# A New Derivation of $E_{o}=m c^{2}$ Explains a Particle's Inertia 

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#### Abstract

The relationship $E_{o}=m c^{2}$ for a resting elementary particle composed of a locally circulating photon of energy $E_{o}$ and momentum of magnitude $p_{o}=E_{o} / c$ is derived from the rate of change of the circulating momentum $\vec{p}_{o}$ and from the acceleration $\vec{a}$ of the circulating photon, using Newton's second law $\Sigma \vec{F}=d \vec{p} / d t=m \vec{a}$ applied to the circulating photon. This derivation gives the inertial mass $m$ of the resting elementary particle as $m=E_{o} / c^{2}$. Therefore $E_{o}=m c^{2}$ for a resting particle composed of a locally circulating photon of energy $E_{o}$. The electron's relativistic energy-momentum equation is derived from the vector momentum relations in a moving particle model.


Key words: inertia, mass, inertial mass, energy, momentum, particle, $E_{o}=m c^{2}$, photon, electron, model

## Introduction

The relation $E_{o}=m c^{2}$ for a stationary object of mass $m$ indicates that this object contains an inherent energy $E_{o}=m c^{2}$. Under appropriate circumstances this energy can be released in the form of photons. For example, an electron and its antiparticle the positron can mutually annihilate to create two or three photons whose total energy is equal to $c^{2}$ times the total mass of the electron and the positron before this annihilation. Many attempts to derive $E_{o}=m c^{2}$ theoretically were made by Einstein and others after the idea that the loss of energy $E$ from an object is accompanied by a loss of mass $\Delta m=E / c^{2}$ by the object was introduced by Einstein [1] as a consequence of the special theory of relativity. Though the proof of this mathematical formula is well established experimentally, the road to its theoretical proof has been rocky, as described by Ohanian [2]. Also, theoretical derivations of $E_{o}=m c^{2}$ for a particle have not explained the origin or nature of the inertial property of the mass $m$ of a particle.

The fact is that the origin of the inertial property of the mass of elementary particles of matter is not understood. Inertial mass is represented by $m$ in Newton's second law of motion $\Sigma \vec{F}=m \vec{a}$. The inertial mass $m$ of an object is defined as the quantitative measure of the resistance of an object to acceleration by an external net force, that is, $m=\Sigma \vec{F} / \vec{a}$. The nature or origin of this resistance to acceleration of an object by an external force has been obscure since the time that Newton first formulated the idea of the inertial mass $m$ of an object.
$\Sigma \vec{F}=m \vec{a}$ applies down to the level of individual electrons and other elementary particles with a measured inertial mass $m$. A resting electron has a measured inertial mass or invariant mass $m=9.108 \times 10^{-31} \mathrm{~kg}$. This mass $m$ can be used to calculate a particle's acceleration $\vec{a}$ when an external force $\vec{F}$ is applied to the particle. But the nature or origin of this resistance of an individual particle's mass $m$ to acceleration $\vec{a}$ by this force $\vec{F}$ is unknown.

## Derivation of the resting electron's inertial mass

A resting particle's inertial mass can be easily derived if the particle such as an electron is composed of a hypothesized locally circulating photon of energy $E_{o}$ and momentum $p_{o}=E_{o} / c$. It is an experimental fact that if a photon's energy is $E_{o}$, the magnitude of the photon's momentum is $p_{o}$. The photon's momentum $\vec{p}_{o}$ is a vector quantity pointing in the same direction as the velocity vector $\vec{c}$ of the photon. If the photon is circulating to form an electron, then the directions of both the momentum $\vec{p}_{o}$ and the velocity $\vec{c}$ of the photon are continually changing. According to Newton's second law $\Sigma \vec{F}=m \vec{a}$, an external force $\vec{F}$ is required to change the momentum of the circulating photon that forms an electron. The photon's momentum $\vec{p}_{o}$ and velocity $\vec{c}$ keep changing direction, but the magnitudes of $\vec{p}_{o}$ and $\vec{c}$ remain constant.

Let us first make a simple model of a resting elementary particle as a photon of energy $E_{o}$, momentum $p_{o}=E_{o} / c$ and speed $c$ moving in a circle of radius $R$. The circling photon has angular velocity $\omega=c / R$. Newton's second law defines a force $\vec{F}$ as $\vec{F}=d \stackrel{\rightharpoonup}{p} / d t$, the time rate of change of the momentum of an object. For this photon of momentum $\vec{p}_{o}$ moving in this circular orbit of radius $R$, the time rate of change of the photon's momentum is given by $\vec{F}=d \vec{p}_{o} / d t=\omega p_{o} \hat{r}$ where $\hat{r}$ is a unit vector pointing towards the center of the circle. This force $\vec{F}$ on the circling photon continually points towards the center of the circle as the photon moves around the circle. There is also a centripetal acceleration $\vec{a}_{c}$ of the circling photon, of magnitude $a_{c}=c^{2} / R=\omega^{2} R$. The photon's centripetal acceleration vector $\vec{a}_{c}$ also points towards the center of the circle.

Starting with Newton's second law $\vec{F}=d \vec{p}_{o} / d t=m \vec{a}_{c}$, where $m$ is the inertial mass of the circulating photon which composes the electron (whose inertial mass $m$ is by assumption the inertial mass $m$ of the electron composed of the circling photon), we have

$$
\begin{aligned}
m & =\vec{F}^{\prime} \vec{a}_{c} \\
& =\left(d \vec{p}_{o} / d t\right) /\left(\omega^{2} R \hat{r}\right) \\
& =\left(\omega p_{o} \hat{r}\right) /\left(\omega^{2} R \hat{r}\right) \\
& =p_{o} / \omega R \\
& =p_{o} / c \\
& =\left(E_{o} / c\right) / c \\
& =E_{o} / c^{2}
\end{aligned}
$$

Since the elementary particle is proposed to be composed of this circulating photon, $m=E_{o} / c^{2}$ is the inertial mass of this particle. So we have derived the relationship $E_{o}=m c^{2}$ for an elementary particle composed of a circling photon by deriving the inertial mass $m=E_{o} / c^{2}$ of the circling photon of energy $E_{o}$ and momentum $p_{o}=E_{o} / c$ that forms the particle model.

The above derivation of inertial mass applies to any other elementary particle composed of a circling photon or other light-speed particle having a resting energy $E_{1}$ and momentum $p_{1}=E_{1} / c$.

While a circular orbit of the photon was used for simplicity in the above inertial mass calculation, other smoothly curving trajectories of a photon of energy $E_{o}$ forming a resting particle would lead to the same result $m=E_{o} / c^{2}$. This is because at any point on the photon's circulating trajectory, there would be an instantaneous value of the angular velocity $\omega$ of the photon and an instantaneous value of $R$ for the radius of curvature of the photon's curving trajectory, such that $\omega R$ equals $c$, the speed of the photon, thus leading to the same result above: the particle's inertial mass is $m=E_{o} / c^{2}$.

In the above derivation of $m=E_{o} / c^{2}$, it is only necessary that the elementary particle composed of the circulating photon is resting, i.e. that the elementary particle's velocity is zero, and that the photon composing the elementary particle is circulating at speed $c$ with energy $E_{o}$ and circulating momentum $p_{o}=E_{o} / c$. If the particle formed by the circulating photon has a non-zero velocity, the calculated inertial mass of the particle will no longer be the invariant mass of the particle, but will be larger, and proportional to the total energy $E$ of the moving particle. The circulating photon continues to move at the speed c while forming a moving particle, while the speed of the moving particle itself will always be less than c .

The above derivation doesn't require the trajectory of the elementary particle modeled by a circulating photon of energy $E_{o}$ to have a particular radius. However, if an elementary particle such as an electron were to be modeled, other properties of an electron would have to be taken into consideration in the modeling process, such as the spin and the magnetic moment of the electron. If a resting electron of mass $m=E_{o} / c^{2}$ were to be modeled by a circulating photon of energy $E_{o}$, then the frequency of the circulating photon would be given by $E_{o}=h \nu=m c^{2}$. The circulating photon's wavelength would then be given by $h \nu=h c / \lambda=m c^{2}$. This gives $\lambda=h / m c$, which is called the Compton wavelength and equals $2.43 \times 10^{-12} \mathrm{~m}$.

## Justifications for modeling elementary particles by a circulating photon

It may be argued that since photons don't circulate, how can a particle be composed of a circulating photon? Also, since a photon has spin of $1 \hbar$, or spin 1 in units of $\hbar$, how can a photon circulate to form an electron that has spin of $\hbar / 2$, or spin $1 / 2$ in units of $\hbar$ ? And how can a photon, which is uncharged, circulate to form an electrically charged
electron? The answer to these questions may be that there exists a previously unobserved variety of photon that is electrically charged, has spin $1 / 2$ and can circulate to form an electron or another electrically charged elementary particle. Why then have physicists never observed this variety of photon? It may be because it is generally curled up and called an electron, or another name if it is another particle. A model of a relativistic electron composed of a spin- $1 / 2$ circulating charged photon was proposed by Gauthier [3], and it was shown by Gauthier [4] that this electron model, when resting, generates the electron's mass as in the above calculation. The momentum relationships in this electron model generate the electron's relativistic energy-momentum equation.

One indirect source of support for the proposal of a spin- $1 / 2$ charged photon composing an electron comes from Paul Dirac. In his Nobel Prize lecture Paul Dirac [5] said in reference to the Dirac equation: "It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment."

Dirac did not propose that the electron is a circulating spin- $-1 / 2$ charged photon. But the light-speed spin- $-1 / 2$ electron that he describes as a solution to the Dirac equation sounds very much like the proposed circulating spin- $1 / 2$ charged-photon model of the electron. This electron model has, besides internal light-speed, other properties of the electron described by Dirac: its internal small amplitude $R_{o}=\hbar / 2 m c$, its internal high frequency $v_{\text {zitt }}=2 m c^{2} / h$ called the zitterbewegung frequency, and its spin $1 / 2 \hbar$. It has one-half of the Dirac electron's magnetic moment. And it generates the relativistic electron's de Broglie wavelength $\lambda_{\text {deBroglie }}=h / \gamma m v$.

In further support of the possibility of a spin- $1 / 2$ charged photon, it was recently announced by Ballentine [6] that a new variety of photon, having spin $1 / 2$ instead of the normal spin 1 , has just been discovered. Though this new photon variety is uncharged, it is still a surprising discovery about photons. It suggests that other varieties of photon such as the spin- $1 / 2$ charged photon may also be discovered.

## Comparison with derivations of $E_{o}=m c^{2}$ from the special theory of relativity

A new derivation of the relation $m=E_{o} / c^{2}$ of the inertial mass $m$ of a resting particle to its resting energy $E_{o}$ has been presented here using Newton's $2^{\text {nd }}$ law of motion $\Sigma \vec{F}=m a$, which defines inertial mass. The derivation assumes that the resting particle is composed of a circulating photon (or other lightspeed particle) having energy $E_{o}$ (the rest energy of the particle) and momentum $p_{o}=E_{o} / c$.

The relationship $E_{o}=m c^{2}$ for an object is normally derived from Einstein's special theory of relativity. Such relativistic derivations of $E_{o}=m c^{2}$ for a resting object, including Einstein's own derivations, analyze mass and energy relations as observed in different inertial frames of reference, and make use of the relativistic energy and momentum formulas for Doppler-shifted photons. The results of such relativistic derivations of an object's mass, energy and momentum relations are summarized by the relativistic energy-momentum equation $E^{2}=p^{2} c^{2}+m^{2} c^{4}$ for the relation of the total energy $E=\gamma m c^{2}$ of a moving object (as measured in some inertial coordinate system) to its linear momentum $p=\gamma m v$ and mass $m$. (The mass $m$ is called the object's invariant mass because it has the same value in any inertial coordinate system.) In the case where $p=0$ and therefore $E=E_{o}$ (for a resting object in some inertial coordinate system), these relativistic equations reduce to $E_{o}=m c^{2}$. In relativistic calculations, objects are considered to be either point masses or extended objects with internal structure. A photon is never at rest since it always moves at speed $c$ in any inertial reference frame, and it has an invariant mass $m=0$. Okun $[7,8]$ discusses the history, derivations and usage of the equation $E_{o}=m c^{2}$ as well as the more commonly known equation $E=m c^{2}$. This equation implies that the mass of an object is proportional to the object's total energy.

The relativistic energy-momentum equation $E^{2}=p^{2} c^{2}+m^{2} c^{4}$ works for a photon moving in a straight line, where a photon's energy-momentum relationship is $E=p c$. In this case the relativistic energy-momentum equation gives $m=0$ for the mass of a photon, as expected. If an elementary particle is modeled by a circulating photon of energy $E=E_{o}$ and momentum $p=p_{o}=E_{o} / c$, the energy-momentum equation for this circulating photon forming the elementary particle also gives $m=0$. This is wrong for the mass $m$ of an elementary particle, because the mass $m$ of an elementary particle having experimental rest energy $E_{o}$ is $m=E_{o} / c^{2}$ and not zero. The relativistic energymomentum equation $E^{2}=p^{2} c^{2}+m^{2} c^{4}$ was not designed for particles composed of a circulating photon, unless this equation can be understood in a way that takes into account the resting particle's internal circulating momentum $p_{o}=E_{o} / c=m c$.

## The relativistic energy-momentum equation derived for a particle composed of a circulating photon

As mentioned above, the relativistic energy-momentum equation for an elementary particle or other physical object is $E^{2}=p^{2} c^{2}+m^{2} c^{4}$. Okun [8] calls this equation "the most fundamental equation of relativity theory". This equation applies both to particles without mass (like the photon which always travels at $c$ ) and to particles with mass (which travel at speeds always less than $c$ ). But as shown above, this equation does not give the correct mass of the circulating photon composing a resting particle, since, as we have seen above, the circulating photon composing a resting particle has mass $m=E_{o} / c^{2}$ and not zero.

Let us take a simple example and suppose that a resting elementary particle is composed of a photon of energy $E_{o}$ and momentum $\vec{p}_{o}$ of magnitude $p_{o}=E_{o} / c=m c$
where $m=E_{o} / c^{2}$. The photon moves in a circle in the $x-y$ plane. Let us now suppose that the elementary particle as a whole has a velocity $\vec{v}$ in the $z$-direction and therefore has a relativistic linear momentum $\vec{p}=\gamma m \vec{v}$ also in the $z$-direction. The circular motion of the photon now becomes a helical motion in the $z$-direction. Both $\vec{v}$ and $\vec{p}$ are perpendicular to the $x-y$ plane and therefore perpendicular to $\vec{p}_{o}$. The helically-moving photon's momentum component in the $x-y$ direction is $\vec{p}_{o}$ while the $z$-component of the momentum of the elementary particle is $\vec{p}$. Since $\vec{p}_{o}$ and $\vec{p}$ are perpendicular, they are the two components of the total momentum $\vec{P}_{\text {total }}$ of the moving elementary particle (composed of the helically circulating photon) given by $\vec{P}_{\text {total }}=\vec{p}+\vec{p}_{o}$. The magnitude of $\vec{P}_{\text {total }}$ is given by the Pythagorean theorem as

$$
\begin{aligned}
P_{\text {total }}^{2} & =p^{2}+p_{o}{ }^{2} \\
& =(\gamma m v)^{2}+(m c)^{2} \\
& =\frac{(m v)^{2}}{1-v^{2} / c^{2}}+(m c)^{2} \\
& =\frac{(m v)^{2}}{1-v^{2} / c^{2}}+\frac{(m c)^{2}\left(1-v^{2} / c^{2}\right)}{1-v^{2} / c^{2}} \\
& =\frac{(m v)^{2}+(m c)^{2}-(m v)^{2}}{1-v^{2} / c^{2}} \\
& =\frac{(m c)^{2}}{1-v^{2} / c^{2}} \\
& =(\gamma m c)^{2} \\
& =\frac{\left(\gamma m c^{2}\right)^{2}}{c^{2}} \\
P_{\text {total }}^{2} & =\frac{E^{2}}{c^{2}}
\end{aligned}
$$

since the particle's total energy $E$ is given by $E=\gamma m c^{2}$ This shows that for a moving fundamental particle modeled by a helically-moving photon, the particle's momentum relations described by $P_{\text {total }}{ }^{2}=p^{2}+p_{o}{ }^{2}$ also correspond to $\frac{E^{2}}{c^{2}}=p^{2}+(m c)^{2}$, which is another form of the relativistic energy-momentum equation for an elementary particle: $E^{2}=p^{2} c^{2}+m^{2} c^{4}$ 。

The energy $E$ of a helically circulating photon is proportional (by $E=P_{\text {total }} c$ ) to the magnitude of the total linear momentum $\vec{P}_{\text {total }}$ of the helically circulating photon composing the elementary particle. This total momentum $\vec{P}_{\text {total }}$ is the vector sum of the particle's "internal" or transverse circling momentum component $\vec{p}_{o}=m \vec{c}$ and the particle's "external" or longitudinal momentum component $\vec{p}=\gamma m \vec{v}$ that is due to the velocity of the particle as a whole.

Three experimentally observed properties of a moving elementary particle are the particle's mass $m$, the moving particle's linear momentum $\vec{p}=\gamma m \vec{v}$, and the moving particle's total energy $E$. These are the quantities that go into the particle's relativistic energy-momentum equation, which is now can be understood to also express the vector momentum relations within the modeled elementary particle. Two of these vector quantities, $\vec{p}_{o}=E_{o} / c=m c$ and $\vec{P}_{\text {total }}=E / c=\gamma m c$, are unobserved experimentally. The particle's unobserved circling linear momentum vector $\vec{p}_{o}=E_{o} / c=m c$ is proposed to generate the particle's inertial mass $m$. The moving particle model's unobserved totalmomentum vector $\vec{P}_{\text {total }}=E / c=\gamma m c$ generates the relativistic electron's "transverse" inertial mass $\gamma m$ (see Gauthier [4] p. 5 for the derivation) and "longitudinal" inertial mass $\gamma^{3} m$ from its transverse and longitudinal momentum components respectively. Other experimentally observed properties of an elementary particle, such as an electron, are its quantized electric charge, its quantized spin, its magnetic moment, and its wave properties such as its de Broglie wavelength. A model of a specific elementary particle composed of an internally circulating photon should attempt to account for these other observed properties of the particle as well.

A model of the relativistic electron, based on the simple circling-photon model described above, was developed by Gauthier [3]. In this model, a proposed electricallycharged spin- $1 / 2$ photon travels along a double-looped circle for each Compton wavelength $h / m c$ distance along its circular trajectory, in a resting electron. This doublelooping spin- $1 / 2$ electrically-charged photon gives the electron model the correct electron spin of $\hbar / 2$ at relativistic speeds as well as at low speeds. In the moving electron model, the circular trajectory of the spin $-1 / 2$ charged photon in the resting particle model becomes a helical trajectory. The relativistic electron's external linear momentum $p=\gamma m v$ is derived from the linear momentum of the helically circulating spin- $1 / 2$ charged photon. Also, the relativistic electron's de Broglie wavelength $\lambda_{\text {deBroglie }}=h / \gamma m v$ is derived from the wavelength of the helically-circulating spin- $1 / 2$ charged photon moving along its helical trajectory. This relativistic electron model, through its internal or transverse momentum $p_{o}=m c$, its external or longitudinal momentum $p=\gamma m v$ and its total momentum $P_{\text {total }}=E / c=\gamma m c$ also accounts for the electron's relativistic energymomentum equation as explained above.

## The inertial mass of a photon is $m=E / c^{2}$

In the present derivation of a fundamental particle's inertial mass $m=E_{o} / c^{2}$, the inertial mass of the particle is calculated from the momentum $p_{o}=E_{o} / c$ of a proposed circulating photon composing the resting particle. The motion of the circulating photon is not limited to circular motion as long as an instantaneous angular frequency $\omega$ and instantaneous radius $R$ can be associated with the trajectory of the circulating photon, where $\omega R=c$. Since the resting particle composed of the circulating photon is found to have inertial mass $m=E_{o} / c^{2}$, this means that the circulating photon also has inertial mass $m=E_{o} / c^{2}$, since in this modeling approach the particle composed of a circulating photon IS the circulating photon.

The circulating photon's inertial mass $m$ is calculated to be the same value at every point along the trajectory of the circulating photon. It is not an average value calculated for the complete trajectory of the circulating photon. The circulating photon is not even required to have a closed or periodic trajectory as long as the photon maintains lightspeed c and the particle being modeled remains at rest. In the mathematical limit as the trajectory of the circulating photon becomes straighter and straighter, so that the instantaneous value of $\omega$ for the rotational frequency of the photon approaches zero while the radius of curvature $R$ of the photon increases indefinitely, the relationship $\omega R=c$ still holds and the inertial mass $m$ of the photon continues to be $m=E_{o} / c^{2}$. In the mathematical limit, a photon will still have inertial mass $m=E_{o} / c^{2}$ while moving in a straight trajectory, as well as when moving in a curved trajectory. In this view, a photon with energy $E=h v$ always has inertial mass $m=E / c^{2}$. If a linear-moving photon curves to form a fundamental particle, the photon gains an invariant mass $m=E_{o} / c^{2}$ instead of having an invariant mass equal to zero when the photon is moving linearly.

It may be claimed that photons do not move in circulating trajectories as in the present hypothesis, and therefore the inertial mass of a linear-moving photon cannot be derived from the inertial mass of a circulating photon. But a linear-moving photon can manifest its inertial mass during reflection from a mirror or scattering from another particle (as in Compton scattering of an x-ray photon by an electron in an atom.) Gauthier [9] shows a simple calculation of the change in a photon's vector momentum $\vec{p}$ and its vector velocity $\vec{c}$ during mirror reflection and during Compton scattering. Newton's second law $\Sigma \vec{F}=d \vec{p} / d t=m \vec{a}$ is then applied to this reflection and scattering that is assumed to occur in a very small but finite interval of time. The result is that in both cases the photon's inertial mass is calculated (in the center-of-momentum frame for Compton scattering) to be $m=E / c^{2}$, where $E=h v$ is the energy of the photon. This result is independent of the angle of incidence and reflection of the photon on the mirror. The implication is that if a photon shows inertial mass $m=E / c^{2}$ when being reflected or scattered, even at increasingly small glancing angles, then a photon also has inertial mass $m=E / c^{2}$ even when it is not being reflected or scattered. This is the case even though a photon's invariant mass is zero when its trajectory is straight. This is another way to show that a photon always has inertial mass $m=E / c^{2}=h \nu / c^{2}$.

## The force on a circulating photon forming an elementary particle

It may finally be objected that for a photon to form an electron by circulating in a local spatial region approximately the size of the electron's Compton wavelength $h / m c=2.43 \times 10^{-12} \mathrm{~m}$, a very large force on the photon would be required to curve its trajectory appropriately. For the electron modeled by a double-looping spin- $1 / 2$ charged photon in Gauthier [3], the central force required to act on the charged photon to curve its trajectory into a circle is calculated in Gauthier [4] to be 0.424 Newtons. What could be the source of such a large force on the electric charge of a circulating spin $-1 / 2$ charged photon? It may be that a force related to the principle of conservation of electric charge is what keeps the charged photon circulating. The reader is encouraged to think about what could cause such a large force on a spin- $1 / 2$ charged photon so that it circulates to form a fundamental particle like an electron.

## Conclusion

A model of a resting elementary particle composed of a circulating photon of energy $E_{o}$ and momentum $E_{o} / c$ provides a short, non-relativistic derivation of the particle's mass-energy relation $m=E_{o} / c^{2}$ or $E_{o}=m c^{2}$. To obtain this result, Newton's second law $\Sigma \vec{F}=d \vec{p} / d t=m \vec{a}$ or $m=(d \vec{p} / d t) / \vec{a}$, which defines inertial mass $m$, is applied to the circulating photon's changing vector momentum $\vec{p}_{o}=E_{o} / c$ and to its changing vector velocity $\vec{c}$. When the moving particle model's circulating inner momentum $\vec{p}_{o}=E_{o} / c=m \vec{c}$ is perpendicular to the particle's external momentum $\vec{p}=\gamma m \vec{v}$, the moving particle's relativistic energy-momentum equation $E^{2}=p^{2} c^{2}+m^{2} c^{4}$ is obtained from $\vec{P}_{\text {total }}=\vec{p}+\vec{p}_{o}$.

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