Wave interference: mechanics of the standing wave component; and the illusion of ‘which way’ information?

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ABSTRACT

Two adjacent coherent light beams, 180° out of phase and traveling on adjacent, parallel paths, remain visibly separated by the null (dark) zone from their mutual interference pattern as they merge. Each half of the pattern can be traced to one of the beams. Does such an experiment provide both “which way” and momentum knowledge?

To answer this question, we demonstrate, by examining behavior of wave momentum and energy in various media, that interfering waves interact. Central to the mechanism of interference is a standing wave component resulting from the combination of coherent waves. We show the mathematics for the formation of the standing wave component and, in particular, for the wave momentum involved in the waves’ interaction.

In water and in open coaxial cable, we observe that standing waves form cells bounded by “reflection zones” where wave momentum from adjacent cells is reversed, confining oscillating energy to each cell. Applying principles observed in standing waves in media to the standing wave component of interfering light beams, we identify dark (null) as regions where wave reflection develops.

Each part of the interference pattern is affected by interactions between other parts, obscuring “which-way” information.

We demonstrated this physical interaction experimentally using two opposing phase beams, ‘destructively interfering’, with one dark zone between them. Blocking one beam “downstream” from the interference region removed the null zone and allowed the remaining beam to evolve to a footprint of a single beam.

Keywords: multiple beam interference, standing waves, non-cancellation. wave-wave reflection, ‘which-way’ information, light-light interaction, zero-slit experiment, optical lattice

INTRODUCTION

In this paper, the third in our series on the nature of light, we develop models to explain the mechanism of standing waves and their essential role in the formation of interference patterns in general.

Our first paper in this series on interference[1] focused on resolving the inadequacy of the superposition principle to account for energy and momentum. Our own work demonstrated that all the energy feeding into an interference pattern was present throughout its length. Also, it was apparent that during interference some form of interaction redirected and concentrated the light.

The second paper[2] investigated possible momentum exchange/reflection in light interactions with light. We compared observations suggesting reflection between light waves with apparent reflection in the well-studied standing wave models in water and air. We found reflection between waves to be essential to the mechanism of standing waves. Our analysis of interference between converging laser beams suggested that each dark (null) zone in the interference pattern acted as a mirror to reflect the path of adjacent bright zones. We also demonstrated that a mirror acted as a null zone by positioning a front-surface mirror so that a ~1 mm diameter laser beam covered its width (~100 mm). The result, in the interaction between the incident and reflected beams, was an interference pattern that resembled one half the diagramed, two-beam, pattern. Importantly, at the surface of the mirror was a null zone, providing an experimental connection between null zones and reflections as described in the Lloyd's Mirror experiment.[3] The visual identity of interference patterns helps to equate the reflected beams from a mirror with the reflected beams from a dark zone of interfering coherent waves.

Of special significance was our observation that standing waves are essential to the mechanism of two-beam interference. The resultant beam from two converging beams can be resolved into two components: one traveling along the bisector of the angle between beams and another, a standing wave, orthogonal to the first.

In a 1992 publication, Dowling and Gea-Banacloche in, "The specular reflection of light off light", saw that interference in light could result from mirror-like reflection between oppositely polarized light waves.[4] They concluded that, despite their strong theoretical argument, they could not prove whether light waves reflected or passed through each other. They observed that the superposition principle and mirror reflection gave identical results.

In this paper we extract mechanisms from standing waves in water (waves with mass), from standing waves in an electrical circuit (waves with charge, current, and magnetism), and in electromagnetic waves (reflection from a mirror). We conclude that all waves carry momentum and, therefore, colliding waves must interact and that equal waves cannot pass through each other.

The result is a generalized mechanistic model with unexpected predictive power. Importantly it is also consistent with much emerging literature. Our observation of standing waves as components of interfering waves was bolstered by an academic presentation on oblique standing waves.[5] We find that the behavior of converging water waves perfectly mimics the behavior of converging laser beams.

Our model of standing waves is based on the principle of pendulum-like harmonic, motion. It is applicable to all types of standing waves. Standing waves often occur in series as seen in figs. 1 and 2. Each segment, or cell, of the standing wave series oscillates between periods when potential energy is stored and periods when kinetic energy surges. The kinetic energy will drive flow and momentum exchange with reflectors and with adjacent waves in a standing wave series. In electromagnetic systems the kinetic energy flow (e.g., current) also produces a magnetic field

We use a laser interference arrangement that produces two opposite-phased beams, which apparently remain completely separated and project to two separate, interference patterns. It would be reasonable to interpret this system as one that provides "which way" knowledge about the light path responsible for each bright interference zone. We examine that possibility by applying our understanding about the process of standing wave interactions and by observing rules about the nature of information.

1. EXPERIMENTAL

Our experimental set up[1,2] is composed of a laser beam directed into a variable density mirror/filter/beam splitter. Beams, one reflecting from the front and one reflecting from the back (mirrored) surface, are controlled with respect to separation, relative intensity, and phase by adjusting the angle of incidence. The beam splitter was chosen and adjusted so that the emerging beams were parallel and clearly separated. Thus, there would be no possibility of interference between the beams within, or at the front surface of, the splitter. Beam intensities were determined by photometry or by measurement with a beam analyzer.

For the "which way" experiment, it was important to determine the distance from the beam splitter at which the beams converged. The beam splitter was adjusted until a clearly 'constructive' interference pattern (a strong central peak with a smaller one on either side) appeared in a distant beam analyzer. The pattern observed close (several inches) to the beam splitter was that of the two separate beams. Then, as the analyzer distance from the splitter increased, the distance at which the constructive interference pattern began to develop was recorded as the beginning of the beam convergence point. The splitter was then adjusted until the appearance of a destructive interference pattern (central dark zone with symmetric bright zones) indicated that the phases of the beams were opposed and the emerging two beams were balanced. We then recorded the appearance of the beams as they emerged from the splitter and as they reached apparent total overlap. From our analysis of standing waves produced in an interference pattern, we can see that beams having opposing phase appear to repel each other leaving a continuous dark zone between them. We also recorded the point where a clear interference pattern had persisted for a meter. We then blocked one of the beams at various points beyond. This work is described in Section 5. However, as background for interpreting those results, it is necessary to establish some of the concepts to be used.

Fig. 1 is the 'instantaneous' pattern of constructive interference between two crossing coherent laser beams. Instantaneous means that one can see the individual peaks (high-field regions or wave fronts) of the travelling waves. Since the peaks of the travelling waves move, a time-integrated image of the pattern would produce continuous bright zones between horizontal dark bands, which have electric fields insufficient to excite electrons to produce illumination in a medium. Thus, the constructive interference pattern of Fig. 1 will provide the same information about interference and reflection from the dark bands that we will be describing and 'proving' below in the context of destructive interference.

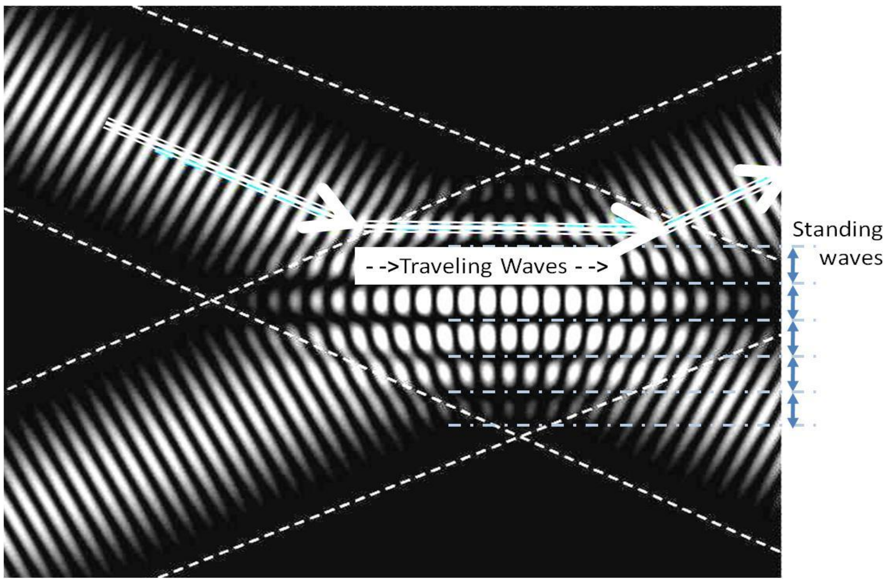

Fig.1. An 'instantaneous' pattern of constructive interference between two coherent laser beams.[6] The heavy arrows indicate traveling wave 'flow'. Standing waves within the interference region reflect between the dark bands (dot-dashed lines).

The dashed lines in the figure indicate the extent of the two beams and the overlap region that defines the interference zone. It is apparent (from the dashed arrows) that the beams are redirected on entering this zone and move along the horizontal regions bounded by the dark zones established by the potentials of the standing waves. The traveling waves then follow new paths, being reflected from, instead of crossing, the dark bands.

Mathematically, bi-directional transmission of equal beams of light through the dark bands is indistinguishable from reflection from these dark zones. Thus, non-interaction and reflection must have the same probability when based on the wave equations alone.[4] Only by recognizing that the momentum is reversed in these dark regions can one prove that reflection is present. This information is presented below in terms of orthogonal components of the light inside the interference region.

2. STANDING WAVE COMPONENT OF INTERFERING WAVES

The result of obliquely[i] incident waves that interfere may be broken into two components. One component is a traveling wave along the bisector of the angle between the two waves. The second component is a standing wave at 90° to the bisector.[2, 4] This second component is 100% of the wave energy when they are anti-parallel[ii] and is reduced to zero as the waves rotate to the parallel orientation. Descriptions of better-known wave phenomena in media are useful for the less obvious light-wave interactions, where the medium is not known.

In the case of reflected water waves within the linear domain, the vertical displacement y of a wave resulting from both incident and reflected waves can be written

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[i] 'Obliquely' indicates neither parallel nor orthogonal. Nevertheless, in 2 dimensions, the analysis actually works in all cases.

[ii] There is no traveling wave in this case and an optical lattice can result (see Sect. 4).

$$h = 2A[\cos(kx\ cos\theta)][\cos(kz\ sin\theta - \omega t)]$$

where θ is the angle of incidence between the waves, A is maximum vertical displacement, k is wave number, and ω is angular frequency. The bracketed expression on the left is a standing wave in x; the one on the right is a traveling wave in z. The equilibrium water surface is in the *x-z* plane. (See Fig. 4).

Standing waves in water

Figure 2, below, represents the surface profile of waves in a wave tank at different times. The x-axis represents horizontal distance from a reflector at the end of the wave tank (opposite end of wave tank not shown). Crests and troughs are at extreme height and depth at t_0. Then water descends from the crests into the troughs, which themselves rise, as potential energy PE from the dropping crests and rising troughs converts to kinetic energy KE and locally builds momentum. As the water flows down either side of each crest, horizontal momentum $\overrightarrow{P_x}$ develops toward either side of each rising trough.

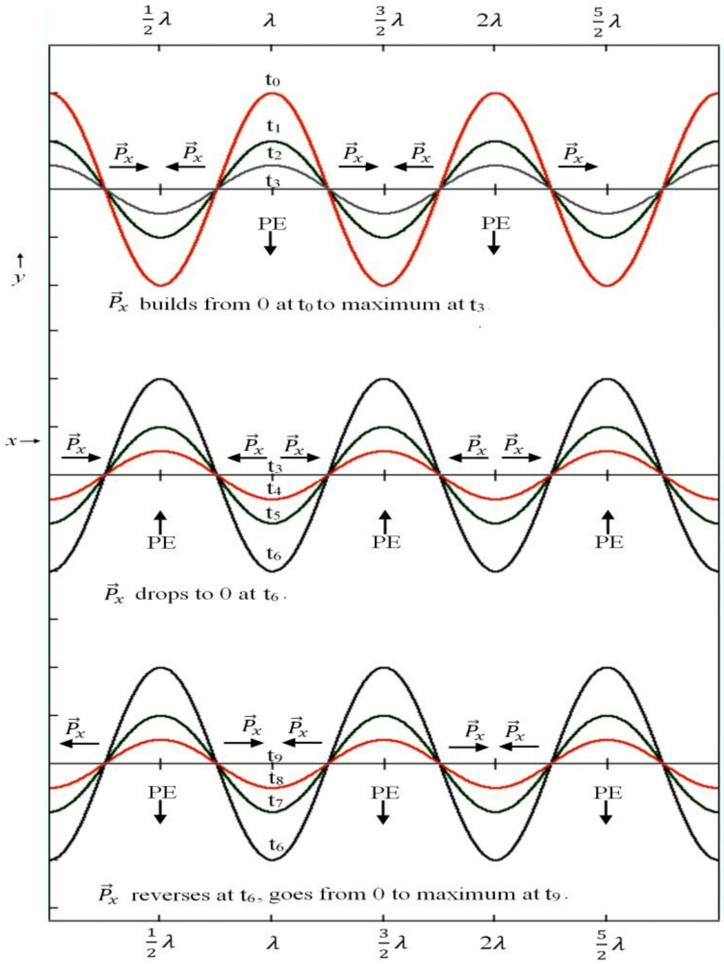


Figure 2. Standing waves in water

At t_0 through t_2, troughs occur at odd half wavelengths λ/2, 3λ/2, 5λ/2… The troughs fill and then rise to become crests at t_4 through t_6 as the inertia of the water flow converts its KE back into PE. Horizontal momentum $\overrightarrow{P_x}$ from either side of the new crests drops to zero and then reverses.

Water swings between reversals in semi-independent half wavelength cells, rising and falling with potential and kinetic energy, much as does a pendulum. The half wavelength points λ/2, 3λ/2, 5λ/2 … are *reflection points*[iii] for horizontal motion. It is these points to and from which water oscillates.

Surface vertical oscillation is maximal at the half wavelengths 0, λ/2, λ, 3λ/2, 2…, along the x-axis. At even wavelengths λ, 2λ …, potential energy drops from maximum at time t_0 to a minimum at t_6. At odd multiples of half wavelength wavelengths λ/2, 3λ/2, 5λ/2 …, potential energy phase is the reverse, rising from minimum at time t_0 to a maximum at t_6.

The quarter wavelength points λ/4, 3λ/4, 5λ/4 … are *null zones* where the surface vertical displacement of the water remains fixed. Nevertheless, at these points, oscillating horizontal water velocity reaches its maximum. Total horizontal momentum of a single cell $P_x^T(t)$ can be computed (See the Appendix *Momentum in Water Waves*, for a mathematical analysis):

$$P_x^T(t) \; = \rho \int_0^{L_Z} \int_0^{\frac{\lambda}{2}} \int_0^{h(x,t)} v_x \; (x, y, t) dy \, dx \, dz$$

(Adapted from Peskin).[7]

Where h is wave surface height, measured above its undisturbed height on y axis, ρ is mass density, and v_x is horizontal velocity of the wave moving in the *x* direction. The momentum of any 'cell' volume (e.g., 'h' tall, λ/2 long, and L_z wide) oscillates and is therefore reversed twice every cycle.

Standing waves in transmission lines

Driving an open coaxial cable with a signal generator produces standing waves at specific resonant frequencies.[8] Fig. 3 illustrates the dynamics of voltage peaks and troughs that occur at half-wavelength points along the coaxial cable during resonance. They follow the same sequence described above for standing water waves. Voltage that starts at peaks at t_o drops through the same time sequence and rebounds to peaks displaced a half-wavelength from the original ones.

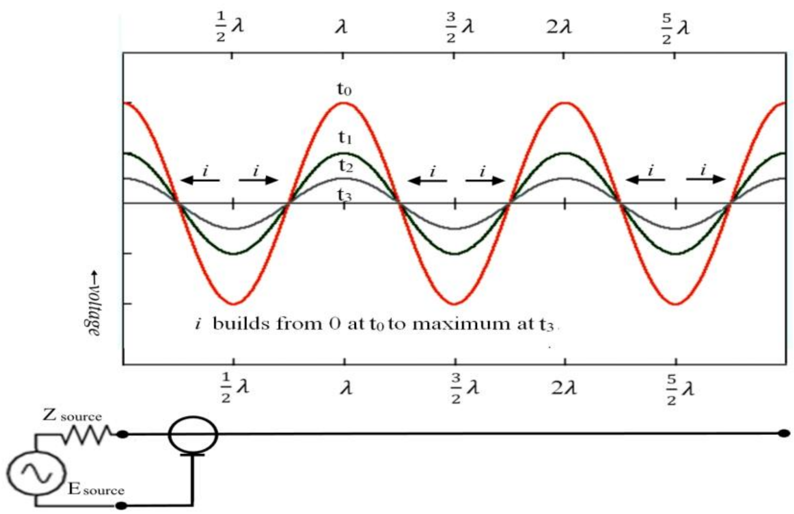


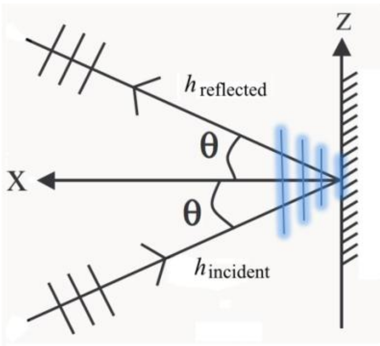
Figure 3. Standing waves in open coaxial cable

Momentum is transferred as current *i* and reverses at half-wavelength points 0, λ/2, λ, 3λ/2, 2λ … These are *reflection points* to and from which currents oscillate. In the hydrodynamic wave, the momentum is seen in the inertia of the water molecules. In the transmission line, the momentum is seen in the currents and inductance of the circuit, which is caused by the motion of the electrons and the magnetic fields they produce.

[iii] More correctly, these points on the axis are centers of reflection regions; but, they still act as solid walls.

Standing waves in coherent light: reflection points

A laser beam, incident on a reflector, produces an interference pattern with alternating dark and light regions. In the left side of Fig. 4, a material reflector is at the *z*-axis. Alternating dark and light regions are parallel to the reflector, which is the bisector of the incident waves assuming that the reflected wave originates within the mirror. Adjacent to the reflector is a dark region. In the rightside diagram, two opposite-phase laser beams are intersecting and the dark zones are again parallel to the bisector, which is the traveling wave within the interference region. The standing wave components establish the dark zones.



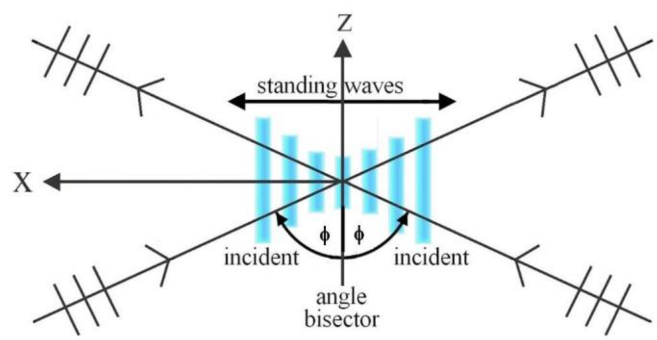


Figure 4. Reflection produces standing light waves along the *x*-axis and convergence of two light waves produces identical results on the one side.

From the dynamics of standing waves discussed above, we observe standing wave components of the interfering beams consisting of cells of energy oscillating between reflection points. Noting the reflector as a reflection point within a dark (null) region, we can identify the alternating dark zones as loci for reflection. The kinetic energy KE observed in the interspersed bright regions is driven by potential energy PE rising and falling in the dark regions.

Results and discussion of standing waves

All three standing wave systems described above have the same general mechanism of momentum interactions between reflecting surfaces and between counter-moving waves in adjacent cells (Figs. 2 and 3). In the reflection regions between oscillating cells, potential energy temporarily builds in one phase of the cycle. The energy resides as gravitational potential in water waves, as voltage in electrical circuits, and as electric potential in electromagnetic waves. However, in electrical circuits, energy is also stored in the magnetic fields. This is probably the case in EM waves also. During the next phase of the cycle, potential energy converts into kinetic energy as water flow in water, etc.

In the light system, there is no mass or charges to be the basis of momentum and inertia in the interference region in front of the mirror; yet, the electric and field energies within the mirror are able to produce the same observable effects. These can be attributed to the interaction of the EM fields of the incident light with the electrons in the mirror. What happens when there are no masses or charges available with which the light can interact?

From our observations, we can extrapolate to the behavior of standing waves in light in vacuum, and suggest that energy must oscillate between electric potentials with opposite polarities (in adjacent null zones). We propose that the energy gradient (the electrical field) between these potentials converts to kinetic energy as a flow (to be determined) that also induces a magnetic field with its stored energy. This concept is developed further in the companion paper.[9]

In each type of standing wave in matter, the flow carries wave momentum that 'collides' with counter-moving momentum in adjacent cells to maintain the standing wave system. In the collision region, motion (KE) is minimal where potential (PE) is maximal.

A theoretical problem arises because there is no established description of what might explain the composition of the electrical potential in the standing light wave. Moreover, during the kinetic energy/flow cycle in the standing wave in light a magnetic field arises without any known particle flow that could cause a field. We believe important clues to the nature of light reside in the identity of what is producing the charge potential and the magnetic field during standing wave oscillations.

We see applications of our observations on standing waves to optical lattices. [10,11]

3. OPTICAL LATTICE

Our understanding of optical interference patterns, from the viewpoint of standing waves, is applicable to optical lattice work that is active today. Optical lattices are standing waves formed from retro-reflected laser beams or by interference fringes from counter-directed coherent light. The standing waves composing these lattices form a series of optical traps for atoms and larger particles. In agreement with the literature,[10,11] our models suggest that the trapping regions are bordered/defined by the oscillating electrical potentials of the null zones. The frequency and depth of the traps is a function of the frequency and intensity of the reflected light source. Alternatively, an optical lattice may be formed from the interference pattern resulting from laser beams crossing at some oblique angle. The character of such a lattice, in terms of frequency and depth, can be modified, as desired, by changing the crossing angle.

The importance of optical lattices to this discussion is twofold. First, they demonstrate that light wave interference can take place independent of a medium (unless the light itself and/or its effects on space-time are considered to be the medium). This interference results in a 'physical' structure that can be controlled at the light source(s) as well as within the structure itself. This structure is stationary in space, although most of it is oscillating in time.

Secondly, it provides a technically important tool that can and will be explored in detail for its properties and nature. Of particular interest is the nature and dynamics of the source of electrical potential that creates the electric and magnetic fields of the standing waves and therefore of photons and light in general. In material systems, we can understand the inertia, momentum, inductance, etc. that build the oscillating potentials from which the fields are derived. All of these require material bodies, molecules, electrons, etc. to meet the requirements. What in the vacuum of space-time is their equivalent?[9] Even in the material systems, some things are not obvious and have produced numerous studies.[12] We know that the momentum of EM radiation (both linear and angular) mimics the physical interaction of momentum in standing waves in the material systems. We hope that this study of optical lattices can lead to a better understanding of 'what' is moving in 'empty' space, why, and how.

4. THE "WHICH WAY" EXPERIMENT

Our 'destructive' interference pattern might appear to provide independent light paths that could be traced to two separate sources. Others may have interpreted similar interference constructs as confirming the presence of "which way" information.[13] Although it might be tempting to make a similar interpretation here, information theory suggests an explanation for this interpretation.[14]

As explained in the experimental section; when approaching identical light beams have opposite polarity or phase there is an opposing force between them that keeps them apart. The opposing forces between them produce continuous standing-wave interactions between their light paths. We demonstrate this by showing that, when one of the beam paths through the interference zone is blocked, the remaining light path migrates from its previous track, back across a previously dark zone to fill the original single-beam footprint. The center of this footprint is the dark-zone at the midpoint between what had been the interference bright zones. We conclude that there is an exchange of momentum and therefore information (cross-talk) during interference.

Figure 5 (below) shows the experimental results with the beam analyzer at various locations.

a. The beams emerging from the beam splitter are amplitude balanced and well-separated.
    - The beam analyzer was placed as close as possible to the beam splitter without interfering with the incident beam.
    - The cross-sectional profiles identified by the cross-hairs are well-separated, near-Gaussian beams with only a very small peak between the beams. This small central peak is a signature of a slight overlap of the constructively interfering beam edges.
    - This beam separation indicates that almost no interference observed downstream takes place in the beam splitter or at its surface. Any interaction must be between the beams alone. No matter is required.

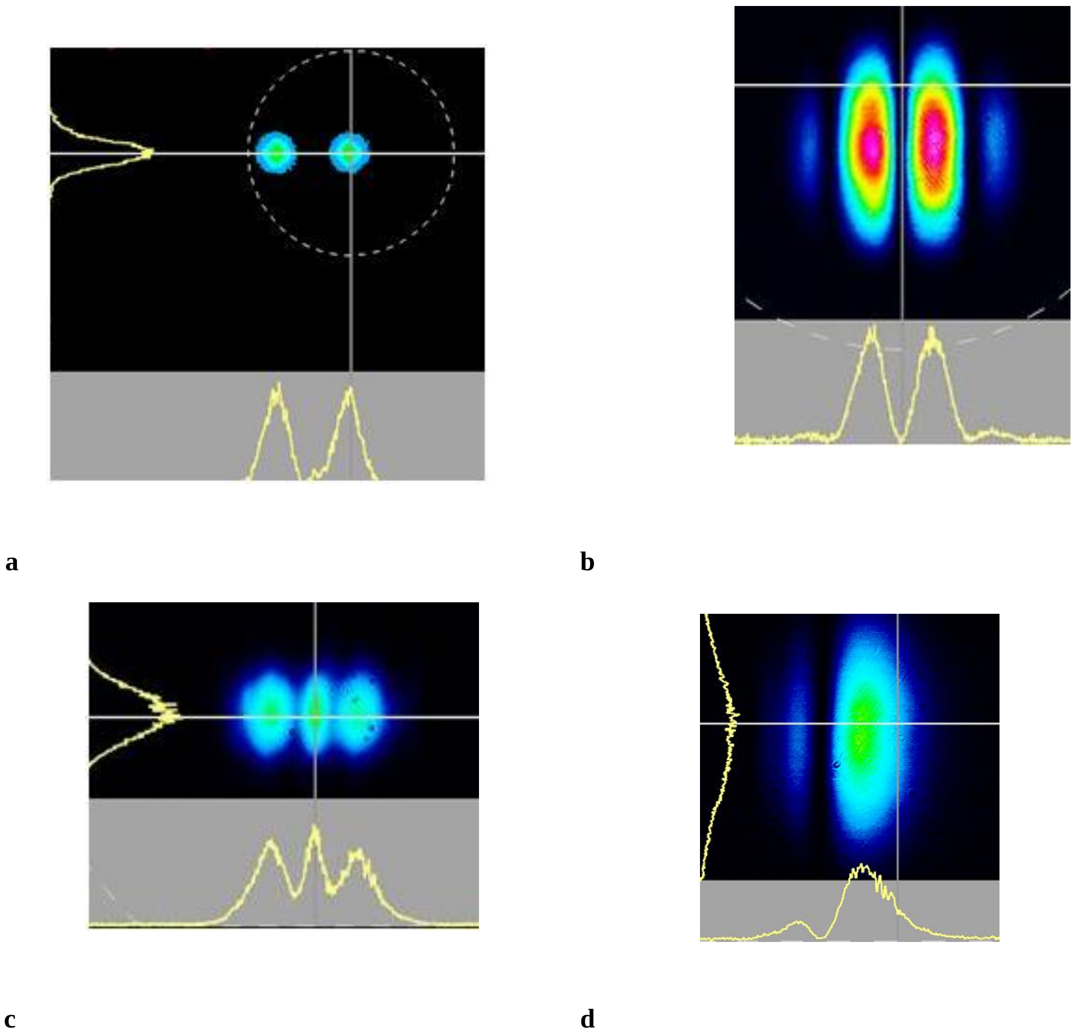


Figure 5. Beam analyzer pictures of the twin laser beams: a) ~5 cm from the beam splitter; b) destructive interference at the beginning of the interference zone; c) constructive interference inside the interference zone; d) destructive interference further inside the interference zone, but with the right-side beam blocked earlier (at b) in the interference zone. See text for details.

b. At ~7 m from the beam splitter and with the light-beam phases opposite, a 'destructive interference pattern' (with a centered null zone) is produced.

- Both laser beams, when measured individually, have expanded to nearly fill a common 'footprint'. When combined, they produce the complete interference pattern.
- The central dark-zone identifies the destructive interference pattern.
- The secondary peaks and null zones result from the individual beam divergence angles of the two parallel laser beams. They disappear only if a beam expander is used to reduce the divergence angle. This increases the spacing between the dark bands and the central dark zone and two bright zones then will spread out to cover the whole foot print.
- The horizontal cross hair has been raised (to a less intense region of the peaks) to keep the profile along that axis from being too large and overlaying the image.
- The profile along the vertical cross hair, placed in the dark zone, is not shown because the values are all near zero.

c. At only ~ ½ m from the beam splitter, the light paths have spread out and their now-larger individual footprints have converged significantly. This figure (5c) indicates an in-phase 'constructive' interference pattern in the 'overlap' region.
    - As the beam analyzer is moved further from the beam splitter, the central peak will continue to grow in magnitude as the beam overlap increases.
    - The location of the central peak is directly below the little peak in Fig. 5a. Thus, that figure must also be in the constructive interference mode

d. The light path through the right bright zone of the interference pattern has been blocked allowing light from the left bright zone to diffuse to the right without interference as measured 2.5 m downstream.
    - The vertical cross-hair defines the center of the central null zone of the far-field pattern if neither side were blocked. It should be aligned with the vertical crosshair of Fig. 5b.
    - The right side of the bright zone is no longer straight because there is no longer any interference on that side.
    - In the far-field, the remaining one-half beam pattern will fill the whole footprint, but at the lower intensity expected from a single beam. There is now no central dark zone separating the two parts of the pattern. Beam divergence has erased most signs of previous interference.

The visibly separated paths within an interference image appear to clearly identify their individual sources (Fig. 1 shows a longitudinal image and Fig. 5b shows a cross-sectional image). However, blocking one of the paths in the interference region (Fig. 5b) altered the other (Fig. 5d). This demonstrates that, although separated by null zones, the two bright regions of the interference pattern require a non-visible interaction across the intervening null zone to maintain the separation. Thus, neither the standard assumption of linear non-interactive crossing of the laser beams, nor the non-crossing picture presented above can provide unique which-way information.

5. CONCLUSIONS

This paper presents a number of ideas, based on the interaction of coherent waves:

1. <u>Some light waves (and photons [15,16,17]), or portions thereof, can reflect from others, if they are coherent.</u>
   Our experiments[1, 2] and references[4, 5] support the conclusions that, when converging or counter-propagating waves collide, they reflect from each other if they have the same frequency, polarity and amplitude. Rather than passing through each other, alternately cancelling and reinforcing in an interference region, their components orthogonal to the bisection of the beam velocity vectors will form a system of standing waves (Section 3). These standing waves, which consist of the non-parallel vector components in the plane of convergence of the incident waves that would otherwise 'cross' each other, are shown (in the appendix) to have momentum reversal and therefore make the waves reflect from each other.

2. <u>Coherent light waves can interact without the presence of matter or charge.</u>
   The configuration that we have characterized (e.g., Fig 1) appears to have provided a clear path change of the light beam from its source as a result of interference. This change in direction depends only on the interaction of coherent wave components and can occur in the absence of matter.

3. <u>Reflection from null zones in an interference pattern is visually indistinguishable from reflection from a front-surface mirror.</u>
   We show in representative examples (Section 3) that, where a reflector is present, there is no effective difference in reflection between coherent (water, light, or electrical) waves and reflection from an appropriately placed physical reflector (wall, mirror, or electrical 'terminal' respectively).

   Reflection between waves results from collision between counter-moving momenta (Section 3 and the appendix). During reflection, potential energy accumulates next to (and within) the reflector. Historically, in light interference, this zone is known to be a null zone, as taught by the 'Lloyd's Mirror Experiment'.[3] Consequently, null zones (dark zones) are locations of potential energy in standing waves.

4. Standing waves generated in the interference region represent alternating energy modes.
   As we have discussed,[2] when wave trains collide at an angle, a standing wave develops that will redirect the incident wave trains into separate parallel-moving wave trains (Fig. 1). They will be separated by the null zones of the standing waves (Fig. 4). In the null zones, momenta that would otherwise move between the wave trains collide and reverse direction, storing potential during the collision that then returns to the wave trains (light energy, if the merging wave trains are laser beams) as kinetic energy. In light, the interactions described are simple physical interactions that do not require the presence of a physical medium. Whatever forms the structure of space appears to be sufficient.

5. The misleading appearance of "which way" information.
   An assumption of which-way information, based on a detector only 'seeing' one of two sources,[13] generally assumes the non-interactive linear crossing of two laser beams. The reflection (non-crossing) picture provided in the present paper can also support an assumption of unique (but different) which-way information. Both pictures, while seeming to be obvious, are shown above (Sect. 5) to be incorrect in these assumptions.

6. Creation of standing EM waves leads to time-varying electrical potentials (with the resultant fields) and, therefore, to effective charge.
   The similarity of the potential-energy region of the standing waves with a physical mirror thus points to light-light interactions creating 'source' terms (charge) of a nature identified with matter. The longitudinal fields of the standing waves even represent the near-source Maxwellian radiation.

   A large percentage (nearly ½) of a laser beam incident on a front-surface mirror never reaches it. It reflects from the interference pattern dark zones in front of the mirror. The potentials in the null-zones mimic that of charge and therefore multiple null zones represent layers of special reflectors created from nothing but space and the energy present in the light beam. A second coherent wave can replace the material mirror and the new interaction can produce the same result, even independent of the environment. The potential-energy mode in a standing EM wave appears to be related to oscillating electric potentials and therefore to optical lattices (Sect. 4).

APPENDIX: MOMENTUM IN WATER WAVES

This appendix defines horizontal momentum density P_x and indicates, conceptually, the procedure to compute horizontal momentum for a standing water wave's oscillating half-wavelength cell, described above.[5] We define the magnitude of a water wave's horizontal momentum density P_x as:

$$P_x\,(x,t) = \rho \int_0^{h(x,t)} v_x\,(x,y,t)dy$$

(adapted from Peskin[7]),

where h is wave surface height, measured above its undisturbed height on the y axis, ρ is density, and v_x is horizontal velocity of the wave moving in the *x* direction.

The momentum density P_x defined above is in units of momentum/area because integration is over the depth (*y*). Later we show integration over *x* and *z* in order to derive total momentum for a portion of the wave.

The total horizontal momentum $P_x^T(t)$ for the half-wavelength cell of a standing water wave is given by integrating over *x* and *z*, as well as *y*:

$$P_x^T(t) \;= \rho \int_0^{L_z} \int_0^{\frac{\lambda}{2}} \int_0^{h(x,t)} v_x\,(x,y,t)dy\,dx\,dz$$

where L_z is the extent of the (uniform) wave along the z axis.

In the linear domain, the height h of the standing water wave can be described[4] by

$$h = 2A\cos(kx)\cos(\omega t)$$

where k is wave number, A is amplitude, and ω is angular frequency. The horizontal velocity component v_x is derived from the velocity potential to be

$$v_x = \frac{2kgA}{\omega}\frac{cosh\ k(y+H)}{cosh\ (kH)}\sin(kx)\sin(\omega t)$$

where g is gravitational acceleration and H is the distance of the water's undisturbed height above a defined bottom.

It is worth noting that the $\sin(kx)$ factor in v_x results in a zero value at the half-wavelengths 0, 1/2λ, λ, 3/2λ, 2λ … These are *reflection points* mentioned above, to and from which water oscillates. Also note that v_x changes sign on either side of the reflection points.

Then

$$P_x^T(t) = \rho\int_0^{L_z}\int_0^{\frac{\lambda}{2}}\int_0^{h(x,t)}\frac{2kgA}{\omega}\frac{cosh\ k(y+H)}{cosh\ (kH)}\sin(kx)\sin(\omega t)\ dy\,dx\,dz$$

Or

$$P_x^T(t) = \frac{2kgA\rho\sin(\omega t)}{\omega}\int_0^{L_z}\int_0^{\frac{\lambda}{2}}\int_0^{h(x,t)}\frac{cosh\ k(y+H)}{cosh\ (kH)}\sin(kx)\ \ dy\,dx\,dz$$

The result after performing the integration is

$$P_x^T(t) = \frac{L_z\,\rho\,2gA\sin(\omega t)}{\omega\,k\cosh(kH)}\left\{\frac{\left[\cosh\left(k(2A\cos(\omega t)+H)\right)-\cosh(kH)\right]}{(2A\,k\cos(\omega t))}-\frac{\sin(kH)}{1}\right\}$$

.

Therefore we see that horizontal momentum in the half-wavelength cell of a standing water wave oscillates as $\sin(\omega t)$ and is reversed (since the horizontal-flow velocity reverses) in the half-wavelength regions as above.[iv]

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