**Does the Inertia of an Elementary Particle Depend on Its Momentum Content?**
**by Richard Gauthier**

Inertia, the resistance of an object to a change in its velocity, is still a mystery more than 500 years after the concept was first proposed by Galileo. But the cause of the inertia of matter has remained unknown.

Einstein in 1905 asked the question "Does the inertia of a body depend on its energy content?", and proposed $m = E/c^2$ as a possible answer, since the mass of an object is found to decrease by an amount $\Delta m$ when it gives off a quantity of energy $\Delta E$ in the form of electromagnetic energy. While this equation established the relationship of mass quantitatively with energy, the question "How does energy give rise to the inertia of matter?" was actually not answered. So physicists continue to search for the origin of inertial mass, such as through the possible existence of the Higgs boson that would give inertial mass to elementary particles, or by the possible effect of the Zero Point Field on an electron, which could also supply it with inertia.

Newton's Second Law defined inertial mass by $F = ma$, where $m$ is the inertial mass, $F$ is the net force acting on it, and $a$ the resulting acceleration. $m$ is the proportionality constant between force and acceleration. A more general way to write Newton's Second Law is $F = \frac{dp}{dt}$ where p is the momentum of an object. Applying a force F to an electron will cause it to accelerate, implying the existence of an inertial mass m of the electron, which Einstein in 1905 associated with $E/c^2$. What momentum is changing when an electron accelerates from rest? An electron with zero velocity is considered to have zero momentum, so why should it have any resistance to changing its momentum, that is, why should it have any inertia? An object with momentum has a tendency to resist change of its momentum, so a moving object has a kind of inertia of its motion (Newton's First Law), but why should a stationary object have inertia (Newton's First Law also).

The answer may lie within the electron. I proposed above that the electron is composed of a circulating superluminal partino, carrying momentum and energy and traveling in a closed double-looped helical trajectory. So although the electron (which is a kind of average of the motion of the electron's point-like partino) may be at rest, the partino is never at rest. The forward speed of the partino along the circular axis of its closed double-looped helix is $c$ and its forward momentum along this axis is $p = m_o c$. This motion of the electron has never been detected experimentally, but it is predicted theoretically by the Dirac equation, whose solution for a free electron is that the instantaneous speed of an electron is $c$ or $-c$, resulting in the theoretical Zitterbewegung or jittery motion of the electron. The electron's Zitterbewegung is accounted for quantitatively in the partino model of the electron.

Does the momentum associated with the electron's Zitterbewegung give rise to the electron's inertia? We can find a mathematical relationship between the acceleration of the electron's partino to an applied force on the partino. Since the partino constitutes the electron, whatever inertia the electron has is contained in the electron's partino, since the partino is postulated to carry or create all the dynamical properties of the electron. Since the partino has a momentum, an applied force to it will create a change in its momentum, and that change in its momentum can be related to the applied force. The constant of proportionality between the applied force $F_p$ to the partino and its resulting acceleration $a_p$ is by definition the inertial mass $m_I$ of the electron, since the electron is constituted by the partino, which itself has no rest mass.

$$F_p = m_I a_p \tag{1}$$

We will use the relationship $F = \frac{dp}{dt}$ to find a relationship between the applied force $F_p$ and the partino's acceleration $a_p$. We will assume that applying a vertical force $F_p$ to the partino's average horizontal forward momentum $p_o = m_o c$ will cause the partino to accelerate vertically with an acceleration $a_p$. During the time $\vartriangle t$ that the force $F_p$ is applied, the partino will move a vertical distance

$$d = \tfrac{1}{2} a_p (\vartriangle t)^2 \tag{2}$$

the distance an accelerated object moves in a time $\vartriangle t$.

In a very short time $\vartriangle t$, the trajectory of an initially horizontally object in response to a vertically applied force is a nearly circular arc (technically it is a parabolic arc but for small angular changes the arc is nearly circular). Before the vertical force is applied, the angle the momentum $p_o = m_o c$ makes with the horizontal axis 0 degrees. After time $\vartriangle t$, a chord from the beginning of the arc of the circular trajectory to the end of the arc makes an angle of $\theta$ with the horizontal while the angle that the partino's momentum makes with the horizontal axis is found, by a simple geometrical construction, to be $2\theta$.

According to the above diagram, the magnitude of $\vartriangle p$, the difference between the momentum vectors at the beginning and at the end of the time $\vartriangle t$, has a magnitude

$$\vartriangle p = 2 p_o \sin\theta \tag{3}$$

During the time interval V$t$, the horizontal distance L that the partino moves forward is $L = c \vartriangle t$, and the vertical distance that the partino moves is, by geometry

$$d = L\tan\theta = c\vartriangle t \tan\theta = \tfrac{1}{2} a_p (\vartriangle t)^2 \tag{4}$$

Solving the above equation for $\vartriangle t$ gives

$$\vartriangle t = \frac{2c\tan\theta}{a_p} \tag{5}$$

Dividing $\vartriangle p$ by $\vartriangle t$ gives

$$\begin{aligned} \frac{\vartriangle p}{\vartriangle t} &= \frac{2 p_o \sin\theta}{(2c\tan\theta)/a_p} \\ &= \frac{a_p p_o \sin\theta}{c\tan\theta} \end{aligned} \tag{6}$$

As $\theta \to 0$

$$\begin{gathered} \frac{\sin\theta}{\tan\theta} \to 1 \\ \lim\ \theta \to 0 \end{gathered} \tag{7}$$

And as $\vartriangle t \to 0$

$$\frac{\vartriangle p}{\vartriangle t} \rightarrow \frac{dp}{dt} \tag{8}$$

So equation (6) becomes

$$\frac{dp}{dt} = \frac{a_p p_o}{c} = \frac{p_o}{c} a_p \tag{9}$$

Since $F_p = \frac{dp}{dt}$ we have

$$F_p = \frac{p_o}{c} a_p \tag{10}$$

Comparing equation(10) with equation (1) we see that

$$m_I = \frac{p_o}{c} \tag{11}$$

So we have derived the inertial mass of the electron from the forward momentum $p_o c$ that circulates at speed $c$ in the partino model of the electron, a model that also explains the electron's Zitterbewegung motion and has both the spin and the magnetic moment predicted by the Dirac equation. We can conclude that the partino model supports the idea that it the electron's circulation of momentum at light speed that is the direct cause of the electron's inertia as measured by its inertial mass $m_I$. The electron's energy content $E_o$ is only indirectly related to the electron's inertial mass $m_I$ , due to the relationship $E_o = p_o c$ or $p_o = E_o / c$ that the electron's energy content $E_o$ has with the momentum value $p_o c$, which gives

$$m_I = \frac{p_o}{c} = \frac{E_o / c}{c} = \frac{E_o}{c^2} \tag{12}$$

Einstein derived the relationship $m = E / c^2$ in his 1905 paper "Does the inertia of a body depend upon its energy-content?" Using a transformation for electromagnetic energy as measured in different reference frames, he showed that radiation of energy $L$ (he used $L$ instead of $E$) from an object in its rest frame causes a reduction in the kinetic energy by $\frac{1}{2}\frac{E}{c^2}v^2$ in the object as measured in a frame moving with uniform velocity $v$, where $v = \ c$. Since the speed $v$ of the object in the moving frame did not change when the radiation was emitted, he concluded that "if the body gives off the energy L in the form of radiation, its mass diminishes by $\frac{L}{c^2}$."

In the above derivation, while Einstein demonstrates that a body contains energy in the proportion $E = mc^2$, he does not attempt to derive the inertial mass $m$ from its contained energy $E$. There is nothing about measuring inertial mass using $F = ma$ , the equation that defines inertial mass. Einstein makes the connection between $E / c^2$ and $m$ through the kinetic energy formula K.E. = $\frac{1}{2}mv^2$, not $F = ma$ as was done above with the partino model of the electron. Neither force nor acceleration appears in his article at all. The last sentence of his paper does refer to inertia for the first time except in the title: "If the theory corresponds to the facts, radiation conveys inertia between the emitting and absorbing bodies." The implication is that since radiation can come from the transformation of mass into energy, radiation has the power to convey inertia from one body to another. But we saw earlier that inertia is related directly with momentum, through the relationship $F = \frac{dp}{dt} = ma$, and only indirectly with radiant energy through the relationship $E = pc$. So according to the partino hypothesis, the inertia of a body—its inertial mass—depends directly on its contained circulating momentum which corresponds to a wavelength, and only indirectly to its contained energy, which corresponds to a frequency. The speed of light is what connects a particle's internal circulating momentum (on which its inertial mass depends) with its internal energy.

A photon also has an inertial mass. This follows directly from the earlier derivation of $m_I$ from the internal circulating momentum of an electron. The obtained relationship in of $m_I$ to $p$ in equation (11) does not depend on whether the momentum $p$ is circulating or not (it only circulates in a closed path in particles with a rest mass.) So the relation for the inertial mass $m_I$ of any photon is

$$m_I = p / c = \hbar k / c \tag{13}$$

According to the equivalence principle in general relativity, the inertial mass of an object equals its gravitational mass. So for a photon as well as an electron

$$m_G = m_I \tag{14}$$

The non-zero gravitational mass of a photon explains why light is bent while passing a star in accordance with the prediction of general relativity, and why the wavelength of a photon moving downward in a gravitational field decreases (indicating an increased momentum) and its frequency increases (indicating an increased energy) as it falls.

The partino models of the electron and photon also explain why a photon's rest mass is zero even while its inertial mass and gravitational mass are non-zero as given in equations (13) and (14) . A particle (such as a photon) has zero rest mass when its linear momentum $p = E / c$, according to the Einstein-de Broglie equation

$$E^2 = p^2c^2 + m_o^2c^4 \tag{15}$$

But the inertial mass $m_I$ is related to all of the particle's momentum, including the momentum $p_o = m_o c$ that is circulating within the particle, and not only to the linear momentum $p$ in the equation above. The total inertial and gravitational mass of an elementary particle (photon or electron) are therefore given by

$$m_I = m_G = \frac{\sqrt{p^2 + p_o^2}}{c} = \frac{\sqrt{p^2 + m_o^2c^2}}{c} = \frac{E}{c^2} \tag{16}$$

We can turn around the connection between the electrons inertial mass $m_I$ and the partino model's internal speed-of-light circulating momentum. Since the inertial mass $m_I$ of the electron is an established experimental fact, we can say that the partino model of the electron is further supported by deriving the electron's actual inertial mass from first principles ( $F = \frac{dp}{dt} = m_I a$ ) and the electron's proposed partino structure.

Inertial mass $m_I$ is characteristic of all objects that consists of sub-atomic particles with mass along with internal photons or other speed-of-light particles such as photons and gluons. According to the partino model, the inertial mass of an object exists when the internal speed-of-light circulation of momentum within sub-atomic particles, along with the linear momentum of each of the particles composing the matter, create a resistance to the acceleration of the object by applied forces.

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