International Journal of Modern Physics A
Vol. 31, Nos. 2 & 3 (2016) 1641002 (11 pages)

DOI: 10.1142/S0217751X16410025

Source of the Kerr-Newman solution as a gravitating bag model: 50 years of the problem of the source of the Kerr solution

Alexander Burinskii

Theoretical Physics Laboratory,
Nuclear Safety Institute of the Russian Academy of Sciences,
Moscow, 115191, Russia
bur@ibrae.ac.ru

Received 24 September 2015
Revised 17 October 2015
Published 19 January 2016

It is known that gravitational and electromagnetic fields of an electron are described by the ultra-extreme Kerr-Newman (KN) black hole solution with extremely high spin/mass ratio. This solution is singular and has a topological defect, the Kerr singular ring, which may be regularized by introducing the solitonic source based on the Higgs mechanism of symmetry breaking. The source represents a domain wall bubble interpolating between the flat region inside the bubble and external KN solution. It was shown recently that the source represents a supersymmetric bag model, and its structure is unambiguously determined by Bogomolnyi equations. The Dirac equation is embedded inside the bag consistently with twistor structure of the Kerr geometry, and acquires the mass from the Yukawa coupling with Higgs field. The KN bag turns out to be flexible, and for parameters of an electron, it takes the form of very thin disk with a circular string placed along sharp boundary of the disk. Excitation of this string by a traveling wave creates a circulating singular pole, indicating that the bag-like source of KN solution unifies the dressed and point-like electron in a single bag-string-quark system.

Keywords: Kerr-Newman solution; Higgs; soliton; bag model; string; Dirac equation.

PACS numbers: 11.27.+d, 04.20.Jb, 04.60.−m, 04.70.Bw

1. Introduction

This year debating of the source of the Kerr and Kerr-Newman (KN) solution marks 50 years. Reason of the complexity of this problem is the emergence in the Kerr solution of a ring-like singularity instead of the pointlike singularity of the non-rotating black hole (BH) solutions. Radius of this ring, $a = J/m$,[a] is proportional to angular momentum J and inverse proportional to mass m of the BH. It leads to

[a]We use the dimensionless units $c = G = \hbar = 1$ and signature $(- + + +)$.

a breaking of the wide spread opinion that the effective range of the gravitational field r_g is short and proportional to mass, $r_g \sim m$. The short distance Kerr-Newman field is extended as $r_g \sim a = J/m$, taking the form of a singular string, for which r_g is the more, than the smaller is mass of the BH. So, that for the elementary particles of very small masses, the effective range of gravity is tremendously expand, taking the form of the circular-string (or gravitational waveguide[1,2]) extending on the Compton zone of the spinning particles. The spin/mass ratio of the elementary particles turns out to be much more than their mass, $a = J/m \gg m$, $a/m \sim 10^{44}$, and the BH horizons disappear. It corresponds to an ultra-extreme KN solution, for which the Kerr singular ring turns out to be naked, and in accord with the censorship principle, the singular region should be replaced by some source covering the naked singularity.

50 years ago, in the appendix to paper by Newman and Janis,[3] it was noted that the simple interpretation of the KN source as a rotating singular ring does not go, because the Kerr singular ring turns out to be a branch line of space into two sheets, and the observer, which passed through this ring, $r^+ \to r^- = -r^+$, turns out to be on a different space with different metric $g^+_{\mu\nu} \to g^-_{\mu\nu}$ and values of the electromagnetic vector potential $A^+_\mu \to A^-_\mu$. The KN metric turned out to be *two-sheeted* and contains the mirror "Aice" world. The second extra important peculiarity was obtained by B. Carter, who noted[4] that the KN solution has the gyromagnetic ration $g = 2$ as that of the Dirac electron. Therefore, using the KN metric with tree parameters e, m, J corresponding to the experimentally observed parameters of the electron, we automatically obtain the external gravitational and electromagnetic field of the electron corresponding to the correct value of the its fourth parameter — the magnetic charge μ.

These features led to intensive study of the problem of source of the KN solution (for review see Refs. 5–7), and the discussions of this problem are still ongoing.[8,9]

Importance of this problem is related with known *conflict between gravity and quantum theory*. One of the points of this conflict is unacceptance for gravity the known quantum statement on the point-like and structureless electron. Gravity requires the soliton-like sources described by a stress-energy tensor. The known bag models for hadrons[10,11] satisfy this requirement, being fully consistent with quantum theory. The problem of the source of the KN solution is extremely important because it could answer the question: *Which structure of the electron could be compatible with gravity?* and hint the way to solution of the principal problem of consistency gravity with quantum theory. In the recent works (Refs. 12, 13) we showed that the regular source of the KN solution has many features of the bag models, and therefore, the structure of dressed electron may be similar to the structure of hadrons. The obtained structure of the KN source forms a combined bag-string-quark system, which answers also the question, why the bare electron looks as a pointlike structureless particle.

2. Dressed Electron as a Bag-String-Point Complex

There were two lines of the development of the problem of source of KN solution: the stringlike source started in our works,[1,2] and the disk-like model of the KN electron started by Israel[5] and then modified by López in Ref. 14 to the oblate bubble model. Both lines of development are joined now into the bag-bubble model,[12,13] which, similar to many soliton models and the known MIT and SLAC bag models, is based on the Higgs mechanism of symmetry breaking.

Structure of the bag-like source is UNIQUELY determined by the requirements:

(R1) Quantum INTERIOR: no Gravity INSIDE the source!
(R2) Gravitating EXTERIOR: classical exact KN solution OUTSIDE!
(R3) A smooth transition between internal and external regions.

The KN metric in the Kerr-Schild form is[15]

$$g_{\mu\nu}^{(KN)} = \eta_{\mu\nu} + 2H_{(KN)} k_\mu k_\nu \, , \tag{1}$$

where

$$H_{(KN)} = \frac{mr - e^2/2}{r^2 + a^2 \cos^2 \theta} \, , \tag{2}$$

and k_μ is the null vector field tangent to the Kerr congruence, see Fig. 1.

Shape of the bubble is uniquely determined by the requirements **R1** and **R2** as the surface of "zero gravity potential"

$$H_{(KN)} = 0 \tag{3}$$

corresponding to matching of the external KN solution with a flat core of the source. The corresponding disk-like surface $r = R = e^2/2m$, where r is the Kerr oblate spheroidal coordinate, is the surface of the zero gravitational potential. For parameters of an electron, thickness of the disk $R = e^2/2m$ corresponds to known

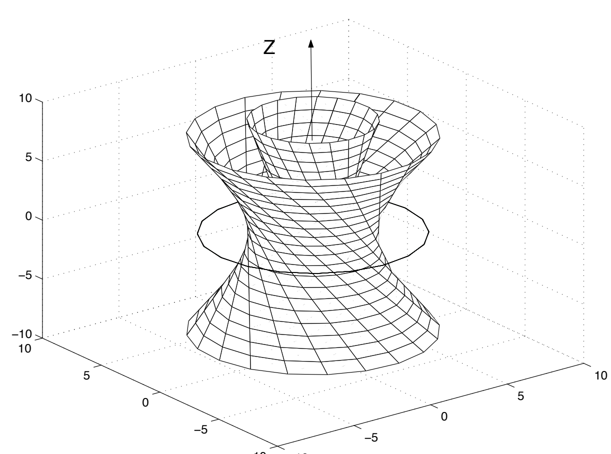

Fig. 1. The Kerr congruence focuses at the Kerr singular ring forming a branch line of space.

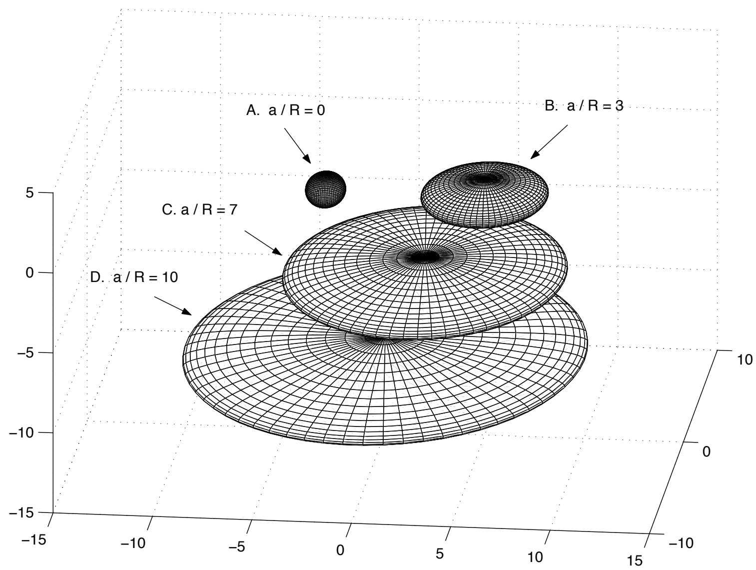

Fig. 2. Radius of the disk depends on angular momentum $J = a/m$. Shape of the disk-like bubble for different ratios a/R.

classical radius of electron, while radius of the disk, which is slightly exceeds the value $a = \hbar/2m$, corresponds to its Compton length. So, the ratio $R/a = 137^{-1}$ is the fine structure constant. How the classical KN solution knows the basic quantum parameters? Is it coincidence? The KN disk-like source takes the Compton size, and thus, we should identify it as a *dressed* electron.

The em field is concentrated in the equatorial plane, $\cos\theta = 0$, at the border of the disk, $r = r_e$, forming a closed relativistic string very close to the regularized Kerr singular ring. Boundary of the disk-like source $R = r_e = e^2/2m$ plays the role of a cut off parameter. Vector potential of the KN solution

$$A_\mu dx^\mu = -Re\left[\left(\frac{e}{r + ia\cos\theta}\right)\right](dr - dt - a\sin^2\theta d\phi) \tag{4}$$

contains the longitudinal ϕ component which forms the closed Wilson loop along boundary of the disk. Interplay of this loop with phase of the Higgs field leads to quantization of the angular momentum of the KN source.

Shape of the source is compliant to deformations, and a circular string-like structure is formed of the edge border of the disk, size of which is proportional to angular momentum (see Fig. 2). There appear the specific features which allow to identify the KN source as a *bag model*:[12,13]

– the source is formed by the Higgs mechanism of symmetry breaking,
– the source is compliant, and its deformations create stringy structure,
– the source admits consistent implementation of the Dirac equation,
– mass of the confined fermion is created by the Higgs field and is variable function of the space-time distribution of the Higgs condensate.

3. Supersymmetric Domain Wall Phase Transition

The usual non-linear self-interaction of the Higgs field described by the potential $V(r) = \lambda(|H|^2 - \eta^2)^2$ conflicts with external KN solution, and we shell use the *supersymmetric scheme of phase transition* based on the triplet of the chiral fields $\Phi^{(i)} = \{H, Z, \Sigma\}$, where H is taken as the Higgs field. As it was shown in Refs. 12, 16, 17, the corresponding Higgs-Landau-Ginzburg (HLG) Lagrangian with a special superpotential $W(\Phi^{(i)})$ (suggested by J. Morris[18]) provides two vacuum states:

(I.VacIn)- a supersymmetric false-vacuum state inside the bubble; the Higgs field is nonzero $|H| = \eta$, and space-time is flat, but the gauge symmetry is broken, and **(II.VacExt)**- the external vacuum state with vanishing Higgs field $|H| = 0$; $Z = 0$; $\Sigma = \eta$ leading to the unbroken gauge symmetry of the external KN solution.

The field model of regularization in zone **(I.VacIn)** coincides with the well known Nielsen-Olesen field models for the vortex string in superconducting media. Interaction of Higgs field $H(x) = |H|e^{i\chi(x)}$ with the KN vector potential inside the bubble is determined by the equation

$$\nabla_\nu \nabla^\nu A_\mu = I_\mu = \frac{1}{2} e|H|^2(\chi,_\mu + eA_\mu)\,. \qquad (5)$$

As a consequence of the rhs of (5), the gradient of the phase of the Higgs field $\chi,_\mu$ compensates potential A_μ and the current I_μ is expelled from interior of the bubble to its boundary. The EM field is regularized, indicating superconducting nature of the internal vacuum state. As was shown in Refs. 16 and 12, Eq. (5) leads to two important peculiarities of this solitonic source:

(i) spin of the source is quantized – the flux of the vector potential in ϕ-direction forms a quantum Wilson loop $\oint eA_\phi d\phi = -4\pi ma$, leading to relation $J = ma = n\hbar/2,\ n = 1, 2, 3...$

(ii) the source has an oscillon structure – the Higgs condensate forms a coherent vacuum state oscillating with the frequency $\omega = 2m$.

The consequent treatment in Ref. 13 showed that the bag-bubble represents a supersymmetric, BPS-saturated soliton formed by the supersymmetric domain wall (DW) phase transition. We should note that the supersymmetric chiral field models where earlier considered mainly for the case of planar domain walls, and the corresponding reduction of the Hamiltonian to Bogomolnyi form was performed by using an artificial "trick" (see for example Refs. 19 and 20) connected with introduction of the constant phase factors, physical meaning which was very uncertain. In the case of DW for the KN source, this transformation turns out to be still more complicated, because the DW forms a bubble with a very curved profile (see Fig. 3) and the Bogomolnyi equations related to twisted coordinate system of the Kerr geometry.

Reduction of the chiral DW equations to Bogomolnyi form, performed in Ref. 13 for the curved Kerr DW source, showed important peculiarity that one of the phases, that have been considered in previous models as constant, in the Kerr DW model

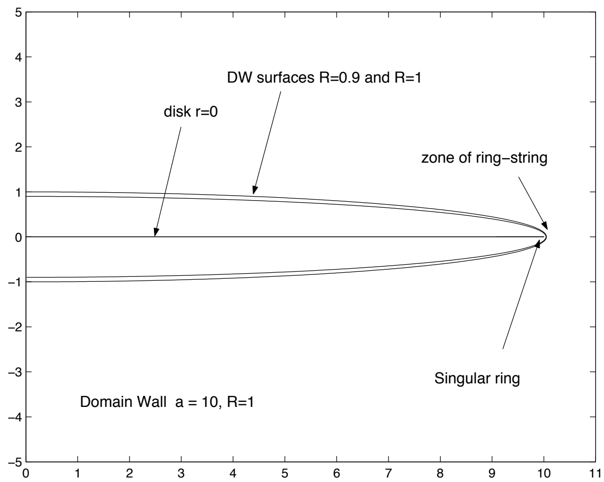

Fig. 3. Profile of the axial section of the spheroidal domain wall phase transition.

turned out to be related with the phase of the Higgs field, and therefore, it became dependent on the angular coordinate and time. Therefore, the transformation to Bogomolnyi form, which previously looked rather artificial, acquired in the Kerr geometry the clear physical meaning, by showing how the obscure constant phases should work at full capacity.

It was obtained that the mass-energy of the supersymmetric interior of the bubble, together with its DW boundary, is determined by the incursion of the superpotential crossing the DW boundary $\Delta W = W(R+\delta) - W(R-\delta) = -\mu\eta^2$. Integration over the Kerr radial coordinate r on the positive sheet of Kerr geometry, $r \in [0, R]$, yields

$$M_{ch} = 4\pi(R^2 + \frac{1}{3}a^2)\Delta W \,. \tag{6}$$

However, as it was recently obtained (unpublished), the total contribution should vanish, because integration over r may analytically be extended to negative sheet, $r \in [-R, 0]$, where ΔW gives opposite contribution. Therefore, the KN source forms a type of "breather", the bubble-antibubble analog of the 2D kink-antikink solution. As a result, the total mass of the source is determined only by the surface currents and by the external EM field.

4. Embedding of the Dirac Equation in the KN Bag Model

The massive quark-fermion, described by the Dirac equation, is confined inside the bag, trying to get energetically advantageous position with minimum mass. Thus, in the bag models mass is variable function determined by vev of the Higgs condensate. In the KN bag, the Higgs condensate is enclosed inside the bad[b] (**I.VacIn**), and

[b]Note, that in the MIT bag situation is opposite that is unacceptable for KN gravity.

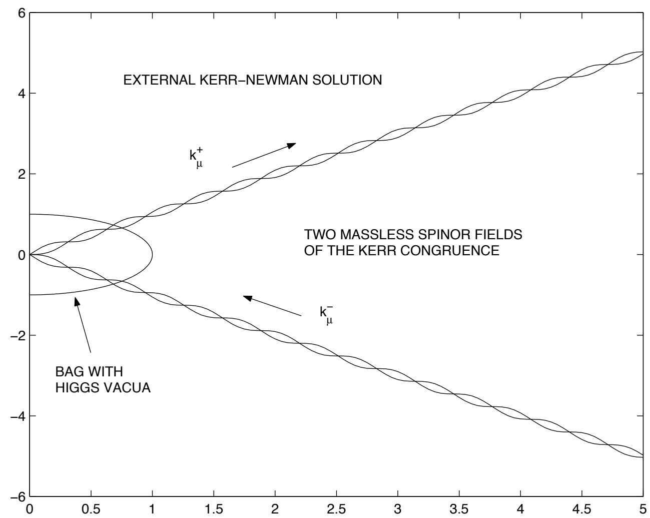

Fig. 4. Spinors ϕ_α and $\bar{\chi}^{\dot{\alpha}}$ are placed on different sheets controlled by the Kerr theorem. Inside the bag the Weyl spinors are joined into the Dirac field, acquiring mass via Yukawa coupling.

the Dirac equation acquire mass inside the bag, while outside it is massless, and the splits into two equations for the massless Weyl spinors. The Weyl spinors have to be collinear to the Kerr congruence.[c] The later is determined by the **Kerr Theorem** in terms of the projective twistor coordinates[12,15,17]

$$T^a = \{Y, \quad \zeta - Yv, \quad u + Y\bar{\zeta}\}\,. \tag{7}$$

We note that the first projective twistor coordinate $Y = \phi_1/\phi_0$, represents in fact the Weyl spinor ϕ_α, and thus, this *Twistor/Spinor correspondence* provides consistency of the Dirac field with KN gravity.

The two Weyl spinor solutions, ϕ^α and $\bar{\chi}_{\dot{\alpha}}$, generated by the Kerr theorem are consistent with KN solution. However, they are not equivalent, since they are collinear to two different Kerr congruences and turn out to be related to different sheets of the Kerr geometry. Only one of them is "retarded" and compatible with the external KN solution. Another one is "advanced" and should live on another (unphysical) sheet of the Kerr geometry, see Fig. 4. Inside the bag the space-time is flat, and the both spinors turns out to be consistent with metric. They are joined into a Dirac bispinor, satisfying the Dirac equation

$$(\gamma^\mu \partial_\mu + m)\Psi(x) = 0\,, \tag{8}$$

which acquires the mass from the Higgs condensate $H(x)$. Similarly to the other bag models, the mass turns out to be a variable function $m(x) \equiv gH(x)$ depending on the concentration of the Higgs field.

The mentioned non-equivalence of the "left" and "right" Weyl spinors is consistent with basic conceptions of the electroweak sector of the Standard Model.

[c]It requires the algebraically special structure of the Kerr geometry.

5. Stringy Deformations of the KN Bag

Taking the bag model conception, we should also accept the dynamical point of view that the bags are to be soft and deformed acquiring excitations, similar to excitations of the dual string models. By deformations the bags may form stringy structures with radial and rotational excitations, forming the open strings or flux-tubes.[11]

The bag-like source of the KN solution without rotation, $a = 0$, has the classical electron radius $R = r_e = e^2/2m$, and is similar to the suggested by Dirac[21] the spherical "extensible" electron model.[d] The KN rotating disk-like bag, may be considered as the spherical bag stretched by rotation to the disk of the Compton radius, $a = \hbar/2mc$.

It has been obtained long ago that the Kerr geometry is related with the dual string models. The Kerr singular ring was associated in Refs. 1, 2, 22, 23 with a closed ring-string which may carry traveling waves like a waveguide. In the soliton-bag model the Kerr singularity disappears, but its role is played by the sharp border of the disk-like bag. Like the Kerr singular ring, it serves as a carrier of the traveling waves. It was shown in Ref. 24 that field structure of this string is similar to the structure of the fundamental string, obtained by Sen as a solitonic string-like solution to low energy string theory. The EM and spinor excitations of the KN solution are concentrated near the Kerr ring, forming the stringy traveling waves.[1,2,22] For the stationary KN solution the EM field forms a *frozen* wave, located along the border of the disk-like source. Locally, this frozen string is the typical plane-fronted EM wave propagating along the Kerr singular ring. In the regularized KN solution, the Kerr singular ring is regularized by the cut-off parameter $R = r_e$, which is constant because of the axial symmetry of the stationary KN solution, as shown in Fig. 5A. The Kerr circular ring is formed as the focus line of the Kerr null congruence, and represents a closed light-like curve. The corresponding closed KN string, positioned along border of the bag, turns out to be relativistically rotating, and any external observer will see it reduced by Lorentz contraction.[25] The simple cinematic relations allow estimate the Lorentz factor, $\gamma^{-1} = \sqrt{1 - v^2} \approx \alpha = 137^{-1}$, and therefore, the *observable radius of the closed string in the source of stationary KN solution is reduced by Lorentz contraction to the classical size* r_e.

A new effect appears when the "frozen" EM excitation of the stationary KN solution is completed by the lowest excitation of the traveling wave. All the exact solutions for the EM field on the Kerr background were obtained in Ref. 15, and they are defined by analytic function $A = \psi(Y, \tau)/P^2$ where $Y = e^{i\phi} \tan \frac{\theta}{2}$ is a complex projective angular variable, $\tau = t - r - ia \cos\theta$ is a complex retarded-time parameter and $P = 2^{-1/2}(1 + Y\bar{Y})$ for the Kerr geometry at rest. Vector potential

[d] In fact it was a prototype of the bag model, which had in rest the classical radius, but radial excitations turned it into muon.

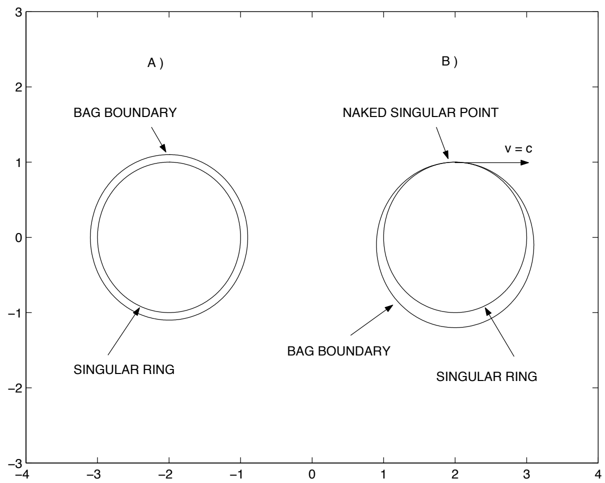

Fig. 5. Regularization of the KN EM field. Section of the disk-like bag in equatorial plane. Distance from positions of the boundary of the bag from position of the (former) singular ring acts as a cut-off parameter R. (A) Axially symmetric KN solution gives a constant cut-off $R = r_e$. (B) The boundary of the bag is deformed by a traveling wave, creating a circulating singular point of tangency (zitterbewegung).

is determined by function ψ as follows[15]

$$A_\mu dx^\mu = -Re\left[\left(\frac{\psi}{r + ia\cos\theta}\right)e^3 + \chi d\bar{Y}\right], \quad \chi = 2\int(1 + Y\bar{Y})^{-2}\psi dY\,. \tag{9}$$

The simplest function $\psi = -e$ corresponds to the frozen EM field of the stationary KN solution leading to the function (2). The lowest excitation is given by combination

$$\psi = e\left(1 + \frac{1}{Y}e^{i\omega\tau}\right). \tag{10}$$

The EM traveling waves will deform the bag surface, and it is easy to find a back-reaction of this excitation. Like the stationary KN solution, function ψ acts on the metric through the function H, which has in general case the form

$$H = \frac{mr - |\psi|^2/2}{r^2 + a^2\cos^2\theta}\,. \tag{11}$$

Boundary of the disk is very close to position of the Kerr singular ring, and regularization by the *constant* cut-off parameter $R = r_e$, is replaced now by the boundary of the deformed bag, which again can be determined from the condition $H = 0$, which yields the cut-off parameter for EM field $R = |\psi|^2/2m$. The corresponding deformations of the bag boundary are shown in Fig. 5B. One sees that solution (10) takes in equatorial plane ($\cos\theta = 0$) the form $\psi = e(1 + e^{-i(\phi-\omega t)})$, and the cut-off parameter $R = |\psi|^2/2m = \frac{e^2}{m}(1 + \cos(\phi - \omega t)$ depends on $\phi - \omega t$. Vanishing R at $\phi = \omega t$ creates singular pole which circulates along the ring-string together with traveling wave of the string excitation, reproducing the known zitterbewegung of the Dirac electron. This pole may be interpreted as a point-like bare electron forming a single end point of the light-like circular string, or as a light-like quark associated with fermionic sector. This model unites the dressed and point-like electron in a single bag-string-quark structure.

6. Conclusion

Supersymmetry determines the structure of source of the KN solution almost uniquely, leading to Bogomolnyi equations which determine its stability.[13] Any deviation of the domain wall boundary from the surface of "zero gravity potential", determined by the equation (3), will break supersymmetry, adding gravitating terms to the supersymmetric vacuum states and breaking the Bogomolnyi bound for the domain wall phase transition. This works for any value of the gravitational constant, contrary to the weakness of the gravitational interaction.

Answering the question: which structure of the electron would be consistent with gravity, the Kerr-Newman gravity shows striking awareness of the many basic features of the quantum theory, such as the values of the classical and Compton parameters of the electron, and its gyromagnetic ratio, leading to a geometric meaning to the fine-structure constant $\alpha = R/a$, as the degree of oblateness of the disk-like source of the KN solution. The obtained bag-like structure of the source of the KN solution shows its relationships with string theory, and leads us to the gravitating bag model, suggesting that electron structure should not be far from the structure of hadrons. The gravitating bag model of the source of KN solution unifies quantum theory with gravity, indicating that gravity plays fundamental role in the structure of elementary particles.

Acknowledgments

Author thanks Yu. Obukhov and O. Teryaev for the recent discussion on the possible role of the weak gravitational constant in the strong interactions. This work is supported by the RFBR grant 13-01-00602.

References

1. A. Ya. Burinskii, *Zh. Eksp. Teor. Fiz.* **66**, 406 (1974) [*Sov. Phys. JETP* **39** 193 (1974)].
2. D. D. Ivanenko and A. Ya. Burinskii, *Izv. Vuz. Fiz.* **5**, 135 (1975).
3. E. T. Newman and A. I. Janis, *J. Math. Phys.* **6**, 915 (1965).
4. B. Carter, *Phys. Rev.* **174**, 1559 (1968).
5. W. Israel, *Phys. Rev. D* **2**, 641 (1970).
6. A. Krasinski, *Ann. Phys. (N.Y.)* **112**, 22 (1978).
7. A. Burinskii, E. Elizalde, S. R. Hildebrandt and G. Magli, *Phys. Rev. D* **65**, 064039 (2002).
8. Ch. Heinicke and F. W. Hehl, *Int. J. Mod. Phys. D* **24**, 1530006 (2015).
9. I. Dymnikova, Electromagnetic source for the Kerr-Newman geometry, to appear in *Int. J. Mod. Phys. D*.
10. A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn and V. F. Weisskopf, *Phys. Rev. D* **9**, 3471 (1974).

11. W. A. Bardeen, M. S. Chanowitz, S. D. Drell, M. Weinstein and T.-M. Yang, *Phys. Rev. D* **11**, 1094 (1974).
12. A. Burinskii, *JETP* **148**, 228 (2015).
13. A. Burinskii, Stability of the lepton bag model based on the Kerr-Newman solution, to appear in *Zh. Eksp. Teor. Fiz.*
14. C. A. López, *Phys. Rev. D* **30**, 313 (1984).
15. G. C. Debney, R. P. Kerr and A. Schild, *J. Math. Phys.* **10**, 1842 (1969).
16. A. Burinskii, *J. Phys. A*: *Math. Theor.* **43**, 392001 (2010).
17. A. Burinskii, *Grav. Cosmol.* **21**, 28 (2015).
18. J. R. Morris, *Phys. Rev. D* **53**, 2078 (1996).
19. M. Cvetic, F. Quevedo and S. J. Rey, *Phys. Rev. Lett.* **67**, 1836 (1991).
20. G. W. Gibbons and P. K. Townsend, *Phys. Rev. Lett.* **83**, 1727 (1999).
21. P. A. M. Dirac, *Proc. R. Soc. Lond. A* **268**, 57 (1962).
22. A. Burinskii, *J. Phys.*: *Conf. Series* **361**, 012032 (2012).
23. A. Burinskii, *Theor. Math. Phys.* **177**, 1492 (2013).
24. A. Burinskii, *Phys. Rev. D* **52**, 5826 (1995).
25. L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Pergamon, Oxford, 1975).