A Photon Has Inertial Mass $h\nu / c^2$ in Mirror Reflection and Compton Scattering

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Abstract

Using Newton's second law $\vec{F} = M\vec{a}$ where M is the inertial mass of an object, the inertial mass of a photon is calculated to be $M = h\nu / c^2$ for mirror-reflection and for Compton scattering from an electron. A photon's inertial mass M is not zero, which is the invariant mass *m* of a photon according to the relativistic energy-momentum equation $E^2 = p^2c^2 + m^2c^4$. It is concluded that a single photon has an inertial mass $M = h\nu / c^2$.

Key words: inertial mass, invariant mass, photon, mirror-reflection, Compton-scattering, relativistic energy-momentum equation

Introduction

The calculation of the inertial mass of a photon has often been considered problematic because photons do not speed up or slow down along the direction of their motion. Inertial mass is defined as the measure of the resistance of an object to being accelerated by an external force that changes the object's momentum and its velocity, as described by Newton's second law of motion $\vec{F} = m\vec{a}$, where $\vec{F}$ is the external force applied to an object, M is the object's inertial mass, and $\vec{a}$ is the object's acceleration. Photons always maintain a constant speed $c = 3.00 \times 10^8 m / s$ in a vacuum. But a photon may change its direction of motion, such as when it is reflected off a mirror's surface, or when it meets a charged particle such as an electron and is scattered in another direction during the process called Compton scattering. Similarly, a photon can change its momentum by changing its direction of travel during reflection or Compton scattering, even though the size of the photon's momentum, as calculated in what is called the "center-of-momentum" frame for the interaction, is the same before and after the photon changes its direction. This is because momentum and velocity are vector quantities, dependent on both the magnitude of a quantity and the direction in space.

Here is a brief summary about photon reflection from mirrors from *Scientific American* magazine: "The photons of the light reflected from a metal (or a dielectric mirror) are identical to the incident ones, apart from the changed propagation direction. The loss of light in the metal means that some fraction of the photons are lost, while the energy content of each reflected photon is fully preserved."(1). So while a full mathematical treatment of photon reflection from a mirror is complex, one result is that for highly reflecting mirrors, no energy is lost by most of the reflected photons, and so the magnitude of these photons' momenta is unchanged during reflection, though the direction of the reflected photons' momenta and velocities follows the optical rule for reflection: The angle of incidence equals the angle of reflection, and the two angles lie in the same plane. We will assume this "law of reflection" of photons from a mirror surface is valid for the below calculations of a photon reflection from a mirror.

Photons are often considered to have no inertial mass because their invariant mass *m* is zero, as calculated from the relativistic energy-momentum equation $E^2 = p^2c^2 + m^2c^4$. But the invariant mass of a particle is not necessarily the same as its inertial mass. Short calculations are carried out below for a) a photon reflecting from a mirror and b) a photon scattering from an electron in Compton scattering, as calculated in the photon-electron center-of-momentum frame of reference, where the total momentum of the photon-electron system is zero. These calculations both yield a value for the inertial mass *M* of a photon to be $M_{photon} = h\nu / c^2$. The Compton scattering calculation also leads to the value of the inertial mass of a relativistic electron to be $M_{electron} = \gamma m$, and $m = 0.511 MeV / c^2$ is the electron's invariant mass.

Calculation of the inertial mass of a photon reflecting from a mirror

Use the Newton's second law formula for inertial mass $M = \vec{F} / \vec{a} = (d\vec{p} / dt) / \vec{a}$. Here we will work with momentum and velocity components that will be either positive or negative. $F_{av} = (\Delta p / \Delta t)$ is the average force acting on the photon during the reflection process, and $a_{av} = \Delta v / \Delta t$ is the photon's average acceleration during the reflection process. When a photon of energy $E = h\nu$ and momentum $p = h\nu / c$ approaches a reflecting area with an angle of incidence θ to the normal (perpendicular), the photon's component of momentum normal to the reflecting area is

$$p_{normal} = -p\cos(\theta)$$

taking the outward direction perpendicular (normal) to the to the mirror surface as the + direction. After the photon is reflected, the component of its reflected momentum that is normal to the surface is

$$p_{normal} = +p\cos(\theta).$$

The change in momentum of the reflected photon is therefore

$$\Delta p = 2p\cos(\theta) = 2\frac{h\nu}{c}\cos(\theta).$$

Take the very small time interval for the photon to reflect from the mirror surface to be Δt (it will be the same Δt for the average force calculation and the average acceleration calculation.) Then the average force F_{av} acting on the photon during the reflection process is

$$F_{av} = \Delta p / \Delta t = 2\frac{h\nu}{c}\cos(\theta) / \Delta t \,.$$

There is no change in the component of the photon's momentum parallel to the reflection area during the reflection process.

Now calculate the average acceleration a_{av} of the reflected photon during the same reflection time interval Δt. The photon approaches the reflecting area with speed c and incident angle theta, so the component of its incoming velocity in the normal direction is

$$v_{normal} = -c\cos(\theta) \,.$$

After the photon is reflected the normal component of its velocity is

$$v_{normal} = +c\cos(\theta) \,.$$

Therefore the change in the normal component of its velocity during the reflection process is

$$\Delta v = 2c\cos(\theta) \,.$$

The photon's average acceleration during this time is

$$a_{av} = \Delta v / \Delta t = 2c\cos(\theta) / \Delta t \,.$$

There is no change in the component of the photon's velocity parallel to the reflection area during the reflection process.

Putting these two above results F_{av} and a_{av} into Newton's formula for inertial mass

$$M = F / a = (dp / dt) / a$$

gives

$$M_{photon} = F_{av} / a_{av}$$
$$= \{2\frac{h\nu}{c}\cos(\theta) / \Delta t\} / \{2c\cos(\theta) / \Delta t\}$$
$$= h\nu / c^2$$

So a photon's inertial mass is $M_{photon} = h\nu / c^2$ as found by this simple reflection calculation, and so it is non-zero while the same photon's invariant mass m is zero.

Calculation of photon's inertial mass and an electron's inertial mass in a Compton photon-scattering experiment

Another situation in which a photon's (as well as an electron's) inertial mass can be calculated simply is the scattering of a photon from an electron as in Compton scattering (2). Here an x-ray photon is scattered off of an electron that lightly bound (compared to the energy of the incoming x-ray) in an atom. The scattered photon moves away at an angle from the electron with a lower energy and momentum while the electron recoils at another angle during the scattering process, escaping from its atom and gaining some kinetic energy and some momentum from the photon. Compton scattering experiments demonstrate that a photon acts somewhat like a billiard ball as it hits another billiard ball at an angle, where both balls move off in different directions with each ball carrying some energy and some momentum. The total energy and total momentum of the photon-electron system in Compton scattering are conserved in the scattering process.

The Compton photon scattering process can be viewed in the center-of-momentum reference frame of the photon and the electron. In the center-of-momentum frame the vector sum of the momenta of the two particles is zero. The photon and the electron are moving in opposite directions along a straight line. The incoming photon with momentum $p_{1photon} = +h\nu / c$ (let us say along the +*x* axis) and velocity $v_{1photon} = c$ approaches a relativistic electron with momentum $p_{1electron} = -\gamma mv$ and velocity $v_{1electron} = -v$, moving in the negative direction along the x-axis. We have

$$P_{total} = p_{1photon} + p_{1electron} = h\nu / c - \gamma mv = 0$$
$$h\nu / c = \gamma mv$$

When the photon and the electron are each scattered 180 degrees and move away in opposite directions along the x-axis (in this example), the photon now moves in the negative x-direction with momentum $p_{2photon} = -h\nu / c$, while the electron now moves in the positive *x*-direction with momentum $p_{2electron} = +\gamma mv$. Again,

$$P_{total} = p_{2photon} + p_{2electron} = -h\nu / c + \gamma mv = 0$$
$$h\nu / c = \gamma mv$$

because the total momentum before and after the interaction is zero in the center-of-momentum system. Let us assume that the scattering of the photon by the electron takes a very short but non-zero time interval Δt.

We will use Newton's second law $\vec{F} = m\vec{a}$ or $M = \vec{F} / \vec{a}$ to obtain the inertial mass of the photon and that of the electron in this Compton scattering process. For the photon,

$$F_{av} = \Delta p_{photon} / \Delta t$$
$$= (p_{2\,photon} - p_{1\,photon}) / \Delta t$$
$$= (-2h\nu / c) / \Delta t$$

and

$$a_{av} = \Delta v_{photon} / \Delta t$$
$$= (-c - (+c)) / \Delta t$$
$$= -2c / \Delta t$$

So

$$M_{photon} = F_{av} / a_{av}$$
$$= (-2h\nu / \Delta t) / (-2c / \Delta t)$$
$$= h\nu / c^2$$

Similarly for finding the inertial mass $M_{electron}$ of the electron in this interaction:

$$F_{av} = \Delta p_{electron} / \Delta t$$
$$= (p_{2electron} - p_{1electron}) / \Delta t$$
$$= (\gamma mv - (-\gamma mv)) / \Delta t$$
$$= 2\gamma mv / \Delta t$$

and

$$a_{av} = \Delta v_{electron} / \Delta t$$
$$= (v_{2electron} - v_{1electron}) / \Delta t$$
$$= (v - (-v)) / \Delta t$$
$$= 2v / \Delta t$$

So

$$M_{electron} = F_{av} / a_{av}$$
$$= (2\gamma mv / \Delta t) / (2v / \Delta t)$$
$$= \gamma m$$

These will be the results no matter how small the scattering time Δt is as long as it is not exactly zero, in which case the calculations for the photon's inertial mass M_{photon} and the electron's inertial mass $M_{electron}$ are undefined. The inertial mass $M_{photon} = h\nu / c^2$ calculated for a photon undergoing Compton scattering is the same as the inertial mass calculated for a photon reflected off of a mirror shown earlier.

The calculated electron inertial mass $M_{electron} = \gamma m$ above for a relativistic electron is the standard expression for a relativistic electron's inertial mass, when the term "inertial mass" is applied to an electron. This expression $M_{electron} = \gamma m$ is also sometimes called the electron's relativistic mass. It is currently unfashionable in particle physics to use the term "inertial mass" or "relativistic mass", which are both proportional to the total energy $E = \gamma mc^2$ of a relativistic electron. In particle physics the electron's invariant mass $m = 0.511 MeV / c^2$ is now generally considered to be the only mass needed for describing an electron.

Discussion

It may be objected that the above derivation of a photon's inertial mass for a photon being reflected from a mirror oversimplifies the process of photon reflection from a mirror. A full description of photon reflection and transmission requires quantum electrodynamics (QED) treatment. Yet the well-known experimental "law of reflection" for light, where the angle of incidence equals the angle of reflection, is applied to photon reflection, it leads to the photon's inertial mass expression $M_{photon} = h\nu / c^2$ found here.

The description of Compton scattering of a photon from an electron would also require QED treatment to fully predict the photon-scattering results obtained experimentally in the laboratory. Compton scattering of a photon from an electron gives strong experimental evidence that a photon acts like a particle with energy $E = h\nu$ and momentum $p = h\nu / c$, as Einstein had predicted theoretically. In Compton scattering of a photon, unlike with mirror reflection of a photon above, only two particles are involved, the photon and the rebounding electron. As a result, the "center-of-momentum" calculations lead to an expression for both a photon's inertial mass $M_{photon} = h\nu / c^2$, which is the same as the expression found with mirror reflection, and a relativistic electron's inertial mass $M_{electron} = \gamma mv$.

The above calculations for mirror reflection and Compton scattering of a photon were done after a finding that a circulating spin-1/2 charged-photon model of an electron (3) yields the inertial mass $M_{electron}$ of a resting electron equal to the electron's invariant mass $m = E_o / c^2 = h\nu_o / c^2$, as derived from the momentum of the circulating photon $p_o = E_o / c = mc$. (4) The question arose, does the circulating photon in the electron model only have an inertial mass $M_{photon} = h\nu / c^2$ due its circular motion, or does a photon of energy $E = h\nu$ also have an inertial mass $M_{photon} = h\nu / c^2$ when moving in a straight line, even though such a photon has zero invariant mass. In the calculation of the inertial mass of the circulating spin-1/2 charged-photon model of the electron, the inertial mass $M_{photon} = m$ of the circulating charged photon modeling the electron is obtained from Newton's second law $M_{photon} = \vec{F} / \vec{a} = (d\vec{p} / dt) / \vec{a}$ by dividing the rate of change of the circulating photon's momentum mc, circulating at angular frequency ω, by the circulating photon's centripetal acceleration $a_{centripetal} = \omega^2 r$ in its circular trajectory of radius r. This gives

$$
\begin{aligned}
M_{photon} &= (d\vec{p}/dt)/\vec{a}_{centripetal} \\
&= \omega mc/\omega^2 r \\
&= mc/\omega r \\
&= mc/c \\
&= m
\end{aligned}
$$

Why does it matter if a photon has an inertial mass $M = h\nu / c^2$ and not zero, even though the invariant mass of a photon is zero? Two reasons why this matters can be given.

First, if photons contribute to gravitational mass-energy in Einstein's general theory of relativity, then they should also have inertial mass, due to the "principle of equivalence" between gravitational mass and inertial mass in general relativity.

Second, the motion of a photon of any momentum in a straight line is consistent with Newton's first law, the "principle of inertia", that a body moves in a straight line at constant velocity unless acted on by an outside force. The motion of a photon in reflection and in Compton scattering is also seen to be consistent with Newton's second law $\vec{F} = M\vec{a}$, which defines an inertial mass M that can be calculated when a photon is changing its direction and momentum and is also being accelerated in a very small but finite interval of time.

To claim that a photon has no inertial mass just because its invariant mass is zero is not logical since inertial mass, as calculated from Newton's second law, is not the same concept as invariant mass, which is calculated from the relativistic energy-momentum equation $E^2 = p^2c^2 + m^2c^4$ where for a photon, $m = 0$ because $E = pc$. The electron's relativistic energy-momentum equation can itself be derived from the spin-1/2 charged photon model of the electron. (4) In this derivation, the electron's invariant mass m is just part of the invariant circulating transverse component of momentum $mc = E_o / c$ of the total momentum $P_{total} = \gamma mc$ of the circulating charged photon that is modeling a relativistic electron, whose longitudinal component of momentum is the electron's relativistic momentum $p = \gamma mv$.

The origin of the inertial mass of a massive fundamental particle like the electron is still not fully understood despite the discovery of the Higgs boson, whose Higgs field is said to give an unspecified invariant mass to certain elementary particles such as the electron. My hypothesis is that the electron's inertial mass may be actually derived from the circulating momentum within a circulating spin-1/2 charged-photon model of the electron. This electron model generates a resting electron's inertial mass $M_e = h\nu_o / c^2 = m$ through the internal rotation of the momentum $mc = E_o / c$ of the charged photon, as shown above. This origin-of-electron's-inertial-mass hypothesis gains indirect support by the finding in this article that a mirror-reflected or Compton-scattered photon also carries inertial mass $M_{photon} = h\nu / c^2$. It may be that an internally circulating momentum within particles such as the electron is what actually gives these particles their inertial mass.

Conclusion

Although a photon moves with a constant speed *c* in a vacuum and so cannot be linearly accelerated, it can change direction during reflection from a mirror-like surface or scattering from single electron, in addition to following a curved path due to gravitational effects. This reflection and scattering give a photon an applied force vector corresponding to the transverse vector momentum change with time during the reflection or Compton-scattering process, as well as a transverse change in the average vector velocity during reflection or scattering, yielding an average acceleration vector. Calculations were made for the inertial mass of a photon of energy $E = h\nu$ and momentum $p = h\nu / c$ using Newton's second law $M = \bar{F} / \bar{a}$ during mirror-reflection and Compton scattering from an electron. These calculations yielded an inertial mass $M_{photon} = h\nu / c^2$ for photon reflection at any less-than-90^o angle of incidence including very small glancing angles, and for Compton reflection with an electron as seen in the photon-electron center-of-momentum system. These results can tentatively be generalized to give an inertial mass $M_{photon} = h\nu / c^2$ for any linearly moving photon, even though a photon's invariant mass m is zero based on the relativistic energy-momentum equation $E^2 = p^2c^2 + m^2c^4$ as applied to a photon.

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