Origin of the Electron's Inertia and Relativistic Energy-Momentum Equation in the Spin-½ Charged-Photon Electron Model

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Abstract

The electron's inertial mass m is derived from the circling momentum $mc = E_o / c$ of a proposed circulating spin-½ charged photon that is modeling an electron. Newton's second law $F = dp / dt = ma$ is used to derive the electron's inertial mass m. Here a is the centripetal acceleration of the circling charged photon. The electron's relativistic energy-momentum equation $E^2 = p^2c^2 + m^2c^4$ is derived from the charged photon's momentum equation $P^2 = p^2 + (mc)^2$, where $P = E / c = \gamma mc$ is the circulating spin-½ charged photon's total momentum, $p = \gamma mv$ is its longitudinal component of momentum (the external momentum of the electron) and mc is its transverse component of momentum (the inner momentum of the electron).

Key words: inertial mass, electron, photon, energy-momentum equation, model, Dirac equation

Introduction

Matter has the interesting property that it resists a change in its state of motion. This property of matter is called inertia. Newton's first law of motion is sometimes called the law of inertia. It was actually first stated by Galileo, developed by Descartes and later included in Newton's three laws of motion. Translated from the Latin of Newton's Principia, the first law is: "The *vis insita*, or innate force of matter, is a power of resisting by which every body, as much as in it lies, endeavors to preserve its present state, whether it be of rest or of moving uniformly forward in a straight line" or "Every body persists in its state of being at rest or of moving uniformly straight forward except insofar as it is compelled to change its state by force impressed". In modern terminology, Newton's first law of motion has become: "Unless acted upon by a net unbalanced force, an object will maintain a constant velocity." What Isaac Newton called the state of motion of an object is now called the object's momentum.

No innate force in matter was ever discovered to explain its inertia. The term "inertia" has come to mean "the amount of resistance of an object to a change in its velocity". The quantitative measure of the resistance of an object to acceleration *a* upon the application of an external force *F* is now called the body's inertial mass: $m = F / a$.

Newton originally defined the quantity of matter in an object as the product of the object's density and its volume. But since density is mass per unit volume, this is a circular definition of the quantity of matter. The quantity of matter, or mass, of an object can be measured, relative to some standard quantity of matter, but the cause of the inertia of matter is unknown.

The standard international unit today in which mass is measured is the kilogram. The momentum of an object is measured by the mass m times the vector velocity *v* of the object : $p = mv$, if the object is moving very slowly compared to the speed of light, or $p = \gamma mv$ for relativistic objects, where $\gamma = 1/\sqrt{1 - v^2 / c^2}$ and *c* is the speed of light. Momentum, like velocity, is a vector quantity, having both magnitude and direction.

It is commonly said that the mass of an object increases with the object's speed as this speed approaches the speed of light. But this is not how mass is understood today among most physicists. Nowadays the mass *m* of an object is defined as the object's mass when it is not moving. This is called the object's invariant mass *m* or simply the object's mass *m*. The mass *m* of an object is now considered to be independent of the velocity of the object. An object having a mass $m = 1$ kilogram has the same invariant mass when it is moving at half the speed of light as when it is at rest. In Einstein's theory of special relativity, an object of mass *m* has an energy $E_o = mc^2$ when it is at rest. E_o is called the rest energy of the electron. An electron's inertia is quantitatively associated with the mass $m = E_o / c^2$ of the electron. When an electron is moving with a velocity *v* less than *c*, it has total energy $E = \gamma mc^2$. The basic equation relating the total energy *E* of a moving object to its momentum *p* and its mass *m* is the relativistic energy-momentum equation: $E^2 = p^2c^2 + m^2c^4$. Photons have energy and momentum and move at light-speed, but have no invariant mass *m* and therefore have no inertia, according to standard physics. An object is said to have an inertial mass defined in Newton's second law of motion by $m = F / a$. For a resting electron this inertial mass *m* is the same as the electron's invariant mass *m*.

A little background about the electron

Newton did not know about electrons, which are very small particles of electrically charged matter discovered in 1897. Electrons have mass and inertia. The mass *m* of an electron is 9.11×10^{-31} kilograms. Electrons also carry a negative charge $-e = -1.602 \times 10^{-19}$ Coulombs. Electrons have a characteristic spin component $s = \frac{1}{2}\hbar$ where $\hbar = h / 2\pi$ and *h* is Planck's constant and equals 6.626×10^{-34} Joule seconds. So $s = 5.26 \times 10^{-35}\, Js$ for an electron. The electron's resting energy is

$E_o = 0.511 MeV$ where MeV is one million electron volts, and one electron volt is equal to 1.602×10^{-19} Joules in standard energy units. If a photon has the same energy $E_o = mc^2 = h\nu = hc/\lambda$ of the electron, this photon's wavelength λ is called the Compton wavelength and equals $\lambda_{Compton} = h/mc = 1.426 \times 10^{-12}$ meters. An electron also acts like a little magnet and has a property called its magnetic moment μ, which has been measured extremely precisely experimentally. This experimental value has also been predicted theoretically extremely precisely by the theory of quantum electrodynamics (QED). The electron also has wave properties—a slow moving electron is associated with a wave having the de Broglie wavelength $\lambda_{dB} = h/mv$, while for a relativistic electron this value is $\lambda_{dB} = h/\gamma mv$.

The electron model's inertial mass m calculated from its circling internal momentum *Eo/c*

It is a result from basic calculus that if a vector $\vec{A}$ rotates in a plane with a constant angular velocity ω, then the rate of change of vector $\vec{A}$ with time has magnitide $|d\vec{A}/dt| = \omega A$, and this rate-of -change vector $d\vec{A}/dt$ having magnitude ωA is always perpendicular to the vector $\vec{A}$ as $\vec{A}$ rotates, and lies in the plane of rotation . This result will be applied to a charged photon with momentum *mc* moving in a circle with a constant angular velocity ω.

The inertial mass *M* of an object is the mass in Newton's second law: $F_{net} = Ma$ or $M = F_{net}/a$. But $F_{net} = dp/dt$, the rate of change of an object's momentum with time. If an object such a charged photon has momentum *p* and moves in a circle with radius *R* and constant speed *v*, and so with constant angular frequency $\omega = v/R$, then $dp/dt = \omega p$ and points towards the center of the circle. Similarly, the object's centripetal acceleration is $a_c = v^2/R = \omega^2 R$.

Combining these above relationships gives

$$M = (dp/dt)/a_c = \omega p/(\omega^2 R) = p/\omega R = p/v$$

where *p* is the momentum of the circling object and *v* is its speed, which is lightspeed *c* for a photon and less than *c* for a particle with mass.

In the case of a circling photon with photon momentum $p = E_o/c = mc$ where $m = E_o/c^2 = 0.511 MeV/c^2 = 9.11 \times 10^{-31}\ kg$, and photon speed is *v=c*, we have from above:

$$M = p/v = (E_o/c)/c = E_o/c^2 = m.$$

So we have shown that a circling spin-½ charged photon, proposed to compose an electron, does have inertial mass *M* equal to the mass $m = E_o/c^2$ of the electron. We have derived the inertial mass of the electron from the rotating momentum *mc* of the circling spin-½ charged photon.

This is an important result. If the electron that is being modeled as a circulating spin-½ charged photon has inertial mass *m*, this *m* is the inertial mass of the spin-½ circulating charged photon modeling the electron.

An electron's total momentum is derived from its internal and external momentum in the spin-½ charged-photon model of the electron

Recently the author proposed that an electron is composed of a helically-circulating spin-½ charged photon (1). While an electron is considered to be a particle of matter, the spin-½ charged photon composing an electron is proposed to be a new variety of photon. The spin-½ charged photon composing a resting electron makes a closed circular double loop whose total length λ is one Compton wavelength $\lambda_{Compton} = h/mc$.

According to this new hypothesis of the electron, the circling spin-½ charged photon of an electron has an invariant internal circling linear momentum *mc*. One way of looking at the inertia of the electron is to consider that the source of the electron's invariant mass *m* is this invariant internal circulating momentum *mc*. In addition, an electron moving forward with speed *v* has external linear momentum $p = \gamma mv$ and total energy $E = \gamma mc^2$. In this new electron model, the internal momentum *mc* and the external momentum $p = \gamma mv$ are always perpendicular. See Figure 1 below. They combine, according the Pythagorean theorem, to yield a total electron linear momentum $P = \gamma mc$ given by $P^2 = p^2 + (mc)^2$.

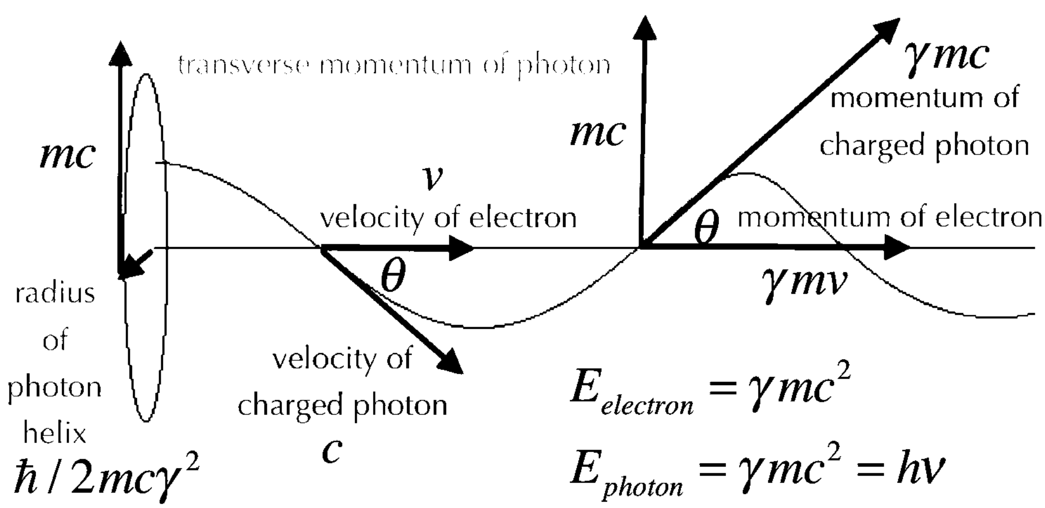

Figure 1. Velocity, momentum and energy relationships for the charged photon model moving along its helical trajectory. The velocity and momentum vectors of the charged photon and its components related to the electron being modeled are indicated.

This total electron linear momentum *P* is the linear momentum of the helically circulating spin-½ charged photon composing the electron. This spin-½ charged photon's energy is $E = Pc$, or $P = E/c$. The circling internal momentum *mc* is the transverse component of the charged photon's total momentum *P*, while the electron's external momentum $p = \gamma mv$ is the longitudinal component of the charged photon's total momentum *P*. This gives the equation

$$(E/c)^2 = p^2 + (mc)^2$$

which when multiplied by c^2 gives

$$E^2 = p^2c^2 + m^2c^4$$

This is the standard relativistic energy-momentum equation for an electron. So the total energy of an electron can be seen as $E = Pc$ where *P* is the total momentum of the helically circulating spin-½ charged photon composing the electron.

The transverse inertial mass of a relativistic electron's charged photon model moving on its helical trajectory

Let us calculate by the above method the transverse inertial mass M_{trans} of the spin-½ charged photon of a relativistic electron moving at a constant speed *v* in its longitudinal direction. This value corresponds to the transverse inertial mass in relativistic dynamics terminology because the acceleration in the helically circulating spin-½ charged photon model of the electron is transverse or perpendicular to the forward velocity of the electron being modeled.

In the spin-½ charged photon model of the electron, the charged photon is moving along a helical trajectory of radius $R = R_o/\gamma^2$, where $R_o = \hbar/2mc$. The charged photon's trajectory makes an angle θ with the forward or longitudinal direction, given by $\cos(\theta) = v/c$. By the Pythagorean theorem, this gives $\sin(\theta) = 1/\gamma = \sqrt{1 - v^2/c^2}$. In the spin-½ charged photon model of the electron, the momentum of the charged photon along its helical trajectory is $P = \gamma mc$. In the spin-½ charged photon model, the angular velocity ω of the helically moving charged photon around its helical axis is $\omega = 2\gamma mc^2/\hbar$.

Using the above information about the circulating charged photon model, the transverse component P_{trans} of the charged photon's momentum that is perpendicular to the longitudinal motion of the charged photon is

$$P_{trans} = P\sin(\theta) = (\gamma mc) \times (1/\gamma) = mc$$

The centripetal acceleration a_c of P_{trans} around its helical axis is

$$a_c = \omega^2 R$$

So

$$\begin{aligned} M_{trans} &= (dP_{trans} / dt) / a_c \\ &= \omega P_{trans} / (\omega^2 R) \\ &= P_{trans} / (\omega R) \\ &= mc / \{(2\gamma mc^2 / \hbar) \times (\hbar / 2\gamma^2 mc)\} \\ &= mc / (c / \gamma) \\ &= \gamma m \end{aligned}$$

This is the traditional value of the calculated transverse inertial mass of a moving electron, which reduces to $M_{trans} = m$ for a stationary electron as in the previous calculation of the inertial mass for a stationary electron.

The longitudinal inertial mass of the charged photon model moving on its helical trajectory

The circulating charged photon's longitudinal momentum is $P_{long} = \gamma mv$, where *v* is the speed of the electron along the longitudinal direction. The longitudinal inertial mass is M_{long} when the spin-½ charged photon is accelerated in the longitudinal direction. Since P_{long} of the charged photon equals the electron's momentum $p = \gamma mv$, the calculation of M_{long} for the spin-½ charged photon model of the electron gives the same result as the standard calculation for M_{long} in relativistic kinematics of the accelerated electron:

$$dP_{long} / dt = d(\gamma mv) / dt = \gamma m \ dv / dt + mv \ d(\gamma) / dt$$

which, in the standard relativistic calculation where a_{long} is the circulating charged photon's (the electron's) longitudinal acceleration, leads to

$$M_{long} = (dP_{long} / dt) / a_{long} = \gamma^3 m$$

So both of the traditional transverse and longitudinal inertial masses M_{trans} and M_{long} for the accelerated relativistic electron are found from the spin-½ charged photon model of the relativistic electron. The spin-½ charged photon model of the relativistic electron, like the electron itself, has a greater inertial mass for a longitudinal direction of acceleration that for the transverse direction because an accelerating force in the longitudinal direction of the electron's motion increases the total energy of the electron, while a transverse acceleration of the electron does not. In other words, a greater force is required to get same acceleration of an electron when the electron's velocity increases than when the electron merely changes direction while its speed remains constant. These equations for the inertial mass for a relativistic electron are of practical value when accelerating electrons in circular or linear accelerators. In modern usage, the terms "transverse relativistic mass" and "longitudinal relativistic mass" are not used so much, though the quantitative relations for accelerating an electron are the same whether these terms are used or not. Instead, the electron's mass *m* is now considered to be invariant and $p = \gamma mv$ is used for the electron's transverse linear momentum rather than $p = Mv$ where $M = \gamma m$. $M = \gamma m$ used to be called the relativistic mass of the electron.

Does a photon have inertial mass?

A photon of energy $E = h\nu$ moving linearly with momentum $p = E/c = h\nu/c$ is not traditionally associated with an inertial mass, since a photon doesn't change its velocity or direction continuously, except under gravitational influences. A photon's inertial mass is said to be zero because it has no invariant mass *m*. But there is no unanimity among physicists today as to whether or not a photon has an inertial mass even though it has no invariant mass. When the proposed spin-½ charged photon moves circularly in the charged photon model of the resting electron, the charged photon is calculated to have an inertial mass $m = E_o / c^2$ where $E_o = 0.511 MeV$ for an electron. This suggests that a free photon would also have an inertial mass $m = E/c^2 = h\nu/c^2$. But the term "inertial mass" for a photon is considered to be redundant because it is always proportional to the energy $E = h\nu$ of the photon. In general relativity, energy has inertia so photons would also have inertia. Still, it is currently unfashionable in physics to speak of the inertial mass of a photon. Perhaps the discussion of the origin of the inertial mass *m* of an electron as derived from the circulating momentum $p = E/c = h\nu/c$ of a charged photon can add something useful to the discussion about the possible inertial mass of a normal photon.

The numerical values of the internal momentum, internal angular frequency, internal centripetal acceleration, and internal force in the spin-½ charged photon model of a resting electron

We have derived the inertial mass of a resting and a moving electron, modeled as a circulating spin-½ charged photon, without mentioning the numerical values of a) the internal angular frequency ω of rotation, b) the circulating internal momentum *mc*, c) the internal centripetal acceleration $a_c = \omega^2 R_o$, and d) the internal force $F = dp/dt$ required to rotate the internal momentum *mc* of the charged photon at this internal angular frequency ω. These quantities are calculated below for the spin-½ charged photon model of the resting electron.

a) **The internal angular frequency.** This internal angular frequency $\omega = \omega_{zitt}$ in a resting electron is given in the charged photon model by $\hbar\omega_{zitt} = 2mc^2$, or $\omega_{zitt} = 2mc^2 / \hbar$. In the spin-½ charged photon model of the electron the *zitterbewegung* ("jittery motion") angular frequency $\omega_{zitt} = 2mc^2 / \hbar$ corresponds to the *zitterbewegung* frequency $\nu_{zitt} = 2mc^2 / h$ found for the internal frequency of the Dirac electron. This *zitterbewegung* frequency is incorporated into the double-looping charged-photon model of the electron. So

$$
\begin{aligned}
\omega_{zitt} &= 2mc2 / \hbar \\
&= (2 \times 9.11 \times 10^{-31} kg)(3.00 \times 10^{8} m/s)^2 / (6.63 \times 10^{-34} Js / 2\pi) \\
&= 1.55 \times 10^{21} rad/s
\end{aligned}
$$

b) **The internal momentum *mc* of circulating charged photon.** The value of *mc* is given by

$$
\begin{aligned}
mc &= 9.11 \times 10^{-31} kg \times 3.00 \times 10^{8} m/s \\
&= 2.73 \times 10^{-22} kg\ m/s
\end{aligned}
$$

c) **The centripetal acceleration.** This is $a_c = \omega^2 R_o$, where $R_o = \hbar / 2mc$ in the double-looping charged-photon model of the electron. So

$$
\begin{aligned}
a_c &= \omega^2 R_o \\
&= (2mc^2 / \hbar)^2 \times \hbar / 2mc \\
&= 2mc^3 / \hbar \\
&= 2 \times (9.11 \times 10^{-31} kg) \times (3.00 \times 10^{8} m/s)^3 / (6.63 \times 10^{-34} Js / 2\pi) \\
&= 4.66 \times 10^{29} m/s^2
\end{aligned}
$$

d) **The force acting on the circulating momentum mc.** This is given by

$$
\begin{aligned}
F &= dp/dt = \omega_{zitt} p = \omega_{zitt} mc \\
&= (1.55 \times 10^{21} rad/s) \times (9.11 \times 10^{-31} kg) \times (3.00 \times 10^{8} m/s) \\
&= 0.424 N
\end{aligned}
$$

The source of this large force and correspondingly large centripetal acceleration within a single electron is not explained in the spin-½ charged photon model of the electron. An accelerated charge normally loses energy, according to standard electromagnetic theory. But the highly-centripetallyaccelerated negative charge of the circulating spin-½ charged photon model of the electron apparently does not lose energy due to this centripetal acceleration. Due to quantum effects, an electron in the ground state of the hydrogen atom does not radiate in spite of its changing speed in the hydrogen atom. Something similar may be going on for the proposed accelerated charge within an individual electron.

The relation of the internal circulating linear momentum mc of an electron to its external linear momentum gamma mv

It is interesting to ask "how fast must an electron be moving so that its relativistic momentum $p = \gamma mv$ equals its circulating internal momentum *mc*?" Setting the two momentum expressions equal gives:

$$\begin{aligned} \gamma mv &= mc \\ \gamma \mathrm{v} &= c \\ v/\sqrt{1-v^2/c^2} &= c \\ v/c &= \sqrt{1-v^2/c^2} \\ v^2/c^2 &= 1-v^2/c^2 \\ 2v^2/c^2 &= 1 \\ v^2/c^2 &= 0.5 \\ v/c &= \sqrt{0.5} = 1/\sqrt{2} \\ v &= c/\sqrt{2} = 0.707c \end{aligned}$$

An electron would have to move at 0.707 times the speed of light to have momentum *mc.* At this speed γ is given by

$$\begin{aligned} \gamma &= 1/\sqrt{1-v^2/c^2} \\ &= 1/\sqrt{1-0.5} \\ &= 1/\sqrt{0.5} \\ &= \sqrt{2} \\ &= 1.414 \end{aligned}$$

for having the electron's external momentum $p = \gamma mv$ equal to the electron's internal momentum *mc*.

This value of gamma corresponds to an electron whose relativistic kinetic energy KE is given (according to the standard formula for relativistic kinetic energy) by

$$KE = (\gamma - 1)mc^2$$
$$= (1.414 - 1) \times 0.511 MeV$$
$$= 0.414 \times 0.511 MeV$$
$$= 0.212 MeV$$
$$= 212{,}000 \text{ electron volts}$$

This electron velocity $v = c/\sqrt{2} = 0.707c$ corresponds to the spin-½ charged photon moving along a helical trajectory whose forward angle θ is given by

$$\cos(\theta) = v/c = 1/\sqrt{2}$$
$$\theta = \cos^{-1}(1/\sqrt{2}) = 45^\circ$$

Discussion

The proposal here is that the inertial mass m of a resting electron is derived from the circling momentum $mc = E_o/c$ of a circling spin-½ charged-photon modeling the resting electron. Also, the electron's relativistic energy-momentum equation $E^2 = p^2c^2 + m^2c^4$ is derived from the charged photon's momentum relationships given by $P^2 = p^2 + (mc)^2$, where the total momentum $P = E/c$ of the spin-½ charged photon modeling the electron is obtained from the vector addition of the charged photon's transverse circling linear momentum *mc* and the charged photon's longitudinal linear momentum $p = \gamma mv$ (the relativistic momentum of an electron.) The total energy *E* of the electron equals the energy $E = Pc$ of the helically-circulating spin-½ charged photon.

The spin-½ charged-photon model of the electron was developed by taking my initial idea of modeling an electron by a circling photon, and then adding to this circling photon model of the electron some of the properties of the Dirac electron found from analyzing the Dirac equation for a free electron such as 1) the electron's spin component $s = \frac{1}{2}\hbar$, 2) its calculated light-speed *c*, 3) its measured sub-light speed v, 4) its *zitterbewegung* internal angular frequency $\omega_{zitt} = 2mc^2/\hbar$ and 5) its vibrational amplitude $R_o = \hbar/2mc$. The full description of the spin-½ charged-photon model of the electron is at (1), where it is also shown that the spin-½ charged-photon model of the electron also generates the relativistic de Broglie wavelength $\lambda_{dB} = h/\gamma mv$ for a moving electron.

One very interesting aspect of these results is that the Dirac Equation was itself derived starting from the above relativistic energy-momentum equation for the electron, without assuming anything about the electron's inertial or invariant mass except that it is a given measured property of an electron. The electron's inertial mass *m* and its relativistic energy-momentum equation $E^2 = p^2c^2 + m^2c^4$ have now been derived from the perpendicular linear momentum components *mc* and $p = \gamma mv$ of a proposed circulating spin-½ charged photon with total momentum $P = \gamma mc$. The spin-½ charged photon embodies several electron properties derived from the Dirac equation, which in turn was derived by Dirac starting from the electron's relativistic energy-momentum equation. The circularity of this relationship is intriguing, and suggests a close relationship of the spin-½ charged-photon model to the relativistic Dirac electron.

Conclusions

The inertial mass *m* of a resting electron is derived from the circulating momentum E_o / c of a proposed circulating spin-½ charged photon composing an electron. Similar calculations of the inertial mass of the spin-½ charged photon model of a relativistic electron are also consistent with standard equations for the calculated transverse inertial mass $M = \gamma m$ of a relativistic electron. If this helically circulating internal transverse momentum *mc* vector is perpendicular to the relativistic electron's linear momentum vector $p = \gamma mv$ as proposed, adding these two momentum components of the charged photon model leads to the electron's relativistic energy-momentum equation $E^2 = p^2c^2 + m^2c^4$. According to the spin-½ charged photon model of the electron, this circling internal momentum *mc* of an electron is currently unobserved when measuring an electron, and is only measured indirectly by measuring the electron's inertial mass *m* that is derived from the electron's circling internal momentum *mc*. The derived inertial mass *m* of a resting electron is seen in the present proposal to be more directly related to the electron's internal circulating momentum $E_o / c = mc$ than to its total resting energy $E_o = mc^2$. The fact that the inertial mass *m* of an electron can be derived from the same spin-½ charged-photon model of the electron that leads to the electron's relativistic energy-momentum equation and to the electron's relativistic de Broglie wavelength $\lambda_{dB} = h / \gamma mv$, as shown in (1), lends strong support to the idea that the spin-½ charged photon model of the electron is a step closer to understanding the electron.

Reference

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