Space, Time and Coordinates in a Rotating World

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Abstract

The peculiarities of rotating frames of reference played an important role in the genesis of general relativity. Considering them, Einstein became convinced that coordinates have a different status in the general theory of relativity than in the special theory. This line of thinking was confused, however. To clarify the situation we investigate the relation between coordinates and the results of space-time measurements in rotating reference frames. We argue that the difference between rotating systems (or accelerating systems in general) and inertial systems does not lie in a different status of the coordinates (which are conventional in all cases), but rather in different global chronogeometric properties of the various reference frames. In the course of our discussion we comment on a number of related issues, such as the question of whether a consideration of the behavior of rods and clocks is indispensable for the foundation of kinematics, the influence of acceleration on the behavior of measuring devices, the conventionality of simultaneity, and the Ehrenfest paradox.

1 Introduction

In his Autobiographical Notes [1], Einstein relates how important Machian empiricist ideas were for his discovery of a theory that could reconcile the idea that all inertial frames are equivalent with the principle that the velocity of light has a fixed value that is independent of the velocity of the emitting source. It was essential, he states, to realize what the meaning of *coordinates* in physics is: they are nothing but the outcomes of length and time measurements by means of rods, clocks and light signals. This idea led Einstein to his famous critique of the classical notion of simultaneity, one of the cornerstones of the special theory of relativity.

It soon turned out, however, that the special theory of relativity was not able to accommodate gravitation, and the principle of equivalence, in a natural way. Einstein fully recognized this problem in 1908, but it took him another seven years before he succeeded in constructing the general theory. As he explains in his Autobiographical Notes, the main reason for the slowness of his progress in this period was the difficulty of *abandoning* again, in the context of the general theory, the idea that coordinates should possess immediate metrical meaning.

From a systematical (as opposed to a historical or psychological) point of view this emphasis on the different meaning of coordinates, in the context of the two theories, is very odd. For the practice of physics before, during and after Einstein's days, even if governed by the severest empiricist norms, does not at all indicate that coordinates should possess a metrical significance, relating to the indications of rods and clocks. Think, for example, of the way coordinates are used in observational astronomy: the essential thing is that the coordinates are assigned to celestial objects in an objective and reproducible way; how the coordinates relate to distances is a matter to be found out subsequently. Coordinates are even routinely attributed to regions of the universe in which rods and clocks could not possibly exist. This is obviously unobjectionable from an empiricist point of view, as long as the method by which the coordinates are assigned is operationally specified. So, even within the framework of special relativity general coordinate systems that do not reflect the indications of rods and clocks are entirely permissible.

What finally led Einstein to abandon his special relativistic analysis of the meaning of coordinates, he tells us, was the lack of metrical significance of coordinates in accelerating frames of reference; the consideration of coordinates on a rotating disc played an important role in reaching this conclusion

[2]. But, as we will see, there is confusion here: the metrical significance of coordinates in accelerating frames can be determined completely through application of the principles of *special* relativity, so there can be no need to revise the meaning of the notion of coordinates, or to invoke a new epistemological analysis.

As it turns out, the difference between inertial and non-inertial frames of reference, and between special and general relativity, is not in the epistemological status of the coordinates. Rather, the difference is that chronogeometric characteristics become globally different. This is a physical rather than a philosophical difference, and has nothing to do with the meaning or permissibility of coordinate systems.

The rotating frame of reference nicely illustrates these points. There is no problem in defining operationally meaningful coordinates in a rotating (and therefore accelerating) frame. Furthermore, relating these coordinates to distances and time intervals, and the behavior of moving objects, can be done by the means provided by special relativity. However, the spatial geometry becomes non-Euclidean, and local Einstein synchrony does not lead to a global notion of time. These latter features constitute the essential differences from the situation in an inertial frame.

In the course of our discussion we will have occasion to comment on a number of related issues, such as the status of rods and clocks, the behavior of accelerating measuring devices, the conventionality of simultaneity, and the Ehrenfest paradox.

2 The rotating frame of reference

Let us start from Minkowski space-time, coordinatized by inertial coordinates r, φ, z and t: r and φ are polar coordinates in a plane, z is a Cartesian coordinate orthogonal to this plane, and t is the standard time coordinate. It so happens that r, z, and t can be thought of as representing the indications of rods and clocks, but that is not important for their role as coordinates, which is just to pinpoint events unequivocally. The choice of coordinates is conventional and pragmatic. In this case we choose polar coordinates because we are going to describe a system that possesses axial symmetry: polar coordinates simplify the description.

Once we have laid down coordinates, the metrical aspects should be introduced via further stipulations. This is ordinarily done through the introduction of the 'line element' $ds^2 = c^2dt^2 - dr^2 - r^2d\varphi^2 - dz^2$, plus a specification of what this mathematical expression represents physically. The traditional approach is to invoke standard rods and clocks: ds/c is the time measured by a standard clock whose r, φ and z coordinates are constant. Furthermore, $\sqrt{-ds^2}$ is the length of a rod with a stationary position in the coordinates and with constant coordinates and differences dr, $d\varphi$, dz between its endpoints, taken at one instant according to standard simultaneity ($dt = 0$). However, it would be a mistake to think that rods and clocks are indispensable to relate the coordinates to metrical concepts. In section 4 below we will discuss an approach that does not make use of rods and material clocks.

We now introduce alternative coordinates for the events in this Minkowski world: $t' = t$, $r' = r$, $\varphi' = \varphi - \omega t$ and $z' = z$, with ω a constant. Since rest in the new coordinates obviously means uniform rotation with respect to the old frame, we call the frame of reference defined by these new coordinates the *rotating frame of reference.*

It is clear that if operational methods are at hand to fix the old coordinates, the same methods can be used to assign values to the new coordinates (we assume ω to be known). So from an empiricist or operational point of view the new coordinates are impeccable. However, from the special theory of relativity we know that material bodies at rest in the new coordinates may not exist (ωr may be greater than c, the velocity of light). It is true, therefore, that the new coordinates will not always have a direct interpretation in terms of co-moving bodies—but this is something to be distinguished sharply from the more general question of whether they have adequate empirical significance at all.

Substitution of the rotating coordinates into the expression for the line element yields $ds^2 = (c^2 - {r'}^2\omega^2)dt'^2 - dr'^2 - {r'}^2d\varphi'^2 - dz'^2 - 2\omega {r'}^2 d\varphi' dt'$. As we already mentioned, it is a basic principle of the special theory of relativity that the line element supplies all information about the physics of the situation, as described in the given coordinates. It was also mentioned above that the traditional link between ds and physical concepts makes use of clocks and measuring rods. However, there is another and more fundamental physical interpretation available that only makes use of the basic laws of motion: as long as no disturbing forces act, point particles follow time-like geodesics and light follows null-geodesics in the metric defined by ds^2. The relation between these dynamical aspects (how particles and light move) and the metrical aspects (rods and clocks) will be the subject of comments in section 4.

3 Rods and clocks

Let us for the moment stay with the physical interpretation of ds in terms of measurements performed with rods and clocks. Concerning time, the coordinating principle is that ds/c represents proper time, measured by a clock whose world line connects the events between which ds is calculated. This principle entails that a clock at rest in the rotating frame will indicate the proper time

$$ds/c = \sqrt{(1 - r'^2\omega^2/c^2)}dt'. \tag{1}$$

Because $t' = t$ and t has the physical meaning of the time indicated by a clock at rest in the old frame, this implies that clocks at rest in the rotating frame are slow compared to clocks in the original ("laboratory") frame.

With regard to spatial distances, the interpretative principle is that $\sqrt{-ds^2}$ gives the length of an infinitesimal rod whose endpoints are simultaneous according to standard simultaneity in the rod's rest frame ([3], p.187). (A rod is a three-dimensional object, so we need a stipulation about the instants at which its endpoints should be considered in order to get a four-dimensional interval for which ds can be calculated.) When we apply this rule to rods that are at rest in the rotating frame of reference, we encounter the complication that $dt' = 0$ does not automatically correspond to standard simultaneity in the rotating frame. The definition of standard synchrony of two (infinitesimally near) clocks A and B is that a light signal sent from A to B and immediately reflected to A, reaches B when B indicates a time that is halfway between the instants of emission and reception, respectively, as measured by A. Suppose that A and B, both at rest in the rotating frame, have positions with coordinate differences dr, $d\varphi$ and dz—from now on we drop the primes of the rotating coordinates. A light signal between A and B follows a null-geodesic:

$$ds^2 = (c^2 - r^2\omega^2)dt^2 - dr^2 - r^2d\varphi^2 - dz^2 - 2\omega r^2 d\varphi dt = 0. \tag{2}$$

This equation gives the following solutions for dt when it is applied to the signals from A to B and back, respectively:

$$dt_{1,2} = \frac{\pm\omega r^2 d\varphi + \sqrt{(c^2 - \omega^2r^2)(dz^2 + dr^2) + c^2r^2d\varphi^2}}{c^2 - \omega^2 r^2}. \tag{3}$$

If t_0 is the time coordinate of the emission event at A, the event at A with time coordinate $t_0 + 1/2(dt_1 + dt_2)$ is standard-simultaneous with the event at B with time coordinate $t_0 + dt_1$. It follows that standard synchrony between infinitesimally close events corresponds to the following difference in t-coordinate:

$$dt = (t_0 + dt_1) - (t_0 + 1/2dt_1 + 1/2dt_2) = (\omega r^2 d\varphi)/(c^2 - \omega^2 r^2). \qquad (4)$$

As was to be expected, it is only for events that differ in their φ-coordinates that $dt = 0$ is not equivalent to standard simultaneity; indeed, the instantaneous velocity of the rotating frame is tangentially directed, and the relativistic dilation and contraction effects only take place in the direction of the velocity.

The spatial distance between two infinitesimally near points, as measured by a rod resting in the rotating frame, is found by substituting the just-derived value of dt, (4), in the expression for ds. The result is the following expression for the 3-dimensional spatial line element:

$$dl^2 = dr^2 + \frac{r^2 d\varphi^2}{1 - \omega^2 r^2/c^2} + dz^2. \qquad (5)$$

We could have found (1) and (5) in a simpler way by making use of the standard expressions for the time dilation and Lorentz contraction undergone by clocks and rods, respectively, that possess the instantaneous velocity ωr. However, the use of the line element as the central theoretical quantity provides us with a unifying framework that makes it easier to discuss the relation between metrical and dynamical concepts.

4 Space and time without rods and clocks

In his Autobiographical Notes, Einstein already points out that from a fundamental point of view it is unsatisfactory to interpret ds via measuring procedures with complicated macroscopic instruments. Indeed, this could create the false impression that rods and clocks are basic entities without which the theory would have no physical content. However, it is clear that rods and clocks themselves consist of more fundamental entities, like atoms and molecules. In principle it would therefore be better to base the interpretation of the theory directly on what it says about the fundamental constituents of matter. It is only because no complete theory of matter was available, Einstein explains, that it was expedient to introduce the theory through measurements by rods and clocks. In principle they should be eliminated at a later stage.

This desideratum, to do without rods and clocks, becomes even more urgent when accelerated frames of reference are considered, as in the case of our rotating world. Obviously the motions of clocks and rods that are stationary in the rotating frame are not inertial. Centrifugal and Coriolis forces will therefore arise, which will distort the rotating instruments. It is not a priori clear that such deformed instruments will keep on functioning as indicators of ds. Indeed, one could easily think of rods or clocks that would be torn apart by centrifugal forces and would therefore certainly not indicate any length or time intervals.

Fortunately, it *is* possible to found the space-time description of our rotating world on a more fundamental level than that of macroscopic measuring devices. In fact, in general space-times one can use the basic principles that time-like geodesics are physically realized by inertially moving point-particles and that null-geodesics represent light rays, to define space-time distances between neighboring events ([4], section 16.4). In our case, Minkowski space-time, we can start by constructing a set of elementary 'light clocks' by letting light signals bounce back and forth between neighboring parallel particle geodesics. If we confine our attention to the plane $z = 0$, we can take the geodesics defined in the laboratory frame (the inertial system we started with) by constant r, φ and $r+dr$, φ, respectively. The thus constructed clock has a constant period (the dt between two 'ticks') of $2dr/c$. In other words, we have here an elementary process that provides a physical realization of t; and we have come to this conclusion on the basis of the dynamical postulates alone (the only ingredient is that light follows null-geodesics). Length can be determined in a similar way: let a light signal depart from A, with fixed r and φ and go to a neighboring position B with $r + dr$ and $\varphi + d\varphi$ from which it returns immediately to A. Let the round trip time measured at A be dt. We can now define the spatial distance dl between A and B as $cdt/2$. From the postulate that light follows null-geodesics it follows that $dl^2 = dr^2 + r^2 d\varphi^2$. In this way the laboratory coordinates obtain metrical significance, without reliance on macroscopic clocks and rigid rods. When such (complicated) systems are introduced at a later stage, we can study their workings on the basis of the fundamental laws of physics governing their constituents and see, on that basis, whether they are indeed suitable to measure the just-defined intervals.

We now turn our attention to measurements performed within the rotating system, i.e. with instruments resting in the rotating coordinates. From Eq. (3) we see that the round trip time dt needed by a light signal between two neighboring points that are stationary in the rotating frame of reference is given by

$$dt = dt_1 + dt_2 = 2\frac{\sqrt{(c^2 - \omega^2 r^2)dr^2 + c^2 r^2 d\varphi^2}}{c^2 - \omega^2 r^2}.$$

If the laboratory coordinate t is used as the measure of time, and if the definition $dl = cdt/2$ is used to fix spatial distances, we arrive at the metric

$$dl^2 = \frac{(1 - \omega^2 r^2/c^2)dr^2 + r^2 d\varphi^2}{(1 - \omega^2 r^2/c^2)^2}.$$

However, it is more natural to link the measure of time intervals in the rotating system to the indications furnished by light clocks that are co-moving, i.e. stationary in the rotating coordinates instead of stationary in the laboratory frame. So let a light ray bounce back and forth between two points that only differ in their r-coordinate, by the amount dr, in the rotating frame. It follows from the expression (2) that the period of the thus defined clock is $2dr/\sqrt{c^2 - \omega^2 r^2}$, whereas the period of the similar and instantaneously coinciding clock in the laboratory frame is $2dr/c$. The period of the rotating light clock is therefore longer, by a factor $1/\sqrt{1 - \omega^2 r^2/c^2}$, than the period of the laboratory clock. When we now define distances as $cd\tau/2$, with τ measured in the new 'co-moving' time units, we have to multiply the distances we found a moment ago by $\sqrt{1 - \omega^2 r^2/c^2}$. The final result is

$$dl^2 = dr^2 + r^2 d\varphi^2/(1 - \omega^2 r^2/c^2).$$

This is the same result as we found in Eq. (5).

5 Accelerating measuring devices

The above sketch shows how we can achieve a physical implementation of the two systems of coordinates, and give them metrical meaning, by the sole use of point-particles and light. The thus defined space-time distances can be used to calibrate macroscopic measuring rods and clocks. Indeed, it is clear that in general such instruments will be deformed by the rotational motion, and that this will introduce inaccuracies in their readings.

The general effect of accelerations can be illustrated by the consideration of a light-clock of the kind mentioned above: a light signal bouncing back and forth between two particle world-lines. Light travelling to and fro between two mirrors resting in an inertial system, with mutual distance L, defines a clock with half period $T = L/c$. When the two mirrors move uniformly with the same velocity $\overrightarrow{v}$, in a direction parallel to their planes, a simple application of the Pythagorean theorem shows that the half period of the moving clock becomes $L/(c\sqrt{1 - v^2/c^2}) = T/\sqrt{1 - v^2/c^2}$. This demonstrates the presence of time dilation in the case of a moving light-clock (by means of the relativity principle this result can be extended to other time-keeping devices). Consider now what happens if the velocity is not uniform but the system starts accelerating when the light leaves the first mirror, with a small acceleration $\overrightarrow{a}$ in the direction of $\overrightarrow{v}$. As judged from the inertial frame, the light now needs a time T' to reach the second mirror; during this time the accelerating mirror system has covered a distance $s \approx vT' + 1/2aT'^2$. Application of Pythagoras now yields $c^2T'^2 = L^2 + s^2$. It follows that

$$c^2T'^2 = L^2 + v^2T'^2 + avT'^3 + 1/4a^2T'^4. \tag{6}$$

The half period T' that follows from this equation obviously depends on a. However, it is also obvious that the extent of the change in the period caused by a depends on the magnitude of T' itself. If we make T' in Eq. (6) very small, by reducing L, we find in the limiting situation $T' = T/\sqrt{1 - v^2/c^2}$, just as in the case of the uniformly moving clock. In other words, the acceleration has an effect, but the magnitude of this effect depends on the peculiarities of the specific clock we are considering (in this case on L). This acceleration-dependent effect can be made as small as we wish, by using suitably constructed clocks (in the example: by reducing L). What remains in all cases is the universal effect caused by the velocity.

This shows in what sense velocities have a universal effect on length and time determinations, but accelerations not. There is no independent postulate involved here; everything can be derived from the dynamical principles of special relativity theory, by considering the inner workings of the measuring devices. It turns out that acceleration-dependent effects are there, but can be varied, and corrected for, by varying the characteristics of the devices. This is the real content of the textbook statement that acceleration has no metrical effects. It should be stressed again that this does not constitute a new hypothesis that has to be *added* to the dynamical principles of the theory of relativity. Quite to the contrary, the effects of accelerations on any given clock or measuring rod can be computed from the dynamical principles applied to these devices.

Of course, that the magnitudes of distortions will depend on the specific constitutions of the rods or clocks in question is only to be expected. Robust rods and clocks will be less affected accelerations than fragile ones. One way of correcting for the deformations is to gauge the accelerating instruments against the light measurements results described in section (4). The expressions (1) and (5) should be understood as applying to the results of space-time measurements performed with thus corrected measuring devices.

6 Space and time in the rotating frame

The spatial geometry defined by the line element (5) is non-Euclidean, with a negative r-dependent curvature (see [5], pp. 330-337). One of the notorious characteristics of this geometry is that the circumference of a circle with radius r (in the plane $z = 0$) is $2\pi r/(1-\omega^2 r^2/c^2)$, which is greater than $2\pi r$. The recognition that the geometry in accelerated frames of reference will in general be non-Euclidean, which through the equivalence principle suggests that the presence of gravitation will also cause deviations from Euclidean geometry, played an important role in Einstein's route to General Relativity. We will restrict ourselves to the special theory, however.

The properties of time in the rotating frame are perhaps even more interesting than the spatial characteristics. Expression (4) demonstrates that standard simultaneity between neighboring events in the rotating frame corresponds to a non-zero difference dt. It follows that if we go along a circle with radius r, in the positive ϕ-direction, while establishing standard simultaneity along the way, we create a 'time gap' $\triangle t = 2\pi\omega r^2/(c^2 - \omega^2 r^2)$ upon completion of the circle. Doing the same thing in the opposite direction results in a time gap of the same absolute value but with opposite sign. So the total time difference generated by synchronizing over a complete circle in one direction, and comparing the result with doing the same thing in the other direction is $\triangle t = 4\pi\omega r^2/(c^2 - \omega^2 r^2)$.

Now suppose that two light signals are emitted from a source fixed in the rotating frame and start travelling, in opposite directions, along the same circle of constant r. We follow the two signals while locally using standard synchrony; this has the advantage that locally the standard constant velocity c can be attributed to the signals. We therefore conclude that the two signals use the same amount of time in order to complete their circles and return to their source, as calculated by integrating the elapsed time intervals measured in the successive local comoving inertial frames (the signals cover the same distances, with the same velocity c, as judged from these frames). However, because of the just-mentioned time gaps the two signals do not complete their circles simultaneously, in one event. There is a time difference $\triangle t = 4\pi\omega r^2/(c^2 - \omega^2 r^2)$ between their arrival times, as measured in the coordinate t. This is the celebrated Sagnac effect (see [6], p. 652 for a related derivation).

The Sagnac effect directly reflects the space-time geometry of the rotating frame; it does not depend on the specific nature of the signals that propagate in the two directions. Indeed, as long as the two signals have the same velocities in the locally defined inertial frames with standard synchrony, the difference in arrival times is given by the above time gap. So the same Sagnac time difference is there not only for light, but for any two identical signals running into two directions. The Sagnac experiment directly probes the space-time relations in the rotating frame.

Because of the difference in arrival times of the two light signals, the velocity of light obviously cannot be everywhere the same in the rotating coordinates. This is a consequence of the fact that in the rotating frame events with equal time coordinate t are not standard simultaneous. So t may appear as an unnatural time coordinate for the rotating frame: it would be desirable to have a time coordinate that *would* reflect standard simultaneity everywhere. The question can therefore be asked whether we could define a coordinate $\tilde{t}$ in such a way that $d\tilde{t} = 0$ would imply standard synchrony in the local inertial frame. Suppose that $\tilde{t} = \tilde{t}(t, r, \varphi)$, then we should have that $d\tilde{t} = 0$ if Eq. (4) holds. This implies that $\omega^2 r^2/(c^2 - \omega^2 r^2)\partial\tilde{t}/\partial t + \partial\tilde{t}/\partial\varphi = 0$ and $\partial\tilde{t}/\partial r = 0$. In view of the axial symmetry in our frame we may assume that $\partial\tilde{t}/\partial\varphi = 0$. The only solution of our partial differential equations is therefore that $\tilde{t}$ is independent of r, φ and t, which clearly is unacceptable. Therefore, it turns out to be a basic characteristic of the rotating frame that the locally defined Lorentz frames do not mesh: they cannot be combined into one frame with a globally defined standard simultaneity. Evidently it *is* possible to define global time coordinates, like t; but the description of physical processes in terms of these coordinates must necessarily differ from the standard description in inertial systems. The non-constancy of the velocity of light in the rotating system furnishes an example. It should be noted that this peculiarity of the description of physical processes in the rotating system is not a consequence of the presence of centrifugal and Coriolis forces:

indeed, in our space-time determinations we have compensated for the effects of such forces. It is the space-time geometry itself that is at issue.

7 Simultaneity, slow clock transport and conventionality

As we saw in the previous section, the Sagnac effect is independent of the nature of the signals that propagate into the two directions on the rotating disc. So, if we transport two clocks along a circle with radius r around the center of the disk, one clockwise and one counter-clockwise, while keeping their velocities the same in the locally co-moving inertial frames, there will be a difference $\triangle t = 4\pi\omega r^2/(c^2-\omega^2 r^2)$ between their return times (measured in the laboratory time t). It is well known that the indications of the clocks will conform to standard simultaneity in the limiting situation of vanishing velocities. That is, if the clocks are transported very slowly with respect to the rotating disc, they will remain synchronized according to standard simultaneity in the local inertial frames. It follows that slow clock transport cannot be used to define an unambiguous global time coordinate on the rotating disc: in the just-mentioned case the result will depend on whether a clockwise or counter-clockwise path is chosen. In general, the result of synchronization by slow clock transport will be path dependent.

With regard to time in inertial frames, there has been a long-standing and notorious debate about whether standard simultaneity ($\varepsilon = 1/2$ according to Reichenbach's formulation) is conventional or not. One of the arguments often put forward against the conventionality thesis is that the natural procedure of slow clock transport leads to $\varepsilon = 1/2$, thus showing its privileged status. In the case of the rotating world, this argument can only be applied locally. Neither the Einstein light signal procedure, nor the slow transport of clocks can be used to establish a global notion of simultaneity on the rotating disc.

More generally, it cannot be denied that in inertial frames standard simultaneity has a special status: it allows a simple formulation of the laws, conforms to slow clock transport and other physically plausible synchronization procedures, and agrees with Minkowski-orthogonality with respect to world lines representing the state of rest [7]. So time coordinates t that correspond to this notion of simultaneity (in the sense that $dt = 0$ expresses simultaneity) may be said to be privileged. In non-inertial frames this still is so, though now the argument applies only locally. The rotating system illustrates the situation very well: in each point on the disc standard simultaneity can be defined just as in an inertial system, but this does not result in a global time coordinate. This supports the general conclusion of this paper, namely that the difference between the status of coordinates in inertial and non-inertial frames of reference, or special and general relativity, is not so much a matter of epistemology—or philosophical analysis of the meaning of coordinates—but rather a matter of physical facts. In global inertial systems privileged coordinates can be chosen that have a global metrical interpretation. In reference frames that are not globally inertial such privileged coordinates do not exist in general. This is not a matter of a different philosophical status of coordinates, but rather a reflection of different global space-time symmetry properties—a factual physical difference rather than a philosophical distinction.

The purpose of *coordinates* is to label events unambiguously, which can be done in infinitely many different ways. The choice between these different possibilities is a matter of pragmatics; though there may be very good reasons to prefer one choice over another. Thus, in inertial frames of reference time coordinates that reflect standard simultaneity lead for many purposes to an especially simple description. In this case there exists a physically significant global temporal relation between events, and coordinates that are adapted to this relation inherit its special status. But in the general case no physically significant simultaneity relation exists. Global "simultaneity" can then only refer to some global time coordinate, which is chosen conventionally. This is true in non-inertial frames of reference, like the rotating disc, and in generally relativistic space-times in which there are no global temporal symmetries. These non-inertial frames of reference, and general relativistic space-times, seem an arena where the thesis that (global) simultaneity is conventional can be defended without controversy.

8 The rotating Ehrenfest cylinder

Not only in its temporal aspects, but also in its spatial physical properties the rotating frame differs globally from an inertial frame. Until now we spoke about a rotating frame of reference as defined by a set of rotating *coordinates*, without discussing a possible material realization of this frame. It is clear from the outset that the special theory of relativity sets limits to such a realization: objects at rest in the rotating frame should not move faster than light as judged from the inertial laboratory frame. This implies that $\omega r < c$ should hold for such an object. In other words, there is an upper bound to the value of r that can be realized materially.

However, even if this condition is satisfied there remain interesting questions, as made clear by Ehrenfest in his famous note on the subject [8]. Suppose that a solid cylinder of radius R is gradually put into rotation about its axis; finally it reaches a state of uniform rotation with angular velocity ω. It would seem that in the final state the cylinder has to satisfy contradictory requirements: on the one hand the Lorentz contraction should make the circumference shorter, on the other hand the radial elements should not contract because their motion is normal to their lengths. From symmetry it is clear that the form of a cross section of the moving cylinder remains a circle, as judged from the laboratory frame; but this would apparently mean that the circumference of the circle has become smaller while the radius has stayed the same. This is inconsistent (remember that Euclidean geometry holds in the laboratory frame).

The solution of this paradox is that the various parts of the cylinder, being fastened to each other, cannot move freely and therefore cannot Lorentz contract as freely moving infinitesimal measuring rods would do. What will happen to the cylinder during its acceleration depends on the elastic properties of the material: tensions will develop because the tangential elements want to shrink, whereas the radial elements do not. A possible scenario is that the tangential elements will be stretched as compared to their natural (i.e. Lorentz contracted) lengths. Another possibility, if the material is sufficiently strong, is that the radius will contract, allowing the circumference to contract too. However, if ω becomes big enough one would have to expect that the tensions and strains grow to such an extent that they cause the cylinder to explode. This makes it clear that the Lorentz contraction can be responsible for clearly dynamical effects—the contractions are not just a matter of "perspective" (see [9] and [10]). (Of course, this whole discussion is rather academical because centrifugal forces will tear the cylinder apart before the relativistic effects become noticeable.)

As long as the cylinder survives, and keeps its cylindrical shape (as judged from the laboratory frame), not all its elements will be free from deformations, tensions or strains. However, the length determinations by measuring rods at rest in the rotating frame, as discussed in section 3, were supposed to be carried out with freely movable rods that are not hampered in their Lorentz contractions. So measuring rods laid out along the circumference of the circle will have undergone a Lorentz contraction, whereas rods laid out along a radius will have retained their rest length (as judged from the laboratory system). The measurement would reveal that the circumference is longer than 2π times the radius, in conformity with equation (5).

The spatial geometry of the disc is therefore non-Euclidean. That means that distance relations must be represented by a metrical tensor that cannot be put into the Euclidean diagonal form everywhere. It remains possible, of course, to choose coordinates locally in such a way that the Euclidean form results at the point in question. The difference from the inertial system concerns global aspects, not local ones. The impossibility to define a global coordinate system in which the metrical tensor reduces to its Euclidean standard form implies that there cannot be coordinates whose differences correspond to distances everywhere. The situation is analogous to the one we discussed in the context of time coordinates: nothing changes in the status and meaning of coordinates when we go from inertial to non-inertial systems. The things that do change are the global characteristics of the physical geometry, which are coordinate-independent.

Conclusion

The transition from inertial to non-inertial frames of reference, and the transition from special to general relativity, does not imply a change in the status and meaning of coordinate systems. It is therefore a misunderstanding to think that general relativity allows a wider class of coordinate systems than classical physics or special relativity. In classical physics and in relativity theory, both in inertial systems and non-inertial systems, coordinates just serve to label events. The choice for a particular coordinate system from the infinity of possible ones is dictated by pragmatic considerations.

What *does* change in the transition from inertial to non-inertial systems, and from special to general relativity, are the global aspects of the physical spatial and temporal relations. Pragmatic arguments for choosing one coordinate system over another may therefore lead to different choices in the different situations: if geometrical relations have become different, coordinate systems with different characteristics, adapted to the new geometry, may lead to a simpler description. But this does not change the conventional nature of the coordinates.

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