The Charged-Photon Model of the Electron Fits the Schrödinger Equation

Richard Gauthier
Santa Rosa Junior College

Abstract

This article shows how the charged photon model of the electron is consistent with the time-dependent and the time-independent forms of the one-dimensional Schrödinger equation, which describes the conservation of total energy of a non-relativistic charged photon in the presence of an electric potential.

Introduction

The charged photon model of the electron, as developed in Gauthier (2014a, 2014b), proposes that an electron can be modeled as a circulating charged photon. The electron's de Broglie wavelength is derived from the charged photon model in a simple way. The present article shows how the charged photon model, in the presence of an electric potential $V(x)$ satisfies the one-dimensional Schrödinger equation, where the wave number k in the Schrödinger equation solutions correspond to the longitudinal wave number of the circulating charged photon, while the angular frequency ω of the wave function solutions corresponds to the difference between the charged photon's total angular frequency $\omega_{total} = \gamma mc^2 / \hbar$ and the angular frequency ω_o of a resting electron where $\omega_o = mc^2 / \hbar$

The derivation of the de Broglie wavelength $\lambda_{deBroglie} = h / \gamma mv$ from the charged-photon model of the electron has implications for the interpretation of quantum mechanical wave functions in general, and specifically for wave functions that are solutions to the Schrödinger equation for bound or unbound electrons. The basic relationships that are built into the Schrödinger equation for non-relativistic electrons are the electron's energy $E = \hbar\omega$ and the electron's momentum $p = \hbar k = h / \lambda_{deBroglie}$. The charged photon model for the electron contains both of these expressions, similarly defined.

Let us consider the charged photon model in the light of the non-relativistic one-dimensional Schrödinger equation.

$$\frac{-\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

We can write the total energy of the charged photon as $E_{total} = \gamma mc^2$ and as $E_{total} = h\nu_{total} = \hbar\omega_{total}$ where $\omega_{total} = 2\pi\nu_{total}$ is the angular frequency $\omega_{total} = \gamma mc^2 / \hbar$

of the charged photon corresponding to an electron with total energy $E_{total} = \gamma mc^2$. In the non-relativistic limit, $E_{total} \to mc^2 + p^2/2m$ where p is the longitudinal component of the momentum of the charged photon (which equals the momentum of the electron). We can also write for the non-relativistic electron: $E_{total} = \hbar\omega_{total} = \hbar(\omega_o + \omega)$ where $\hbar\omega_o = mc^2$ is the resting energy of the electron and $\hbar\omega$ is the difference between the electron's total energy E_{total} and its resting energy $\hbar\omega_o$. If the charged photon (the electron) encounters a potential $V(x)$, the charged photon's changing energy E (because its kinetic energy is changing due to $V(x)$) is then given due to conservation of total energy by the expression

$$E + V(x) = E_{total} = \hbar\omega_{total}$$

or

$$mc^2 + p^2/2m + V(x) = \hbar(\omega_{total}) = \hbar(\omega_o + \omega) = \hbar\omega_o + \hbar\omega$$

subtracting $mc^2 = \hbar\omega_o$ gives

$$p^2/2m + V(x) = \hbar\omega$$

since the charged photon's longitudinal momentum is $p = h/\lambda_{deBroglie} = \hbar k$, where k is the longitudinal component of the circulating charged photon's wave vector k_{total} .

We can substitute $p = \hbar k$ into the previous equation to give

$$\frac{(\hbar k)^2}{2m} + V(x) = \hbar\omega$$

for the charged photon model of the electron associated with a one-dimensional potential $V(x)$.

Now consider the function $\Psi(x,t) = Ae^{i(kx-\omega t)}$ where k and ω are defined as above from the charged photon model of the electron. In a region where $V(x)$ is constant, k is also constant and

$$\frac{\partial\Psi(x,t)}{\partial x} = ikAe^{i(kx-\omega t)} = ik\Psi(x,t)$$

When $\Psi(x,t)$ is substituted into the one-dimensional time-dependent Schrödinger equation

$$\frac{-\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2}+V(x)\Psi(x,t)=i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

it gives

$$\frac{-\hbar^2}{2m}(-k^2)\Psi(x,t)+V(x)\Psi(x,t)=i\hbar(-i\omega)\Psi(x,t)$$

$$\frac{(\hbar k)^2}{2m}\Psi(x,t)+V(x)\Psi(x,t)=\hbar\omega\ \Psi(x,t)$$

$$\frac{p^2}{2m}\Psi(x,t)+V(x)\Psi(x,t)=\hbar\omega\ \Psi(x,t)$$

In the case of the time-independent Schrödinger equation, the n discrete wave function solutions $\Psi_n(x)$ and discrete angular frequency eigenvalue solutions ω_n for a charged photon in a potential well $V(x)$ correspond to the equation

$$\frac{-\hbar^2}{2m}\frac{\delta^2\Psi_n(x)}{\delta x^2}+V(x)\Psi_n(x)=\hbar\omega_n\Psi_n(x)=E_n\Psi_n(x)$$

So $\Psi_n(x)$ and $\Psi(x,t)$, corresponding to the charged photon model of the electron, satisfiy the Schrödinger equation and its requirement for conservation of total energy in the presence of an electric potential $V(x)$.

In the case where the charged photon (the electron) is in a three-dimensional potential energy well $V(\vec{r})$ the value $\omega=0$ corresponds to a charged photon that is free but with total energy $E_{total}=\hbar(\omega_o+0)=\hbar\omega_o=mc^2$ while a bound charged photon corresponds to a negative value of ω_n and a negative bound state energy $E_n=\hbar\omega_n$ for the nth bound state of the charged photon. For example, if the potential well is the Coulomb potential $V(\vec{r})=-e^2/4\pi\varepsilon_o r$ of a hydrogen atom, the $n=1$ bound state energy is $E_1=-13.6eV=\hbar\omega_1$, the ground state energy of the electron in a hydrogen atom. The total energy of the ground state charged photon (the electron) in a hydrogen atom would then be $E_{total}=mc^2-13.6eV$

Consider how an atom emits an uncharged photon quantum mechanically. In a hydrogen atom, a charged photon (an electron) having for example a higher energy configuration with $n=2$ and bound state energy $E_2=-13.6eV/n^2=-13.6eV/2^2=-13.6eV/4=-3.4eV$ reconfigures itself into a lower energy state with $n=1$ and $E_1=-13.6eV$, emitting an uncharged photon of

energy $E = h\nu = E_2 - E_1 = -3.4eV - (-13.6eV) = 9.2eV$. When a ground state $n = 1$ hydrogen atom's charged photon absorbs an uncharged photon of energy $9.2eV$, this process happens in reverse and the ground state charged photon is raised to the $n = 2$ energy level. This process is described by quantum mechanics for the atom, but now the atom is conceptualized as containing charged photons rather than electrons. All of the photon emission and absorption interactions are between two varieties of photon -- charged and uncharged.

Conclusion

The charged photon model of the electron is apparently compatible with the time dependent and time-independent Schrödinger equation that ensures conservation of energy of the charged photon (the electron) in the presence of a spatial electric potential. Furthermore the angular frequencies ω associated with the Schrödinger equation's wave function solutions are clearly related to the total angular frequency and total energy of the charged photon, while the wave number k solutions are the longitudinal component of the wave vector of the charged photon. The Schrödinger equation may be interpreted as a non-relativistic equation that predicts the allowable quantum wave functions and energies for a bound or unbound non-relativistic charged photon or charged photons in a spatial electric potential.

References

Gauthier, R., "The electron is a charged photon", https://www.academia.edu/9171692/The_electron_is_a_charged_photon , 2014a.

Gauthier, R. , The charged-photon model of the electron, the de Broglie wavelength, and a new interpretation of quantum mechanics", http://www.academia.edu/9973842/The_Charged-Photon_Model_of_the_Electron_the_de_Broglie_Wavelength_and_a_New_Interpretation_of_Quantum_Mechanics , 2014b.

Santa Rosa Junior College
www.superluminalquantum.org
richgauthier@gmail.com
Last updated January 19, 2015