Note that the proton and electron have different masses and also different velocities such that, in the bound state, they have exactly the same De Broglie frequency $\omega_B = 2\pi v/\lambda_B$. The finite mass and motion of the proton ($m_p = 1.67 \times 10^{-27}$ kg $\;(938\ \mathrm{MeV/c^2})$) can be taken into account by replacing the electron mass by the so-called reduced mass:

$$\mu = \frac{m_e m_p}{m_e + m_p} \tag{6}$$

The Coulomb energy is $E_{pot} = -\,e^2/4\pi\varepsilon_0 r_0 = -\,\alpha\hbar c/r_0$, with the fine structure constant defined as

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \approx \frac{1}{137} \tag{8}$$

and $E_{pot} = -2E_{kin} = -\mu v^2$ and also $\lambda_B = 2\pi r_0 = h/\gamma\mu v$. It will turn out that $v \ll c$, so that $\gamma \approx 1$, and for our purpose relativistic corrections are small enough to neglect. Now it follows that

$$r_0 = \frac{\lambda_C}{2\pi\alpha} \approx 0.53 \times 10^{-10}\ \mathrm{m} \tag{7}$$

Of course we immediately recognize this as the Bohr radius. While the centrifugal force balances the Coulomb force on the particles, constructive interference of the quantum wave defines the finite energy of the ground state. It is now also possible to calculate the energy that is carried away by radiation: $E_{phot} = E_{bind} = \alpha^2\mu c^2 = 13.6$ eV. An equal amount of kinetic energy, equally shared by proton and electron, is present inside the atom. The hydrogen atom in its ground state is a fraction $13.6\ \mathrm{eV}/(938.272 + 0.511)\ \mathrm{MeV} = 1.45 \times 10^{-8}$ lighter than the original mass of electron and proton when they were at large separation. On top of that, the same fraction of the atom's internal energy is of kinetic nature.

To get an idea of difference of scale, it is interesting to compare the kinetic energy fraction of the hydrogen atom with a gravitationally bound system such as earth orbiting around the sun. The mass of the sun is $m_{sun} = 1.99 \times 10^{30}$ kg and of the earth it is $m_{earth} = 5.97 \times 10^{24}$ kg. The orbital velocity of the earth is $v_{earth} = 29780$ m/s, so that the kinetic energy is $E_{kin} = \frac{1}{2} m_{earth} v_{earth}{}^2 = 2.65 \times 10^{33}$ J, and the total energy is $E_{tot} = (m_{sun} + m_{earth})c^2 = 1.79 \times 10^{47}$ J. This results in a kinetic fraction $E_{kin}/E_{tot} = 1.47 \times 10^{-14}$, a million times less than for the hydrogen atom.

5.2 The deuteron

We will now climb up a step in the hierarchy of forces to arrive at the next smaller structure: the atomic nucleus. The destructive power of a hydrogen bomb shows that the binding energy of nucleons can be substantial: a ^{4}He atom is approximately 28 $\mathrm{MeV/c^2}$ lighter than the separate neutrons and protons from which it is built, and this corresponds to a mass fraction of 0.0075. Because it is a noticeable fraction, it is known as the "mass defect". The nuclear force harnesses a million times more energy than the Coulomb force.

Where the structure of the atom can be described nicely in terms of electron shells around the nucleus, the behaviour of protons and neutrons in the nucleus is more complicated. The nuclear binding may be modelled by the Woods-Saxon potential that looks somewhat like a square well, but it is not very accurate for low atom numbers. Also the strong spin-orbit interaction of the nucleons complicates things further.

To model the nucleus we will have to improvise a bit, but fortunately we can compare the outcome of our calculations to known experimental results. What we certainly want to know are two things. First, the gain of energy per bound nucleon and second the kinetic energy of the nucleons in the ground state. The shape of the potential is not well known, so we cannot simply use the virial theorem to determine the ratio of potential and kinetic energy. Instead, knowing the size of the nucleus from experiment, we can estimate the De Broglie wavelength. There must be a quantum condition such that the De Broglie wavelength fits the size of the nucleus: $2\pi r_{nuc} = \lambda_B = h/\gamma\mu v$, just the way the electron fits inside the atom. If we apply this to the deuteron, a bound state of one proton and one neutron (which have almost the same intrinsic mass ($m_n \approx m_p$), the reduced mass is $\mu \approx m_p/2$ and we find

$$\gamma\beta = \frac{\lambda_B}{2\pi r_{nuc}}\gamma\beta = \frac{\hbar}{\mu c r_{nuc}} = \frac{2\hbar}{m_p c r_{nuc}} = 2\frac{\lambda_{B,p}}{2\pi r_{nuc}}\gamma_p\beta_p \approx 2\gamma_p\beta_p \tag{8}$$

Note that $\beta = v/c$, that $\gamma = 1/\sqrt{1-\beta^2} = \sqrt{1+\gamma^2\beta^2}$, and also that the velocity in the reduced mass frame is twice as high as that for the individual proton and neutron. Note that the boundary condition $\lambda_B = 2\pi r_{nuc} \approx \lambda_{B,p}$ can only be exact in one frame due to relativistic effects but here the difference will appear to be marginal. In general we can write

$$E^2 = E_0{}^2 + p^2c^2 = m_0{}^2c^4 + \gamma^2 m_0{}^2 v^2 c^2 = \gamma^2 m_0{}^2 c^4 = \gamma^2 E_0{}^2 \tag{9}$$

The total kinetic energy is $E_{kin} = E - E_0 = (\gamma - 1)E_0 = (\gamma - 1)m_0c^2 = (\gamma - 1)\mu c^2 \approx (\gamma_p - 1)2m_pc^2$

The experimental value for the deuteron's charge radius is $r_{nuc} = r_D = 2.14 \times 10^{-15}$ m, so that $\gamma_p = 1.00482$. The kinetic energy of the proton and neutron in the deuteron must therefore be $E_{kin,p+n} \approx (\gamma_p - 1)2m_pc^2 \approx 9$ MeV, which is large compared to the measured total binding energy of a deuteron of "only" $E_{bind,D} \approx 2.224$ MeV. If we were allowed to apply the virial theorem in reverse, then $2E_{kin} = nE_{pot}$ and $E_{pot} + E_{kin} + E_{bind} = 0$ gives

$$n = \frac{-2}{1 + E_{bind}/E_{kin}} = -1.61 \tag{10}$$

This would imply a potential with effectively an $r^{-1.61}$ dependence, steeper than the r^{-1} of the Coulomb potential, and at least this doesn't seem unrealistic.

From experiment it is known that the fractional mass defect of the deuteron is $2.224\ \mathrm{MeV}/(939.6 + 938.3)\ \mathrm{MeV} = 0.0012$, which is moderate compared to the 0.0075 for the helium nucleus. In any case, with sufficient margin of error, we can conclude from the calculation above that the kinetic energy of the neutron and proton is larger than their binding energy. About 0.5% of the deuteron's mass of $1875.613\ \mathrm{MeV/c^2}$ must come from internal kinetic energy. Still 99.5% of it may perhaps be made of "stuff"…

That stuff must then sit inside the proton and neutron, and we are ready to climb another step up the ladder of hierarchy of forces, going down one scale of length. Starting from the solar system, going to the hydrogen atom and the deuteron we see that both the fraction of internal kinetic energy and the fractional mass defect rise very rapidly from 10^{-14} to 10^{-8} to10^{-2}. Now one could argue that maybe we have been too rough with our estimates in the last step and perhaps we are wrong by some factor. It wouldn't matter a bit for the final argument because the trend is clear: the fractional kinetic energy increases each step, not by factors but by orders of magnitude, and at some step on the ladder, the fraction must approach unity, and that is the point where no further granular fragments can serve as building blocks and where a continuum made of some "stuff" must underlie the nature of matter. In the next section we will try to deal with the internal kinetic fraction and stability of the proton.

5.3 The proton

Experimentally, the proton is a stable particle with (internal) structure. It is thought to be built of three quarks (up,up,down), each with spin ½ and a tri-polar charge called "color". The "up" quarks have electrical charge +2/3 and the "down" quarks have electrical charge −1/3. The mass of the quarks may be much smaller than the mass of the proton ($938\,\mathrm{MeV/c^2}$), which can be deduced from the mass of the pion ($139\,\mathrm{MeV/c^2}$). The pion is made of two quarks, a positively charged pion consists of one "up" quark and one anti-"down" quark, so if one of those is very light the other may weigh no more than $139\,\mathrm{MeV/c^2}$, unless a large amount of binding energy may have been drained from the free quark. The free quark?! Nobody has ever seen one, and the proton is very (absolutely) stable. So, yes, this is circumstantial evidence that a lot of binding energy may be involved. Nonetheless, advanced calculations in lattice Quantum Chromo Dynamics[11] confirm that the masses of quarks are quite small: $m_{up} = 2\ \mathrm{MeV/c^2}$ and $m_{down} = 4.8\ \mathrm{MeV/c^2}$, see also Ref. 6. Calculations based on QCD are not trivial because convergence of the theory is poor, and this can be appreciated when comparing Fig. 3 and Fig. 4.

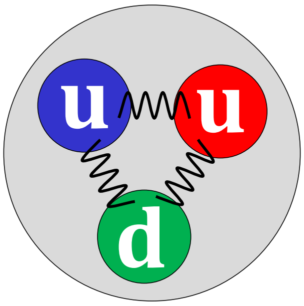

Figure 3. The standard proton model with three quarks (uud) and gluons (wiggly lines). This simple quark model is very powerful in predicting the hadron spectrum of elementary particles.

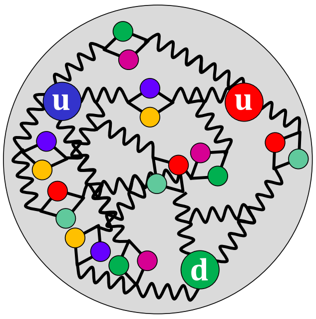

Figure 4. Impression of the distribution of quarks and gluons in the proton as calculated by lattice QCD. The quarks are found to be much lighter than the proton and many gluons and virtual quark-antiquark pairs are required than in the simple 3 quark model, effectively creating a soup.

In Fig. 3, the simple quark model of the proton with three quarks and connecting gluons is shown. The picture of three quarks making a baryon and two making a meson has proven a very powerful model in predicting the hadron spectrum of elementary particles. In Fig. 4 we see what QCD is forced to make of it (see Ref. 11): There are still three valance quarks but there is also a whole network emerging, made of many gluons and virtual quark-antiquark pairs. The quark masses are small and the network carries a substantial part of the energy, momentum and angular momentum (of the proton spin of ½) inside the proton.

A remarkable situation is unfolding. In the previous sections, the sum of the constituting particles invariant mass was always slightly larger than the mass of their bound state, but now the three quarks together weigh hardly $9\ \mathrm{MeV/c^2}$, less than 1% of the proton mass! Again, as for the deuteron, we do not know the binding potential inside the proton, but given the size of the proton we may be able to use a quantization condition to determine the energy level inside the proton, just as we did successfully for the deuteron. This time we propose that the De Broglie wavelength of a single quark must fit on the size of the proton, but because of the spin ½ character of the proton we demand that it should go round twice (this may be debatable but, see Ref. 10 for discussion), so that $2 \times 2\pi r_p = \lambda_{B,q} = h/\gamma_q m_q v_q$ and we find

$$\frac{\hbar}{2m_q c r_p} = \frac{\lambda_{B,q}}{4\pi r_p}\gamma_q\beta_q = \gamma_q\beta_q = \sqrt{\gamma_q{}^2 - 1} \tag{11}$$

With the charge radius of the proton $r_p = 0.842 \times 10^{-15}$ m, it follows that the quarks must be moving at highly relativistic speed: $\beta_q \approx 0.999$, this makes that their De Broglie wavelength fits inside the proton. Note that because of this high speed this is rather independent on the quark mass. We find that $\gamma_q m_q \approx 116\ \mathrm{MeV/c^2}$ so that the kinetic energy of the three quarks together is $E_{kin,uud} \approx 348$ MeV. A single quark carries approximately 12% (116/938) of the internal momentum in the proton and that is in reasonably good accordance with experimental findings from deep-inelastic muon scattering[12]. This means that our simple assumptions are holding well enough so far. Instead of using the charge radius we could also have taken half the Compton wavelength of the proton, $\lambda_C = 1.321 \times 10^{-15}\ \mathrm{m} = 0.784 \times 2r_p$, only a moderate difference but the quarks would have to move even faster.

There is still 590 MeV of energy inside the proton that is unaccounted for. It may be due to the stresses (such as in a loaded spring) to hold the proton together. Or is it the "stuff" we are looking for? The energy in the electric field outside the charge radius (see Appendix B) is found to be $E_{EM,q} \approx 0.856$ MeV, so that is a mere 0.1% of the proton mass. There is also the energy in the magnetic dipole field, $E_{EM,\mu} \approx 0.417$ MeV, which makes a total of 2.5 electron masses of electromagnetic field outside the proton charge radius and this incidentally is the same amount of mass as the mass difference between proton and neutron (but in the opposite direction!). And then there is the total rest mass of the quarks, according to QCD a mere $E_{quarks} \approx 8.8$ MeV.

But isn't that very strange! If the quarks are so light, then inside them there must be a rather weak force at work compared to the strong force. The quarks are then not at the end of the hierarchy ladder but come a step earlier. We have already concluded that they can only fit within the proton if moving very fast, and that may be true for their De Broglie wavelength and orbital motion, but what about their own size, their Compton wavelength size? Too light, hence too big, and the proton would have to be a hundred times larger than it is.

Now what if quarks are actually small, with high masses as compared to the proton mass (and hence QCD would be wrong)? Then a force must be present that is stronger than the strong force, with more energy involved in the binding process and even more kinetic energy to closely bind everything together. Where then should that energy come from? Already a fraction $E_{kin}/E_{tot} = 348/938 = 0.37$ of the total proton energy is kinetic energy.

Starting from the solar system, going to the hydrogen atom, the deuteron and now the proton we see that both the fraction of internal kinetic energy E_{kin}/E_{tot} and the fractional mass defect rise very rapidly from 10^{-14} to 10^{-8} to10^{-2} to ~1. This suggests, on grounds of energetic balance, that there are no particles or structures smaller than the proton that can exist independently, they simply cannot be stable. Hence we must say goodbye to quarks as independent existing particles. Already, it seems to be an experimental fact that quarks cannot exist on their own, so that is fine, but surely we would like to keep the beautiful and powerful quark symmetries that make the full set of hadrons.

We are forced to admit that we have arrived at the end of the ladder, but how can we save the day? It certainly seems that quarks cannot be small and heavy, and at the same time they cannot be light and big, what then? We may want them to be small and light simultaneously, as is the case in QCD. What if the quarks really do not exist as such? Maybe the quarks are strung together to provide some of the symmetries they are expected to. Perhaps the quarks are not coming in granules and have no rest mass, but come together as a continuum of some form of energy that propagates close to (or at) the speed of light. If so, their orbital wavelength (the De Broglie wavelength) can become arbitrarily small without the need for a granule having a size of the Compton wavelength.

An artist impression of a daring looking possibility is given in Fig. 5, it is a trefoil knot of continuous energy flow. It goes around twice at (close to) light speed before closing on itself. The three differently colored loops should be imagined as orthogonal in space. If the underlying structure of the energy flow is a vector field, such as for electromagnetism, each of the loops may have different properties according to their amount of twist (not shown in the figure). If we were to pull one loop it would possibly tighten up the other loops. Both the loop and its knot take part in a spring that binds things together, and may replace the gluon properties. It is all speculation, but it shows there may be a way out. In fact, there must be a way out, the proton really exists!

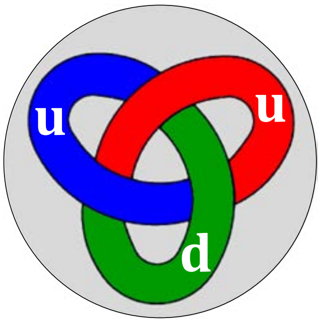

Figure 5. The proton modeled as a trefoil knot of continuous energy flow at (close to) light speed. Note that the flow goes around the proton model twice. The three differently colored loops should be imagined as being orthogonal in space. If the underlying structure of the energy flow is a vector field, such as for electromagnetism, each of the loops may have different properties according to their amount of twist.

6. DISCUSSION

From our exercise we must conclude that we have arrived at the smallest length of stable matter, and it is represented by the proton. This is also an experimental fact; the neutron is only slightly heavier and smaller, but unstable. The electron is stable, but it is much lighter with a much larger Compton wavelength and its classical radius is larger than the charge radius of the proton, see Appendix B. From our exercise it is manifest that the Planck length $l_P = 1.62 \times 10^{-35}$ m cannot play a fundamental role in the structure of (stable) matter. In string theory it is suggested that structures may exist that are of the Planck scale, but according to our view, small structures are energetically impossible.

Coming back to the simple quark model of Fig. 3 and comparing this to the much more complicated QCD quark-gluon soup in Fig.4, we could ask why QCD may be such a poorly converging theory and why does it require so much soup? What if the speculation of Fig. 5 has actually some truth in it? Then, maybe QCD simply starts from the wrong end trying to make a granular structure converge to a knotted continuous flow with closed loops, twist, knots and whirls. In the bag model of atomic nuclei it has always been mysterious why the separate nucleon membranes (of Fig. 4) do not fuse, where the topology of Fig. 5 makes that obvious. Over the years, many papers on the subject of topological electromagnetism have been published[10,13-16]. A most recent paper[17] provides a new theoretical foundation based on the Clifford algebra of space-time that merges relativistic quantum mechanics and topological electromagnetism in which the sources are both given as “knotted” electromagnetic fields and as quantum mechanical wave functions.

Let us look at the general equation for a particle’s energy balance, including the rest mass $E_0 = m_0 c^2$ of internal granularity:

$$E_{tot} = mc^2 = E_{pot} + E_{kin} + E_0 = E_0 - E_{bind} \tag{12}$$

Depending on the binding potential, assuming that it obeys the virial theorem $2E_{kin} = nE_{pot}$, the ratio of kinetic energy and total energy can be expressed as follows:

$$\frac{E_{kin}}{E_{tot}} = \left(1 - \frac{E_0}{E_{tot}}\right) \Big/ \left(\frac{2}{n} + 1\right) = \left(-\frac{E_{bind}}{E_{tot}}\right) \Big/ \left(\frac{2}{n} + 1\right) \tag{13}$$

If we apply this to the proton, with $E_{kin}/E_{tot} = 0.37$ and $E_0/E_{tot} = 0.01$, we find that $n \approx +1.2$, this is very close to the correct value $n = +1$ which is required to make the force on the quarks by the gluons independent of distance. If we substitute $n = +1$ in Eq. (13) we find $E_{kin}/E_{tot} = 0.33$ and $E_0/E_{tot} = 0.01$, still consistent with our argument that the amount of kinetic energy is substantial. Nonetheless, presuming a potential of the form r^n may be too limited a view and we may have to consider the possibility of a so-called pseudo-potential, the strength of which is not only dependent on position r but also on state of motion of the system. An example of such a potential is that of an electric charge in a magnetic field, for which the (Lorentz) force is dependent on the velocity of the charge.

What does the general equation (13) say about pair creation/annihilation? For the electron-positron pair, we can identify that $m = m_e + m_p$ is the total rest mass and $E_{bind} = -2E_\gamma$; the photons need to be absorbed, not emitted to create the a particle pair. There is no additional rest mass ($E_0 = 0$) of any internal granularity:

$$E_{tot} = mc^2 = E_{pot} + E_{kin} + E_0 = E_0 - E_{bind} \tag{14}$$

Indeed, our equation simply applies and we see that the electron and positron consist of potential and kinetic energy without any sign of granularity. Here too, the kinetic energy can be thought to be carried by a continuous internal circulation of energy, the potential energy provides the spring or bag to hold it together, see also Ref. 10.

Eventually, to obtain a picture that is consistent with known physics, the deformation or disentangling of the loops and knots of these light-speed energy vortices must somehow be related to the electromagnetic, the weak and strong force. Now we need to go back to our first answers in Section 3, where we have found that any "stuff" is energy and it must couple to, at least, one of the other forces. Because the positron and electron exhibit electromagnetic and weak interaction, but not the strong interaction, we must assign at least an electro-weak nature to stuff. It seems that, based on the literature[10,13-16] and within the limits of arguments of this paper, the nature of stuff is consistent with some topological form of electromagnetism which may or may not require us to change our view on the structure or nature of space-time. In particular, in this topological electromagnetism it may be so that the linking of electromagnetic field lines and the knotting of energy flow are related to the weak and strong forces.

7. CONCLUSIONS

In this paper we have investigated the nature of "stuff". In Sections 2 and 3 it has been concluded that energy is part of "stuff", or some form of "stuff". Then, it was found that the nature of the "stuff" inside elementary particles requires it to be more than only gravitationally coupled: it must at least couple to electromagnetism or the weak force, otherwise binding of stuff to the particle is too weak and it cannot follow the particle. Further it became clear that "stuff" must be continuous and in case there is only one form of "stuff", that it is some, perhaps alternative or extended, form of electromagnetism. If there is also "anti-stuff", then we may need space-time itself to have a non-trivial structure, something that may be true regardless of our findings.

At all length scales the structure of stable matter is the result of a balance of forces working between some otherwise bound objects, particles or granules. In Section 4 we have argued how the hierarchy of strength of interactions is connected to the length scale of material structures. The stronger forces go first and they make smaller, more tightly bound objects.

In nature, the following is observed (see Fig. 1.): the total mass of a lump of ordinary matter decreases with smaller size, but contrary to that, the mass of subatomic particles increases with smaller size. The electron is at the bottom of the mass scale. Most subatomic particles are unstable, except for the electron and proton.

In this paper two insights about bound systems have been combined. First, that with decreasing size there is a systematical increase of the fraction of kinetic energy (internal dynamics) and binding energy (energy lost at formation) with respect to the total energy (rest mass). Second, that the stronger forces are getting balanced at cost of the larger part of the energy available. In Section 5 we have shown indeed, by example, that the ratio of internal kinetic energy and total energy increases with decreasing size. The consequence is that stable matter cannot exist at smaller length scale than where internal kinetic energy and total energy are of comparable magnitude: $E_{kin}/E_{tot} \approx 1$. At that point the orbiting corpuscular, granular structure with binding forces runs out of internal binding energy to hold the kinetics and from there only a "fluid"-like continuum flow of energy seems consistent with energy conservation. Consistent with experiment and supported by theoretical estimations it can be inferred that this lower limit is given by the size of the proton, and that the proton is the smallest stable particle. Quarks as independent constituents of the proton are ruled out, and so is Planck-scale physics.

Given the large fraction of kinetic energy circulating inside the proton, it is then proposed that the proton's internal dynamics must be essentially a light speed knot of energy. That energy is the "stuff" we were after, and it is continuous and interacts with electromagnetism. Indeed, within the limits of argument of this paper, the nature of "stuff" is consistent with some topological form of electromagnetism[17] which may or may not demand us to change our view on the structure or nature of space-time.

APPENDIX A: THE MASS OF A CLOSED SYSTEM: LIGHT IS HEAVY

Suppose we have a closed black box and inside, so we are told, there are loaded springs, light bulbs, batteries and motors; a lot of motion and the box is filled with gas too. Our assignment is to measure its total energy.

The solution is rather simple; we accurately weigh the box! To do this correctly, to weigh properly, one should put it on a balance to eliminate gravitational differences depending on location on the planet. Also, one should avoid buoyancy effects by either putting the balance in a vacuum or by using a same size box on the other scale. Then, the scale itself may have to be dampened, or made very heavy (symmetrically) to be able to deal with objects bouncing inside our black box. Finally, weigh for long enough so that these fluctuations average out and the balance is in proper balance[5].

The claim is that proper weighing gives the total energy of a closed system by $E = mc^2$. See Fig. 6 for an artist's impression. The meaning of Einstein's famous formula $E = mc^2$ is that *energy is equivalent to mass*. Energy is NOT another form of mass, and the formula does NOT describe a transmutation of mass into energy. Confusion and sloppiness about this (see for example the otherwise excellent paper, p.3 of Ref. 6) seem to justify further explanation.

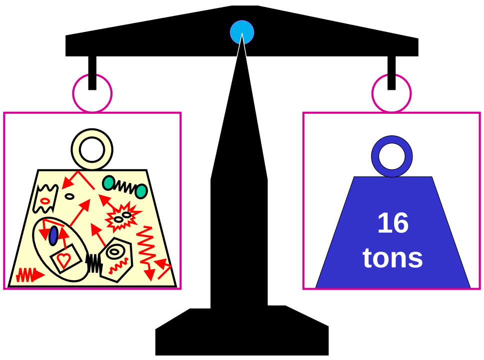

Figure 6. Whatever the matter, we can weigh properly, using a balance, to get the total energy. Springs, boxes, radiation, bouncing balls and gasses…there is energy in the binding, in the kinetics, in the field, etcetera.

A rather instructive example is provided by the proper weighing of a gas[5]. In a closed vessel, the pressure is higher at the bottom than at the top, exactly as is the case for the atmosphere, due to the downward gravitational (de)acceleration:

$$P(h_{top}) = P(h_{bot})e^{-\frac{mg}{kT}(h_{top}-h_{bot})} \qquad \text{(A1)}$$

The difference in pressure $P(h_{top}) - P(h_{bot})$ between top and bottom as a result of the individual impulses of the gas molecules represents exactly the total mass of the gas contained in the vessel. For example if the vessel is a rectangular box with horizontal area A and volume $V = A(h_{top} - h_{bot})$ and is filled with an ideal gas of N molecules of mass m, so that $P = kT\,N/V$ then the force $F = A[P(h_{top}) - P(h_{bot})]$ on the box is:

$$F = -\frac{mg}{kT}VP(h_{bot}) = -Nmg \qquad \text{(A2)}$$

This is exactly the right answer, of course. In case of a very dilute gas, when the molecules do not collide with each other but only with the walls of the box, we could perhaps better have started from the force due to the change in the parallel component of the momentum when a gas molecule bounces from top or bottom per round trip time.

$$F = F_{top} + F_{bot} = \frac{\Delta p_{top} + \Delta p_{bot}}{\Delta t} \qquad \text{(A3)}$$

Instead of a dilute molecular gas, we may now just as well consider a photon gas in a box, and weigh it. Electromagnetic radiation comes in photons of energy $E = \hbar\omega$ and momentum $p = \hbar k = \hbar\,\omega/c$. This implies there is an inertial mas associated to a photon of $m_i = E_{photon}/c^2 = \hbar\omega/c^2$, see also p.204 of Ref. 3. By the principle of equivalence this must correspond to a gravitational mass $m_g = m_i$ of the same magnitude, and indeed light is gravitationally deflected, as was first observed by Eddington during the 1919 solar eclipse.

From general relativity and as proven by the Mössbauer effect it is well known that gravitation will cause a red shift on those photons that go up, and a blue shift on those that come down. The gravitational Doppler shift can be calculated from the instantaneous velocity of top and bottom of the box using special relativity. We determine the mass in the rest frame of the box in which the photon round trip time is $\Delta t = 2t$ and the momentum transfer in the instantaneous frame of the box, that is the rest frame for reflection at the top where $\Delta p_{top} = 2\hbar k_{\parallel}$ (think of it as the point of suspension on the balance) and an accelerated frame at the bottom for which $a = v/t = g$ (the instantaneous velocity is v) and the momentum transfer is $\Delta p_{bot} = 2\hbar(k_{\parallel} - \omega v/c^2)$.

$$F = \frac{\Delta p_{top} + \Delta p_{bot}}{\Delta t} = -\frac{\hbar\omega}{c^2}\frac{2v}{\Delta t} = -\frac{E_{photon}}{c^2}\frac{v}{t} = -m_i g \tag{A4}$$

The Doppler formulas from special relativity combined with the principle of equivalence $m_i = m_g$ shows what should be obvious: that light is heavy!

$$m_g = \frac{E_{photon}}{c^2} \tag{A5}$$

Although true in a sense, there is little meaning in the statement that light has no rest mass, since it is never at rest and always propagating at the speed of light. It may seem very hard to keep light fixed on a scale because it is always on the move. But here we have shown that by putting it in a box one can actually weigh a photon and prove the case that indeed light has gravitational mass.

Confusion arises when the words “mass” and “matter” are interchanged. Matter can be transformed into radiation, but the total energy and mass each remain strictly conserved. Annihilation of an electron-positron pair to two gamma rays in a box on a balance will not show a change in the average mass reading!

Whatever we weigh, if we weigh properly, we will find its total gravitational mass, and this will reveal its total energy.

APPENDIX B: ELECTROMAGNETIC MASS OF A CHARGED SPHERE

This subject is very well treated by Feynman[9]. Imagine a homogeneously magnetized sphere of radius a with mass m_s and surface charge q. The electric field $\vec{E}$ inside the sphere is zero and outside it is radially directed:

$$E_r(r) = \frac{q}{4\pi\varepsilon_0 r^2}, \qquad E_\theta = 0, \qquad E_\varphi = 0 \tag{B1}$$

The magnetic dipole outside the sphere has components:

$$B_r(r,\theta) = \frac{\mu_0 s g \mu_d}{4\pi r^3} 2\cos\theta, \qquad B_\theta(r,\theta) = \frac{\mu_0 s g \mu_d}{4\pi r^3}\sin\theta, \qquad B_\varphi = 0 \tag{B2}$$

with $\varepsilon_0\mu_0 = c^{-2}$, s the spin, g the gyromagnetic ratio and $\mu_d = q\hbar/2m_s$ the corresponding magneton. The electromagnetic field has an energy density of

$$W = \frac{\varepsilon_0}{2}(E^2 + c^2B^2) \tag{B3}$$

So that the total energy in all of the field (all outside the sphere), also called the self-energy, is

$$U = \int_{r>a} W\,dV = \frac{q^2}{8\pi\varepsilon_0 a} + \frac{\mu_0 s^2 g^2 \mu_d{}^2}{12\pi r a^3} = \frac{\alpha m_s c^2}{2}\left[\frac{\lambda_C}{2\pi a} + \frac{s^2 g^2}{6}\left(\frac{\lambda_C}{2\pi a}\right)^3\right] \tag{B4}$$

Because mass and energy are equivalent quantities, the sphere has an extra amount of mass, called the electromagnetic mass $m_{EM} = U/c^2$. This comes on top of the “bare” mass m_s (which may be the mass of “stuff”), hence its total mass is

$$m = m_s + \frac{U}{c^2} \tag{B5}$$

The integral (B4) diverges for $a \to 0$, in other words, for a fixed amount of charge, the energy in the field may be become infinitely large if we let the sphere shrink to a point. Hence the total mass of the sphere may become infinitely large as well. If we take one elementary charge $q = e$, and shrink the sphere such that $a = 1.41 \times 10^{-15}$ meter, then we find that $m_{EM} = 9.11 \times 10^{-31}$ kg, which is the mass of the electron. Historically, the classical radius of the electron is defined as $r_e = 2a = 2.82 \times 10^{-15}$ meter (also $r_e \equiv \alpha\lambda_C/2\pi$, and note that the Compton wavelength of the electron is its quantum mechanical size). Simply speaking, this means that if the electron really has this radius, then half of its mass is of electromagnetic origin. Then the other half may be involved with binding the charge together and give the electron its stability. For example, in the case the sphere is just a charged soccer ball, the leather will stretch a bit due to the repelling charges at its surface. As a result some elastic energy is stored in the ball, and hence there must be a mass of elastic origin too. Note that the magnetic dipole part of U diverges even more rapidly than that of the electric part. In case of the electron, $s = \frac{1}{2}$ and $g = 2.0023$, for the proton, $s = \frac{1}{2}$ and $g = 2 \times 2.79282 = 5.585564$.

Often it is thought that the electron is infinitely small, as small as a mathematical a point. From the above it is clear that there are some problems associated with that, one is known as the self-energy problem, another as the ¾ problem and they are related to the mass and binding energy (the so-called “Poincaré stresses”) respectively. Why does the electron behave as an object without structure, even at length scales down to 10^{-18} meter? Nobody really knows, but an attempt is made in Ref. 10. For further discussion see p.28-12 of Ref. 9, and also Ref. 4.

Acknowledgements

The author would like to thank John Williamson, Meint van Albada, Dick de Boer, Mischa Megens, Julie Fleischer and Martijn Goossens for pleasant and fruitful discussions.

REFERENCES

[1] W. Thomson, “On Vortex Motion”, Trans. Roy. Soc. Edin. **25**, 217-260 (1868).
[2] N. Bohr, “On the Constitution of Atoms and Molecules”, Philos. Mag. **26**, 1-25 (1913)
[3] Hermann Weyl, Space-Time-Matter, page 200 (Dover publications, 1922, 1950).
[4] Frank Wilczek, “The enigmatic electron”, Nature **498**, 31-32 (2013).
[5] M.B. van der Mark and G.W. ’t Hooft, “Light is heavy”, Van A tot Q, NNV november (2000).
[6] Frank Wilczek, “Origins of Mass”, arXiv:1206.7114 [hep-ph] (2012).
[7] M.V. Berry, Principles of Cosmology and Gravitation, page 37 (IOP Publishing Ltd 1989).
[8] R.M. Kiehn, “Periods on manifolds, quantization, and gauge”, J. Math. Phys.**18**, 614 (1977).
[9] R.P. Feynman, R.B. Leigton, M. Sands, “Electromagnetic Mass”, The Feynman Lectures on Physics, Vol. II, Chap. 28, (Addison-Wesley, Reading, 1963).
[10] J.G. Williamson and M.B. van der Mark, “Is the electron a photon with toroidal topology?”, Ann. Fond. L. de Broglie **22**, 133 (1997).
[11] C.T.H. Davies, C. McNeile, K.Y. Wong, E. Follana, R. Horgan, K. Hornbostel, G.P. Lepage, J. Shigemitsu and H. Trottier, **“**Precise Charm to Strange Mass Ratio and Light Quark Masses from Full Lattice QCD**”,** Phys. Rev. Lett. **104**, 132003 (2010).
[12] J. Pumplin, D.R. Stump, J. Huston, H.L. Lai, P. Nadolsky, and W.K. Tung, “New Generation of Parton Distributions with Uncertainties from Global QCD Analysis”, Fig. 1, arXiv:0201195v3 [hep-ph]., (2008).
[13] H. Jehle, “Flux Quantization and Particle Physics”, Phys. Rev. D6, 441 (1972).
[14] A.F. Rañada, “Topological electromagnetism”, J. Phys. A: Math. Gen. **25**, 1621-1641 (1992).
[15] T. Waite, “The Relativistic Helmholtz Theorem and Solitons”, Physics Essays **8**, 60 (1995).
[16] W.T.M. Irvine and D. Bouwmeester, “Linked and knotted beams of light”, Nature Physics **4**, 716 (2008).
[17] M.B. van der Mark, “Quantum mechanical probability current as electromagnetic 4-current from topological EM fields”, paper [9570-35], SPIE Optics + Photonic, San Diego, 9-13 August 2015.