# Fundamental Force and Radius Equations 

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The following are 10 simple equations which show the connection at the level of fundamental particles between the gravitational force ( $F_{g}$ ), the electrostatic force $F_{E}$, the particle's Schwarzschild radius $R_{\mathrm{s}}$ and the particle's Compton wavelength $\not A_{\text {c }}$. The gravitational force equations will assume two of the same mass (m) particles which both have a reduced Compton wavelength of $\mathcal{A}_{c}=\hbar / m c$ The electrostatic force equations will assume that both particles have Planck charge $q_{p}=\sqrt{4 \pi \varepsilon_{o} \hbar c}=\sqrt{\alpha} e \approx 11.7 \mathrm{e} \approx 1.876 \times 10^{-18}$ coulomb. The electrostatic force between two Planck charges is: $F_{E}=\mathrm{q}^{2} / 4 \pi \varepsilon_{0} \mathrm{r}^{2}=\hbar c / r^{2}$. The conversion between $F_{E}$ and the force between two charge $e$ particles $\left(F_{e}\right)$ is: $F_{E}=F_{e} / \alpha$. Planck charge is actually the more fundamental unit of charge. Planck charge is derived from the permittivity of free space $\varepsilon_{o}$ and Planck charge has a coupling constant to photons equal to 1 . Charge $e$ has a coupling constant of $\alpha$, the fine structure constant. Therefore, charge $e$ can be thought of as a weakened version of Planck charge with a coupling constant of $\alpha$.

For any separation distance the ratio $F_{g} / F_{E}=\left(G m^{2} / r^{2}\right)\left(\mathrm{r}^{2} / \hbar c\right)=G m^{2} / \hbar c$. If we assume a hypothetical particle with an electron's mass and Planck charge the dimensionless ratio is: $F_{g} / F_{E} \approx 4.18 \times 10^{-23}$

The first of the equations below shows that the force ratio equals the ratio of two radii. The spacetime particle's radius is equal to its reduced Compton radius is: $\quad \lambda_{c}=\frac{G m}{c^{2}}$

The spacetime particle is a Planck length distortion of spacetime rotating at the speed of light around a spherical volume with radius $\mathcal{A c}$. This means that it is "maximally rotating" and would have a Schwarzschild radius of:
$R_{S}=\frac{G m}{c^{2}}$

A non-rotating mass would have a Schwarzschild radius twice this value. The first of 10 equations shown that the force ratio is equal to the radii ratio:

$$
\frac{F_{g}}{F_{E}}=\frac{R_{S}}{\lambda_{C}} \quad \text { This can also be written as: } \quad F_{g} \lambda_{c}=F_{E} R_{S}
$$

This simple equation is actually implying a deep connection between a particle's gravitational and electrostatic properties. The gravitational properties are represented by $F_{g}$ and $R_{s}$. The electrostatic properties are represented by $F_{E}$ and $\lambda_{c}$. The spacetime particle's radius $\lambda_{c}$ also represents wavelength characteristics which generate the particle's electric field. This equation is actually
derived from the spacetime model of particles and forces, but the derivation is lengthy and beyond the scope of this summary of equations. The equation is correct for any separation distance and is independent of the model used to derive it.

The next equation shows the connection between the force ratio and the ratio of Planck length $L_{p}$ divided by $A_{c}$. Planck length $L_{p}=\sqrt{\hbar G / c^{3}} \approx 1.616 \times 10^{-35} \mathrm{~m}$. Planck length is the distortion of spacetime allowed by quantum mechanics and the uncertainty principle. According to the spacetime model of the universe, the distance in the vacuum between any two points is being modulated by $\pm$ Planck length. This is the displacement amplitude of "vacuum fluctuations". This modulation can be thought of as the background "noise" of the vacuum which gives rise to many of the probabilistic characteristics of quantum mechanics. The ratio of $L_{p} / A_{c}$ is the dimensionless strain amplitude of the dipole waves which form a spacetime particle. Therefore it is not surprising that this ratio also relates to the ratio of forces $F_{g} / F_{E}$.

The next equation relates the force ratio $F_{g} / F_{E}$ to the strain amplitude of the dipole wave in spacetime that forms the spacetime particle. This strain amplitude is the dimensionless ratio $L_{p} / A c$. It is the maximum slope of the dipole wave. Here is the equation showing how this strain amplitude connects to the force ratio:
$\frac{F_{g}}{F_{E}}=\left(\frac{L_{p}}{\lambda_{c}}\right)^{2} \quad$ This can also be written as: $\quad F_{\mathrm{g}}{\lambda_{\mathrm{c}}}^{2}=F_{\mathrm{E}} L_{\mathrm{p}}{ }^{2}$

The next equation shows an interesting relationship between $R_{s}, L_{p}$ and $\lambda_{c}$.

$$
\frac{R_{S}}{L_{p}}=\frac{L_{p}}{\lambda_{C}} \quad \text { This can also be written as: } \quad \lambda_{\mathrm{c}} \mathrm{R}_{\mathrm{s}}=L_{p}^{2}
$$

It is possible to illustrate this equation by showing where $R_{s}, L_{p}$ and $\mathcal{A}_{c}$ would be on a logarithmic scale of length. As shown below, Planck length ( $L_{p}$ ) is exactly half way between a particle's Schwarzschild radius $R_{s}$ and its Compton radius $\mathcal{A}_{\text {c }}$. If the three lengths are placed on a log scale of length, then Planck length is exactly half way between $R_{s}$ (gravity) and the spacetime particle radius $\mathcal{A}$.


The next equation shows an important connection between $F_{g}, F_{E}, F_{p}$ and the number $N$ of reduced Compton wavelengths separating two of the same mass particles. These force relationships become obvious when the separation distance between particles is expressed as the number $N$ of radius units separating the particles rather than expressing the separation distance using meters.
$N=r / A_{c}$
$\frac{F_{g}}{F_{E} \mathcal{N}}=\frac{F_{E} \mathcal{N}}{F_{p}} \quad$ This can also be written as: $\quad F_{g} F_{p}=\left(F_{E} \mathcal{N}\right)^{2}$
This equation can be illustrated by a $\log$ scale of force showing $F_{g}, F_{E}$ and $F_{p}$. The gravitational force $F_{g}$ is the smallest force possible between two of the same particles and Planck force $F_{p}$ is the largest possible force. Exactly half way between these two extremes of force lies $\boldsymbol{F}_{\boldsymbol{E}} \boldsymbol{N}$. This is the electrostatic force between two Planck charges times the number $\boldsymbol{N}$ of reduced Compton wavelengths separating the two particles (the number of radius units). If two charge $e$ are assumed, then the force will be represented by $F_{e}$ and the force half way between the extremes is $F_{e} N / \alpha$.

| $\uparrow$ | $\uparrow$ | $\uparrow$ |
| :---: | :---: | :---: |
| $\boldsymbol{F}_{\boldsymbol{g}}$ | $\boldsymbol{F}_{\boldsymbol{E}} \boldsymbol{N}$ | $\boldsymbol{F}_{\boldsymbol{p}}$ |

This next equation illustrates that the gravitational force can be expressed as the square of the electrostatic force when the magnitude of both forces is expressed in dimensionless Planck units (underlined) and the separation distance is given using $N$.

$$
\underline{\boldsymbol{F}}_{\mathbf{g}} \mathcal{N}^{2}=\left(\underline{\boldsymbol{F}}_{\mathrm{E}} \mathcal{N}^{2}\right)^{2}
$$

where $\underline{\boldsymbol{F}}_{\mathrm{g}}=\frac{F_{g}}{F_{p}}=\frac{G m^{2}}{r^{2}} \frac{G}{c^{4}}$ and $\underline{\boldsymbol{F}}_{E}=\frac{F_{E}}{F_{p}}=\frac{\hbar c}{r^{2}} \frac{G}{c^{4}}$
The square relationship between the forces is particularly easy to see when the separation distance is assumed to be equal to $\lambda_{\mathrm{c}}$ which is a separation of one radius. If $r=\lambda_{\mathrm{c}}$ then $N=1$

$$
\underline{\boldsymbol{F}}_{\mathrm{g}}=\boldsymbol{\boldsymbol { F }}_{E}{ }^{2}
$$

The use of " $N$ " to represent the separation distance also gives the actual force between particles when the force and the particle's internal energy $E_{i}=m c^{2}$ is expressed in dimensionless Planck units which are shown with an underline such as $\underline{\boldsymbol{F}}_{E} \underline{\boldsymbol{F}}_{g}$ and $\underline{\boldsymbol{E}}_{i}$

$$
\begin{aligned}
& \underline{\boldsymbol{F}}_{E} \mathcal{N}^{2}=\underline{\boldsymbol{E}}_{\mathrm{i}}^{2} \quad \text { where } \underline{\boldsymbol{E}}_{i}=\frac{E_{i}}{E_{p}}=m c^{2} \sqrt{\frac{G}{\hbar c^{5}}} \quad \text { for electron } \underline{\boldsymbol{E}}_{i}=4.18 \times 10^{-23} \\
& \underline{\boldsymbol{F}}_{g} \mathcal{N}^{2}=\underline{\boldsymbol{E}}_{\mathrm{i}}^{4}
\end{aligned}
$$

Finally, the actual gravitational and electrostatic forces between particles can be expressed using the particle's Schwarzschild radius $R_{s}$ and the number $N$ of radius units ( $\lambda_{c}$ ) separating the particles.
$\underline{\boldsymbol{F}}_{E} \mathcal{N}^{2}=\underline{\boldsymbol{R}}_{\mathbf{s}}^{2} \quad$ where $\underline{\boldsymbol{R}}_{\mathbf{s}}=\frac{R_{S}}{L_{p}}=\frac{G m}{c^{2}} \sqrt{\frac{c^{3}}{\hbar G}}$ for electron $\underline{\boldsymbol{R}}_{\mathbf{s}}=4.18 \times 10^{-23}$
$\underline{\boldsymbol{F}}_{g} \mathcal{N}^{2}=\underline{\boldsymbol{R}}^{4}$

All of these equations are correct and reveal fundamental relationships which give strong hints about the structure of particles, gravity and electric fields. Perhaps some people might view these fundamental equations as just mathematical curiosities. However, any model of a fundamental particle must eventually be able to explain why these relationships exist. For example, I believe that these equations are completely incompatible with the gravitational force being transferred by gravitons.

I suggest that the group start incorporating Planck length into your models. You already are using $\mathcal{A c}_{c} / 2$ (double loop radius) which is a vastly superior to assuming a point particle. I will be writing a new technical paper incorporating these equations.

John M.

