On the nature of the photon and the electron

J.G.Williamson[a]

[a] University of Glasgow, College of Science & Engineering, Glasgow G12 8LT, Scotland;

ABSTRACT

A new theory, describing both light and material particles, is proposed. A principle is adopted which brings the underlying relativistic properties of space and time into the theory at a fundamental level. An equation encompassing the usual free-space Maxwell equations but similar in form to the Dirac equation is proposed. This equation has new kinds of solutions. Propagating, pure-field solutions may have arbitrary energy, but must all have the same angular momentum. These are identified with the physical photon. Solutions with rest-mass force the field into re-circulating vortex-like solutions. The minimum energy configuration "rectifies" the oscillating electric field of light into a uni-directional, radial (inward or outward directed) configuration. The resulting apparent external charge may readily be calculated and is found to be close to the elementary charge. The spin may, likewise, be calculated, and is found to be half integral with a double-covering internal symmetry. Charge is then not a fundamental quantity in the theory - but is a result of the way field folds from a bosonic to a fermionic configuration. The simplest such charged, fermionic particles are identified with the electron and positron.

Keywords: Electromagnetism electron positron photon

1. INTRODUCTION

A companion paper has outlined many aspects of the theoretical basis to be used here.[14] This paper starts where the companion paper left off. In the preparation of that paper and in a discussion with many physicists all over the world, primarily in the context of the 2015 SPIE conference on the nature of light for which both are submissions, it became clear that the understanding of many terms in common usage in physics carried almost as many different interpretations as there were physicists. These were not esoteric terms (which tend in fact to be better defined) , but common concepts such as "energy", "rest-mass" and "electromagnetic field". This led to many enjoyable discussions as to just what these terms really meant. Such discussions generally tended to generate more heat than light. Accordingly here, for the avoidance of doubt, a new nomenclature has been made-up to allow the precise and rigorous local definition of terms within the limited context of this paper - without bringing in too many pre-conceptions of what these terms may or may not mean.

Space-time, at its simplest, is described by four and only four "linear" degrees of freedom. It has been fashionable in science, for the past half-century or so, to take complicated starting positions involving an extensive a-priori mathematical and conceptual structure. Complex groups have been taken as the starting points of many theories too numerous to mention. Further, there has been a tendency to wish to "quantise" everything from the beginning. It seems to me self-evident that, in doing that, one loses the possibility of finding out why such things may be quantised at all. Putting in a quantisation observed experimentally as a starting axiom has some merit of course, but is a poor starting choice if one wants to understand the origins of that quantisation. In this paper an attempt will be made to avoid any superfluous complexity, keep everything possible continuous and linear and keep the a-priori basis as simple as possible. Accordingly, all that will be introduced a-priori are four basis "directions" in space and time, their properties under multiplication and division, and how energy (introduced a square-root energy density and denoted here "vot") flows between these line directions and such things as planes and volumes derived from them. If, on top of this certain quantised values derived from experiment are then introduced on top of this, such as the elementary angular momentum for example, it will be shown how certain others may be derived in turn from them, such as the elementary charge.

Further author information: (Send correspondence to J.G. Williamson)
J.G. Williamson: E-mail: john.williamson@glasgow.ac.uk, Telephone: +44 141 330 4923

2. OUTLINE OF THE STARTING BASIS

Four unit vector components $\alpha_0, \alpha_1, \alpha_2, \alpha_3$ representing one temporal and three spatial unit lines respectively are introduced. These are labelled α_0 for time, and α_1, α_2 and α_3 for three perpendicular directions in space. These latter three may conveniently be thought of as being the unit "x", "y" and "z" of Cartesian space. The Maxwell equations (and any other linear equations) are, however, equally valid an any proper, conformal, orthonormal system of co-ordinates. Here we will use the right-handed ordered triple$(1, 2, 3)$ for the general case, (x, y, z) for Cartesian, (r, ϕ, z) for cylindrical, (r, θ, ϕ) for spherical and (ρ, θ, ϕ) for toroidal co-ordinates.

Under strict definitions[13,14] of "multiplication" and "division" these lead to self-consistent relativistic properties of further unit elements derived from them. These further elements represent elementary points, planes, volumes and 4-volume derived from their products and quotients. Because the space-time basis has four linearly-independent base unit elements, under multiplication twelve further such unit geometrical objects are generated. Provided that "division" maps to the same set, this leads to sixteen linearly independent units elements in all. This may seem dreadfully complicated, even larger than the 11 dimensions in some string theories, but these unit objects are quite simple, most of them being quite familiar to a child of three. They are just such things as unit planes and volumes, with only the unit scalar (point), quadrivector (4-volume) and spatio-temporal planes requiring much more thought and mathematical explanation.

It is neither desirable nor necessary to introduce a-priori any group on these other than that implicit in the algebra of points, lines , planes, volumes and a 4-volume. In the usual parlance field one might talk of introducing the Lorentz group at this point (but perhaps not the Poincare group - since general translations lie outside the allowed light-cone of the restrictions of absolute relativity). The closest match would be to introduce the Clifford group corresponding to the space-time algebra Cl(1,3), but that is not a division algebra, which is widely thought to cause problems.[5] Imposing that may lead to confusion in other ways, however, as many associations and effective projections have become common in the literature.[?,2]

An algebra may be designed, however, such that it is strictly simpler than it would be were the general consensus of the meaning of a "group" were imposed. It is intended that the theory to be developed should be local, causal, linear and unitary in each and every proper Lorentz frame. Right up to the limit of the speed of light. It would be nice if theory could be made both unitary and relativistic - a prize which has proven elusive in standard representations even of just the Lorentz group. It is the view of the author that the technical problems inherent in this lie in the imposition of a mathematics which differs from the minimal observed physically. In practice, one allows definitions of multiplication and division into the popular definition of such groups which may not have a proper underlying parallel in reality. Just what is the proper meaning, for example, of the "division" of one component of a 4-vector by another component of the *same* 4-vector? For example by dividing space by time? This is the velocity, but a consensus on the meaning of "velocity" itself provokes strong discussion, even within a narrow group of professional scientists, on such things as the meaning of the velocity of light.

Further, is the direction of velocity, or is this not, in the same direction as if one calculates its inverse and divides time by space? the only proper arbiter to this question is experiment and, here, the answer to this question would be yes - at least in the narrow field of experiment well described by the Maxwell equations, since in the Maxwell equations divisions and their inverses turn out to be equated. The intention here is to keep the mathematics as minimal as possible. This will mean that, for example "multiplication" and "division" will be defined for the individual unit elements but "addition" and " subtraction" will not. These properties will be ascribed to a further degree of freedom, independent of the geometrical space-time form and represented by real number quantities. If the reader finds keeping in mind this kind of separation too uncomfortable, read no further. The judgement of whether this distinction has had any value must be considered in retrospect.

To introduce the proper (Lorentz-like) properties of space and time, multiplication of the temporal component α_0 by itself is defined such that it yields the positive invariant scalar unity $\alpha_0^2 = \alpha_P$ and the three spatial components α_1,α_2 and α_3 square to the negative scalar unity $\alpha_i^2 = -\alpha_P$. Here and in the sequel, Greek indices run from 0 to 3 and Roman from 1 to 3, with 0 representing the time "direction". That is, it is taken that the square (or quotient) of the base line elements with themselves yields a unit "point" element. This is a point as opposed to a line or a plane, not in the common meaning of an infinitesimal place. In English the word "point" is often used to denote a place with zero extent - sometimes, perhaps incorrectly, called a mathematical point.

Such points are (usually) frame dependent. In any event, trying to imagine a place which is as close as you like to not there at all is likely to lead to wooly thinking at best and to total confusion at worst. Here, it is considered that a place, for any given observer, is better defined by a relative position vector with respect to that observer. This is a line vector. Vectors are not invariant under a general Lorentz rotation. For example, a simple, positive 90-degree planar rotation in the 1-2 plane will convert a vector in the 1 direction wholly to one in the 2 direction and one in the 2 direction to the negative 1 direction. A Lorentz boost will convert elements of time into elements of space and vice-versa. It is the author's view that a physical point, especially the point corresponding to the point of view of an observer, is better defined with respect to its property of invariance under (general) Lorentz rotations than as the limit of a line (or volume) tending to zero length (or capacity). Here, the unit scalar point α_P is not taken to be a place or an event, it is simply that frame-independent unit sized element invariant under a general Lorentz rotation or boost (and also under several other transformations such as an order inversion).

In previous work the (square-root) rest-mass component, which takes the geometrical form α_P in the new theory, has been called the pivot.[12,13] To make a connection with the earlier work a new word has been invented, which may be used either alone or as a suffix. Many particles end with the suffix -on, this new concept is designed to express an underlying nature (of energy) even more elementary than that of particles - energy as conditioned by the space-time nature of elementary forms. The new word (and suffix) to be used here is "vot" and the nomenclature for the scalar element here will be the p-vot. The word has been chosen, not only because of its convenience in making the connection with earlier work, but also because it has no adverse meaning in any of the languages of which the author is aware. Apologies are extended in advance if this should prove to be so in any of the (majority) of languages of which the author is unaware. The closest existing conceptual description to an element of p-vot is to think of it as a square-root mass-energy density. This is similar in some ways to the probability density (wave-function) term in non-relativistic quantum mechanics often denoted Ψ. To derive the energy, such forms must be squared and integrated over a relevant volume. Anything taking the same geometrical form as the p-vot form is strictly always invariant under a Lorentz transformation; the symbol "Ψ" and the concept of a wave-function are often used in the literature (perhaps confusingly) to describe things which are not. The p-vot may take, just as is the case for any field or density component, any magnitude.

Fundamentally, anything with the geometrical form of a vector, in the directions of α_0,α_1,α_2 and α_3, transforms as a 4-vector. Charge-current (square root) 4-vector density or 4-vector potential may be denoted c-vot and a-vot respectively. Six unit "areas", properly denoted bi-vectors, are derived by multiplying mutually perpendicular unit lines: $\alpha_{23}, \alpha_{31}, \alpha_{12}, \alpha_{10}, \alpha_{20}, \alpha_{30}$. The algebra forces these to transform relativistically as do the (1,2,3,) components of the magnetic (B) and electric (E) field respectively.[6] Because of this, vot field with this form is denoted here f-vot with the three components of the electric field denoted E-vot and those of the magnetic field B-vot. Once again the base form of the fields is of a square-root mass-energy density, as is anyway conventional for fields. One must square them and integrate over a volume to get an energy. Again, this may seem unnecessarily complicated at this point, but it will transpire that the Maxwell equations are vector differentials of the field - properly with vector and tri-vector components as will be discussed below. To get the total field energy one must (and always did), translate to the right units, square the basic field quantities (f-vot) and integrate over the relevant volume. The 4 tri-vectors (the dual of the vectors) represent unit "volume" elements (α_{123},α_{023}, α_{031}, α_{012}). The latter three are a momentum density multiplied by a perpendicular unit vector, and therefore transform as the components of an angular momentum density. These are denoted t-vot. Finally, there is a single quadri-vector, (α_{0123}) which, just as the scalar, is invariant under a Lorentz transformation but, similarly to the unit imaginary i squares to negative unity $\alpha_{0123}^2 = -\alpha_P$. Square-root energy with this form would be denoted q-vot.

Using this kind of algebra proves to provide a better parallel of the physical way energy flows through space and time. The principle of absolute relativity is that no quantity may appear without its proper geometrical form. For those wishing to gain further insight into the properties of the sub-algebra than that expressed in the companion paper, the sub-algebra of the space-time unit elements amongst themselves parallels the Clifford algebra $Cl(1,3)^2$ championed by Hestenes as the space-time algebra in internal models of the electron structure[11] and is a simplification, in some ways, of certain Dirac algebras.[1] The closest algebra discussed in the literature is sometimes called the Clifford-Dirac algebra and has been well-described in recent popular books[5] or more extensively in recent textbooks.[2]The precise algebra used here, however, encompasses certain extra restrictions not usual in standard approaches with either. The reader is referred elsewhere for details.[6,13,14] In this paper, the consequence of adding a seventh (and eventually eighth) term to the six components of the electromagnetic field in this formalism, corresponding to a dynamical scalar (or pseudo-scalar) rest-mass, will be discussed. The resultant effect on the electromagnetic field is to cause it to deviate in the direction of the electric (or magnetic) field. A solution with radial electric field then allows a solution with a re-circulation of electromagnetic energy. An electromagnetic vortex in momentum space. The re-configuration of the field forces the electric field - oscillating in the photon, to become rectified in the sense that it becomes either radially inward-directed (electron-like) or outward directed (positron-like). The method and topology of the confinement, allows pure-field to re-configure in the process of the creation of particle-antiparticle pairs of opposite charge and half-integral spin from uncharged photons with integral spin. Conversely, the annihilation of particle-antiparticle systems to photons cancels the topological vortex-anti-vortex pair in the recirculating system - allowing the particle-antiparticle system to decay to two free photons and conserving both charge and angular momentum. It will be claimed that the new insight and the new solutions parallel more closely that which is observed in experiment than do alternative approaches.

3. EXTENSION OF THE MAXWELL THEORY

The unit elements give the proper form and the relativistic transformation properties of the vector and hence the scalar and the the bi- tri- and quadri- vectors, but to represent an actual quantity these must also have a magnitude. For this real numbers are used - keeping an explicit separation between the amount (of a scalar energy for example) or magnitude (of a vector force for example) and its proper form. This is not merely a question of style, it is this rigour that allows the imposition of absolute relativity at all levels and that gives rise to the new results of light quantisation and the formation of charged fermions from uncharged bosons in the following.

A four vector is written ($\mathbf{v} = a_0\alpha_0 + a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3$). The a_μ are (positive definite) real number quantities expressing a magnitude and the α_μ are fundamental, invariant unit lines which may take at most the two values $\pm\alpha_\mu$. Here and in the sequel, Greek indices run from 0 to 3 and Roman from 1 to 3, with 0 representing the time "direction".

For Cartesian co-ordinates a 4-vector 4-differential is defined as:

$$\begin{aligned} \mathcal{D}_4 &= \frac{\partial}{\alpha_\mu \partial x_\mu} = \partial_\mu/\alpha_\mu \\ &= \alpha_0\partial_0 - \alpha_1\partial_1 - \alpha_2\partial_2 - \alpha_3\partial_3 = \alpha_0\partial_0 - \alpha_i\vec{\nabla} \end{aligned} \tag{1}$$

Where the subscript on the left denotes the number of distinct unit elements in the expression. The over-arrow denotes a conventional three-vector. Note the change of sign of the space components due to the implicit quotient of the unit vectors and the fact that the three spatial vectors α_1,α_2 and α_3 square to the negative scalar unity $\alpha_i^2 = -\alpha_P$.

The 4-differential of a 4-vector potential yields field components. Writing a 4-vector potential as:

$$A_4 = \alpha_\mu A_\mu = \alpha_0 A_0 + \alpha_1 A_1 + \alpha_2 A_2 + \alpha_3 A_3 = \alpha_0 A_0 + \alpha_i \vec{A} \tag{2}$$

It should be noted that there is a possibility, in the full theory to be developed below, that the 4-trivector may also yield field components.Although it is always possible to find a 4-vector whose derivative gives the field components, it is also possible to find a tri-vector term which does so. Note that, in the new equations as derived, both vector and tri-vector play an equal role.

The 16 ($= 1 + 3 + 3\cdot 2 + 3\cdot 2$) terms of the 4-derivative of the 4-potential $\mathcal{D}_4 A_4$ may be gathered together and written as:

$$\mathcal{D}_4 A_4 = \alpha_P(\partial_0 A_0 \quad + \quad \vec{\nabla}\cdot\vec{A}) - \alpha_{i0}(\partial_0\vec{A} + \vec{\nabla}A_0) - \alpha_{ij}\vec{\nabla}\times\vec{A} = P\alpha_P + F\alpha_{\mu\nu} \tag{3}$$

which is the sum of a scalar (pivot) part $P\alpha_P$ and a bivector (field) part $F\alpha_{\mu\nu}$. The corresponding 4-derivative of the 4-trivector potential, $\mathcal{D}_4 T_4$, would yield additional field components as well as a quadri-vector term $Q\alpha_{0123}$.

Before proceeding it is worth digression on the question of units. It has been noted that the magnitudes referred to here have the properties of a sort of square-root energy density, similar to the "probability density" in normal quantum mechanics. These are inconvenient units without a proper name. While one could invent a name for the concept, the field quantities already have this sort of form. The energy density in the electric field E is usually written in S.I. units as $\frac{1}{2}\epsilon_0 E^2$ and that in the magnetic field B may be written $\frac{1}{2}\epsilon_0 c^2 B^2$ where ϵ_0 is the electric constant and c is the speed of light respectively. One may therefore consider the Maxwell equations to written in terms of derivatives over Tesla, Vm^{-1}, $\sqrt{J}$ or $\sqrt{kg}$ as these differ only by a constant transforming the physical units between them. Of these the units for magnetic field, Tesla, seem least complicated, are described by a single familiar word and have the added advantage that, properly the p-vot component has the same S.I units as the magnetic field (B-vot) components. This convention will be adopted implicitly here, though the reader may think of them in any appropriate units they desire.

Over each of the sixteen multivector-quantities defined above, a general dynamical multi-vector field G_{16} is defined over a scalar term P, a vector term A_4 as defined above a field term $F_6 = F\alpha_{\mu\nu} = E_i\alpha_{i0} - B_i\alpha_{jk}$, a trivector term $T_4 = \alpha_{123}T_0 + \alpha_{023}T_1 + \alpha_{031}T_2 + \alpha_{012}T_3$ and an eventual quadri-vector potential Q such that: $G_{16} = P\alpha_P + A_0\alpha_0 + A_i\alpha_i + E_i\alpha_{i0} - B_i\alpha_{jk} + T_k\alpha_{0ij} + T_0\alpha_{123} + Q\alpha_{0123}$.

Writing, by analogy with the form of the free-space Maxwell equation $\mathcal{D}_4 F_6 = 0$, $\mathcal{D}_4 G_{16} = 0$, and again using the conventional 3-space patterns for reference, one obtains a generalisation of the Maxwell equations as:

$$\alpha_0(\vec{\nabla}\cdot\vec{E} + \partial_0 P) = 0\alpha_0 \tag{4}$$

$$\alpha_{123}(\vec{\nabla}\cdot\vec{B} + \partial_0 Q) = 0\alpha_{123} \tag{5}$$

$$\alpha_i\left(\vec{\nabla}\times\vec{B} - \partial_0\vec{E} - \vec{\nabla}P\right) = 0\alpha_i \tag{6}$$

$$\alpha_{0ij}(\vec{\nabla}\times\vec{E} + \partial_0\vec{B} + \vec{\nabla}Q) = 0\alpha_{0ij} \tag{7}$$

$$\alpha_P(\vec{\nabla}\cdot\vec{A} + \partial_0 A_0) = 0\alpha_P \tag{8}$$

$$\alpha_{0123}(\vec{\nabla}\cdot\vec{T} + \partial_0 T_0) = 0\alpha_{0123} \tag{9}$$

$$\alpha_{i0}\left(\partial_0\vec{A} + \vec{\nabla}A_0 + \vec{\nabla}\times\vec{T}\right) = 0\alpha_{i0} \tag{10}$$

$$\alpha_{jk}\left(\partial_0\vec{T} + \vec{\nabla}T_0 - \vec{\nabla}\times\vec{A}\right) = 0\alpha_{jk} \tag{11}$$

With P and Q zero the first four equations reduce to the free-space Maxwell equations. The second set of four equations express a similar set of constraints between current-like and angular momentum-like quantities. Both sets of four equations may play a role in constraining the properties of allowed solutions. In this paper it is primarily the first four that will be considered. In particular the possible physical effect of the two new dynamical terms P and Q will be investigated. Both of these are invariant under a Lorentz transformations. the first of these P transforms as an invariant mass. It is the effect of this term a scalar, invariant mass-energy term that will prove most important in deriving a possible self-confinement mechanism for the extended field theory. It interacts with the electric field component to provide a force capable of rolling the photon field into a double looped configuration with an external charge and half-integral spin

Within the new formalism, the physical effect of the new scalar invariant mass term P is to allow a curvature of the momentum transport direction. If non-zero, this leads to the possibilities of a pivoting of the field flow around the mass leading to new kinds of self-confined circulating solutions with rest-mass.[9,13]

4. THE PHOTON WAVE-FUNCTION

In the companion paper a left circularly polarised electromagnetic wave, travelling in the the $+z$-direction and transmitting a quantum of energy $\mathcal{E}$ in the centre of mass frame was written:

$$F_L = H_0 U_F R\mathcal{E}(\alpha_{10} + \alpha_{31})e^{\frac{\mathcal{E}}{\hbar}R(\alpha_3\frac{z}{c} - \alpha_0 t)\alpha_{012}} = F_0 R(\alpha_{10} + \alpha_{31})e^{R(k\alpha_3 z - \omega\alpha_0 t)\alpha_{012}} = \mathcal{F}\mathcal{W} \tag{12}$$

This has a pre-factor part representing the initial (or final) field configuration $\mathcal{F}$, and a hypercomplex exponential wave-function $\mathcal{W}$. The wave-part $\mathcal{W}$ has 4 parts p-vot field, electric and magnetic fields (E-vot and B-vot parts) and a q-vot part.

This Wave-function is a solution of the more general set of equations above such that $\mathcal{D}_4\mathcal{W} = 0$. In the product $F_L = \mathcal{F}\mathcal{W}$, the p-vot and q-vot terms cancel if and only if the field components conform to that which is observed physically, so that the whole expression is then a solution of the free-space Maxwell equations $\mathcal{D}_4 F_L = 0$. What this means, physically, is that, if $\mathcal{W}$ is an element of the proper relativistic wave-function of the emitter and absorber, such functions propagate field configurations of arbitritary total energy. Such fields are propagated at lightspeed only if the initial (emitter) and final (absorber) fields have equal and perpendicular magnetic field components and are such that the total energy in their frames (note carefully that these are usually different) is proportional to the local wave-function frequency ν such that $\mathcal{E} = h\nu$. This is exactly what is observed experimentally.

The real-number constants c are the (scalar) speed of light and $\mathcal{E}(= \hbar ck = \hbar\omega)$ the (scalar) quantum of energy transmitted in the centre-of-momentum frame respectively. U_F is a universal constant converting to field units. This is a universal factor, which takes the same value for all photons. H_0 is a distribution function representing the spread of field or energy over phase, whose square integrates to unity, as the number of cycles in phase. This is an invariant and is the same in all frames, right up to the limit of light-speed where the integrated energy goes to zero. The single parameter R is that factor which determines the scales of energy, frequency, length and time. This function, though a major result in itself, has been discussed in more detail in the companion paper and is used, here, merely as the starting point for the development of solutions to charged, material particles.

Equation (12) may be readily be expanded in any particular frame. For the conditions corresponding to experimentally observed photons, the non field (scalar and quadri-vector) terms in the exponential part cancel. Setting $F_1 = H_0 U R \mathcal{E}$ and $ck = \omega = \frac{\mathcal{E}}{\hbar}$ one obtains:

$$F_L = F_1[(\alpha_{10} + \alpha_{31})\cos(kz - \omega t) + (\alpha_{23} - \alpha_{20})\sin(kz - \omega t)] \tag{13}$$

This describes electric (α_{i0}) and magnetic (α_{ij}) fields rotating in time in a plane perpendicular to the direction of momentum transport and transforming in space from magnetic to electric and vice-versa. The resultant field configuration is that shown in Fig. 1. It appears identical that found in any elementary textbook on electromagnetism for a left-handed circularly polarised wave.

5. NEW SOLUTIONS: THE ELECTRON AND POSITRON

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6. DISCUSSION

7. EXPERIMENTAL TESTS

8. CONCLUSIONS

8.1 Acknowledgments

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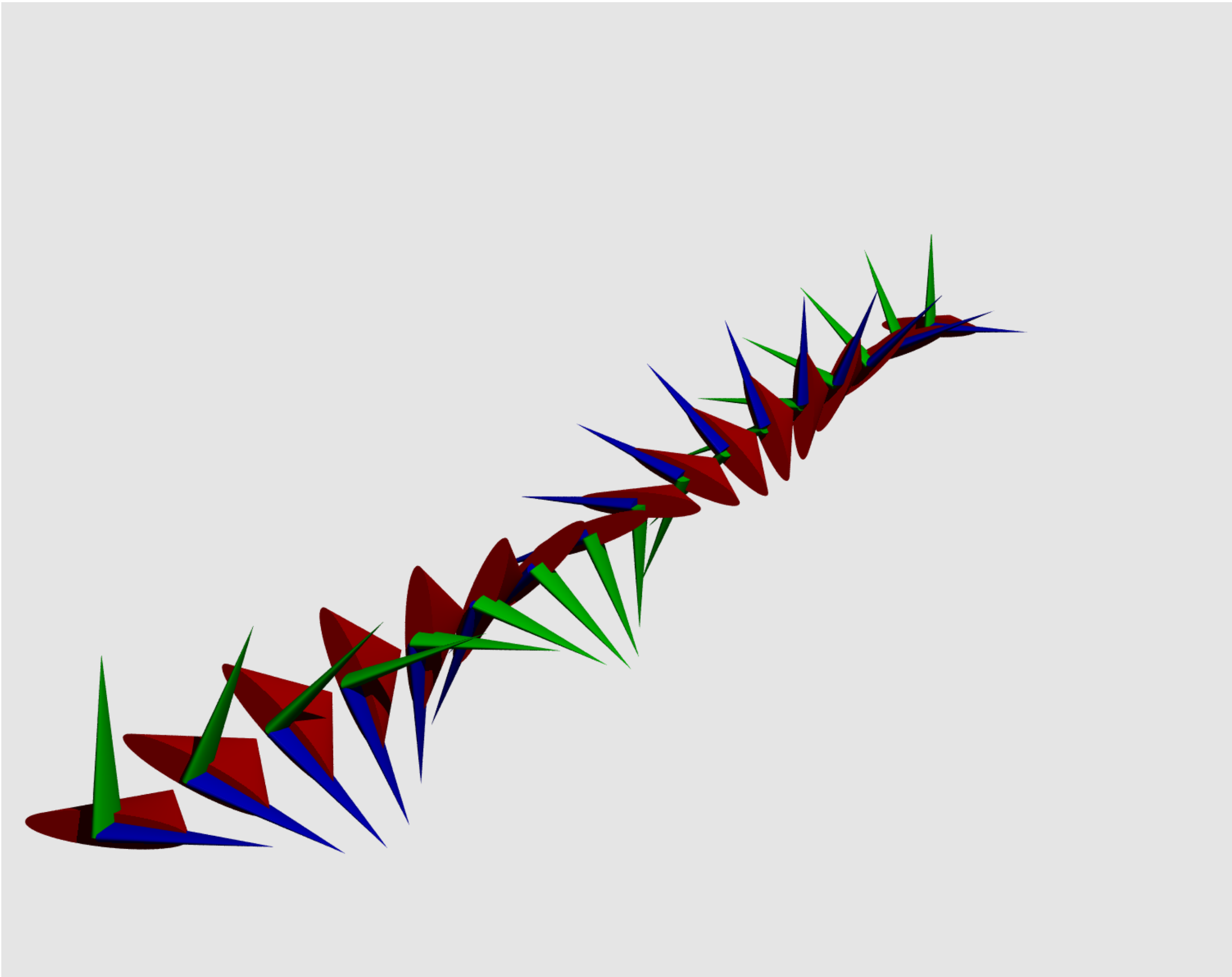

Figure 1. Representation of a single wavelength of a circularly polarised photon of equation (12). The electric field direction is represented using green arrowheads, the magnetic field blue and the momentum density red.

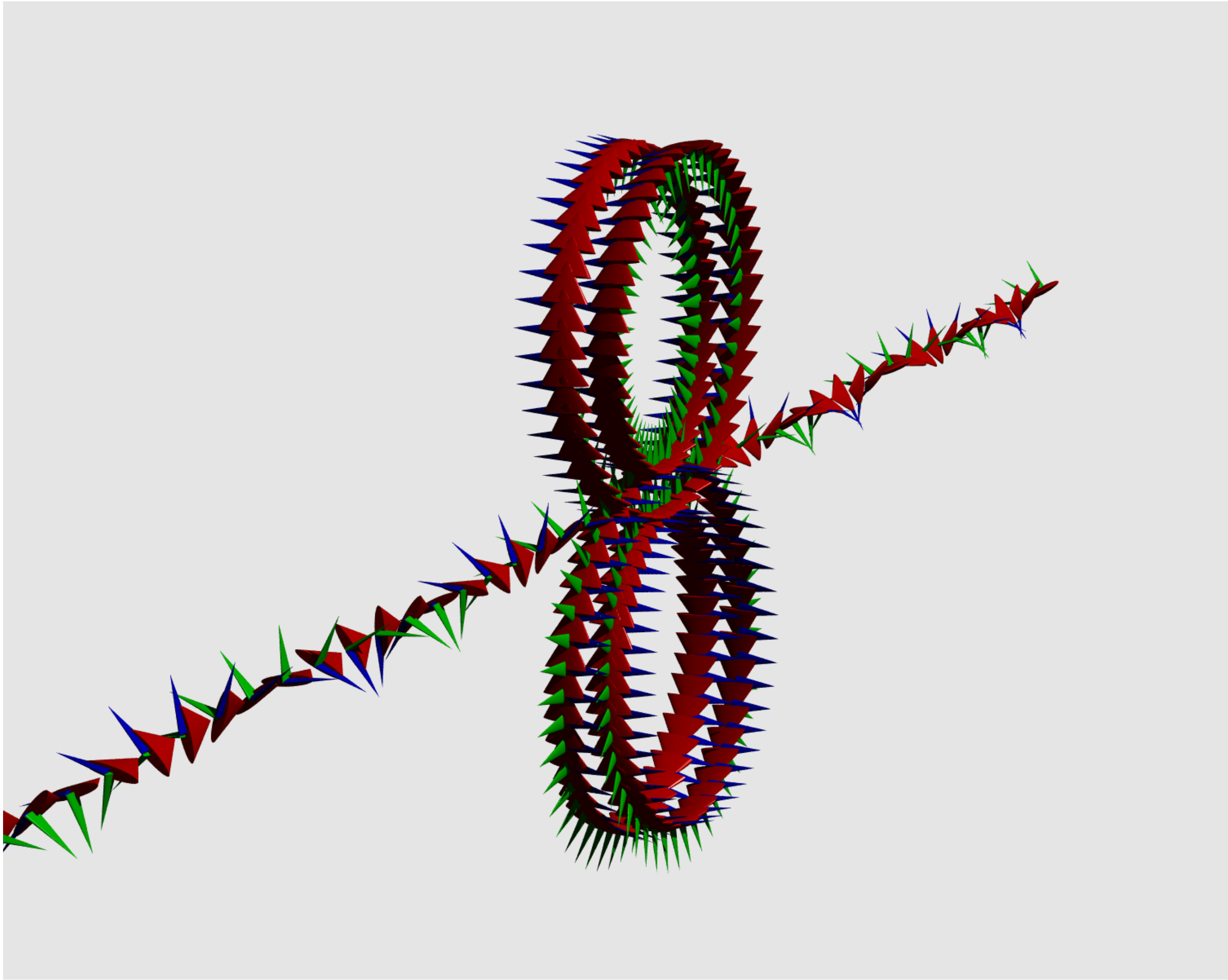

Figure 2. The process of electron-positron pair creation or annihilation in phase (momentum) space. The incoming photons are shown on a different scale to the electron and positron vortices. The vortex radius is a factor of 4π smaller than the photon wavelength.

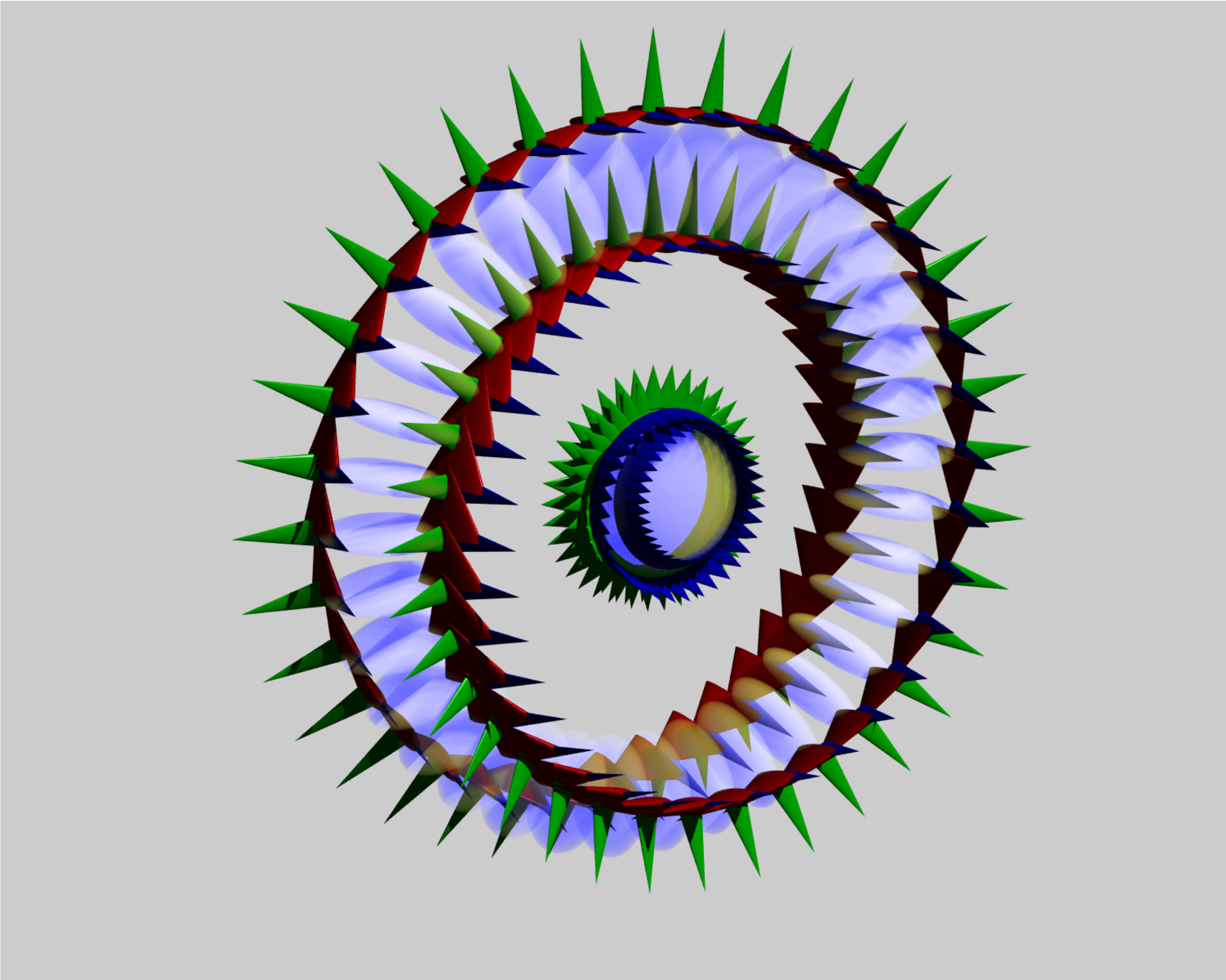

Figure 3. The toroidal field distribution in momentum space and the projection to the sphere in normal space. This makes manifest both the internal double-loop and its relationship to the toroidal topology. The smoke disks representing p-vot in the momentum representation correspond to slices through the hypersphere in spce-time. These each project to the same sphere of p-vot. An artists impression of what this would look like if one rotated at the same rate as the vortex is given in the core of the figure. Externally, one should observe a radial electric field and zero (on average) magnetic field.

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