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GRAVITATING LEPTON BAG MODEL

A. Burinskii*

Nuclear Safety Institute, Russian Academy of Sciences
115191, Moscow, Russia

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The Kerr–Newman (KN) black hole (BH) solution exhibits the external gravitational and electromagnetic field corresponding to that of the Dirac electron. For the large spin/mass ratio, $a \gg m$, the BH loses horizons and obtains a naked singular ring creating two-sheeted topology. This space is regularized by the Higgs mechanism of symmetry breaking, leading to an extended particle that has a regular spinning core compatible with the external KN solution. We show that this core has much in common with the known MIT and SLAC bag models, but has the important advantage of being in accordance with the external gravitational and electromagnetic fields of the KN solution. A peculiar two-sheeted structure of Kerr's gravity provides a framework for the implementation of the Higgs mechanism of symmetry breaking in configuration space in accordance with the concept of the electroweak sector of the Standard Model. Similar to other bag models, the KN bag is flexible and pliant to deformations. For parameters of a spinning electron, the bag takes the shape of a thin rotating disk of the Compton radius, with a ring-string structure and a quark-like singular pole formed at the sharp edge of this disk, indicating that the considered lepton bag forms a single bag–string–quark system.

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1. INTRODUCTION AND OVERVIEW

It has been discussed for a long time that black holes (BH) are to be related to elementary particles [1]. The Kerr–Newman (KN) rotating BH solution was of especial interest in this respect because, as was shown by Carter [2], its gyromagnetic ratio $g = 2$ corresponds to the Dirac electron, and therefore the four measurable parameters of the electron (spin J, mass m, charge e, and magnetic moment μ) indicate that gravitational and electromagnetic fields of the electron should be described by the KN solution. In recent paper [3], Dokuchaev and Eroshenko considered a solution of the Dirac equation under BH horizon, and suggested that this model may represent a “ ... particle-like charged solutions in general relativity ... ”. On the other hand, we note that the model of a Dirac particle confined under a BH horizon can also be considered a type of gravitating bag model, and it acquires special interest because this bag is to be gravitating, leading to a progress beyond the known MIT and SLAC bag models [4, 5]. However, the spin and charge of elementary particles are very high with respect to their masses, which prevents formation of the BH horizons. In particular, the KN solution with parameters of the electron (charge e, mass m, and spin parameter $a = J/m$) exceeds the threshold value $e^2 + a^2 \le m^2$ for the existence of the horizons by about 21 orders. Similar ratios for other elementary particles show that besides the Higgs boson, which has neither spin nor charge, none of the elementary particles may be associated with a true black hole, and they should rather be associated with the over-rotating Kerr geometry, with $|a| \gg m$.

The corresponding over-rotating KN space has a topological defect, the naked Kerr singular ring, which forms a branch line of space into two sheets described by different metrics: the sheet of advanced and sheet of retarded fields. The Kerr singular and related two-sheeted structure created the problem of a mysterious source of the Kerr and KN solutions, which has received considerable attention during more than four decades [6–14]. For the story of this investigation, we refer the reader, e. g., to [15]. Long-term attempts to resolve the puzzle of the source of Kerr geometry led first to the model of the vacuum bubble — a rotating disk-like shell [8, 9]. The vacuum state inside the bubble turned later into a superconducting bulk formed of a false-vacuum condensate of the Higgs field [13, 14]. The structure of the source acquired typical features of

*E-mail: burinskii@mail.ru

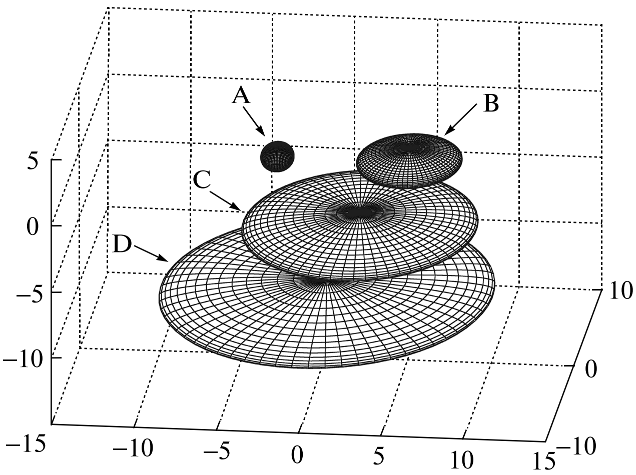

Fig. 1. Spherical bag with zero rotation, $a/R = 0$ (A), and the rotating disk-like bags for different ratios $a/R = 3$ (B), 7 (C), 10 (D)

the soliton and Q-ball models, becoming similar to the known bag models [4, 5].

Recent analysis of the Dirac equation inside the KN soliton source [16] confirmed that the regularized KN solution shares much in common with the known MIT and SLAC bag models. However, the gravitating bag formed by the KN bubble source should have specific features associated with the need to preserve the external KN field.

On the other hand, the semiclassical theory of the bag models [5] includes elements of quantum theory that are based on a flat space–time without gravity, and we are faced with the known conflict between gravity and quantum theory. Our solution to this problem in [13, 14] is based on two requirements.

I. The space–time should be flat inside the bag.

II. The space–time outside the bag should be the exact KN solution.

Thus, the quantum–gravity conflict is resolved by separation of their regions of influence. Remarkably, these requirements determine features of the KN bag unambiguously. First of all, they uniquely determine the border of the KN bag, showing explicitly that, in accordance with the general concept of bag models [5, 17], the KN bag has to be flexible and its shape depends on the rotation parameter $a = J/m$ and on the local intensity of the electromagnetic (EM) field.

As a result, for parameters of an electron, the rotating bag takes the shape of a thin disk of ellipsoidal form (see Fig. 1). Its thickness R turns out to be equal to the classical radius of the electron $r_e = e^2/2m$, while the radius of the disk corresponds to the Compton wavelength of the disk[1)], which allows identifying it with a dressed electron.

The degree of oblateness of this disk is $a/R = \alpha^{-1} = 137$, and the fine structure constant α thus acquires a geometrical interpretation.

The next very important consequence of these requirements is the emergence of a ring-string structure on the bag border, and further the emergence of a singular pole associated with traveling-wave excitations of the string [18, 19]. This pole can be associated with a single quark, and the KN bag finally takes the form of a coherent "bag–string–quark" system.

Finally, these requirements determine that the Higgs condensate should be enclosed inside the bag, contrary to the standard treatments of the bag as a cavity in the Higgs condensate, [4]. This requirement cannot be realized with the usual quartic self-interaction potential of the Higgs fiield [4, 5], and requires a more complicated field model, based on a few chiral fields and a supersymmetric scheme of the phase transition [20].

At this point, we have to mention the important role of the Kerr theorem, which determines the null vector field $k_\mu(x)$, the Kerr principal congruence that forms a vortex polarization of Kerr–Schild (KS) metric

$$g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu. \qquad (1)$$

The Kerr theorem gives two solutions for this congruence $k_\mu^\pm$, which determine two sheets of the KN solution corresponding to two different metrics $g_{\mu\nu}^\pm$. Solutions of the Dirac equation on the KN background should be consistent with the metric corresponding to one of these congruences.

We show that two solutions of the Kerr theorem generate two massless Weyl spinor fields that are coupled into a Dirac field consistent with the Kerr geometry. However, the null spinor fields of the Kerr congruences are massless, and there appears the question of the origin of the mass term. The answer comes from the theory of bag models [5], where the Dirac mass is a variable depending on the local vacuum expectation value (vev) of the Higgs condensate.

This gives a direct hint to a consistent embedding of the Dirac equation into the regularized KN background, indicating that both sheets of the KN solution are necessary as carriers of the initially massless leptons. This is in agreement with the basic concepts of the Glashow–Salam–Weinberg model [21], in which the lepton masses are generated by the Higgs mechanism of symmetry breaking.

[1)] This was determined by López [9].

As a result, we conclude that two-sheeted Kerr's structure is an essential element for the space–time realization of the electroweak sector of the Standard Model consistent with gravity.

2. OVER-ROTATING KERR GEOMETRY: TWO-SHEETED STRUCTURE AND REGULAR SOURCE

The KN solution in the KS form [22] has the metric

$$g_{\mu\nu} = \eta_{\mu\nu} + 2H k_\mu k_\nu, \tag{2}$$

where $\eta_{\mu\nu}$ is metric of auxiliary Minkowski space, $x^\mu = (t, x, y, z) \in M^4$ [2], and

$$H = \frac{mr - e^2/2}{r^2 + a^2 \cos^2\theta}. \tag{3}$$

The vector field k_μ is null, $k_\mu k^\mu = 0$, and determined by the differential form

$$k = k_\mu dx^\mu = dr - dt - a\sin^2\theta\, d\phi, \tag{4}$$

where t, r, θ, ϕ, are the Kerr oblate spheroidal coordinates:

$$x + iy = (r + ia)e^{i\phi}\sin\theta, \quad z = r\cos\theta, \quad t = \rho - r. \tag{5}$$

The field $k^\mu(x)$ forms a principal null congruence (PNC) $\mathcal{K}$ [23], which determines polarization of the Kerr space–time. The PNC is focussed at the Kerr singular ring, $r = 0$, $\cos\theta = 0$, which is the branch line of the Kerr space into two sheets $r > 0$ and $r < 0$ [3].

Extending the Kerr congruence to the negative sheet of the KS space ($r < 0$) along the lines $\phi = \mathrm{const}$, $\theta = \mathrm{const}$ creates another congruence with a different radial direction, and the congruence which is outgoing by $r > 0$ turns into the ingoing one on the negative sheet [4]. Thus, the Kerr solution in the KS form describes two different sheets of space–time, determined by two different congruences

$$k^\pm_\mu(x) dx^\mu = \pm dr - dt - a\sin^2\theta\, d\phi \tag{6}$$

and two different metrics

$$g^\pm_{\mu\nu} = \eta_{\mu\nu} + 2H k^\pm_\mu k^\pm_\nu \tag{7}$$

on the same Minkowski background $x^\mu \in M^4$.

[2] We use the signature $(-+++)$.

[3] These are Riemannian sheets of the Kerr complex radial distance $\tilde{r} = r + ia\cos\theta$.

[4] Relations (5) also change [23].

This two-sheetedness created the problem of the source of Kerr geometry, and there appeared two lines of investigation. One of them [10, 11, 24], accepted the two-sheetedness as an indication of its plausible connection with a spinor structure of the Kerr space–time and with the two-sheeted structure of the topologically nontrivial "Alice" strings introduced by Schwarz and Witten [25].

An alternative line of investigation was related to truncation of the KN negative sheet, and to a consistent replacement of the excised region by a source in agreement with the Einstein–Maxwell field equations [6–9, 12–14].

There is a freedom in choosing the truncating surface, and in the most successful version of the model suggested by López [9], the KN source formed a bubble, whose boundary was determined by matching the external KN metric (2) with a flat metric inside the bubble. According to (2) and (3), this boundary has to be placed at the radius $r = R = e^2/2m$.

We see from (5) that r is indeed the oblate spheroidal coordinate,

$$\frac{x^2 + y^2}{a^2\sin^2\theta} - \frac{z^2}{a^2\cos^2\theta} = 1, \tag{8}$$

and the source of the KN solution takes the form of a very oblate disk of the radius $r_c \approx a = 1/2m$ with the thickness

$$r_e = e^2/2m, \tag{9}$$

which is the classical radius of the electron. Thus, the fine structure constant acquires a geometrical meaning as the degree of oblateness of the disk-like source, $r_e/r_c = e^2 \approx 137^{-1}$.

As a result of the regularization, the disk-like region surrounding the Kerr singular ring is excised and replaced by flat space, which acts as a cut-off parameter — an effective minimal distance $R = r_e$ to the former Kerr singular ring. We note that in the case without rotation, $a = 0$, the disk-like bubble takes the spherical form and the size of the classical electron, Eq. (9).

The López model was later transformed into a soliton–bubble model [13, 14], in which the thin shell of the bubble was replaced by a field model of a domain wall providing a smooth phase transition between the external KN solution and the flat internal space. This phase transition was modelled by the Higgs mechanism of symmetry breaking, and the flat interior of the KN bubble was formed by a false-vacuum state of the Higgs condensate.

The field model of broken symmetry is similar to the Landau–Ginzburg model of superconductivity [26], and regularization of the singular KN solution can be viewed as an analogue to the Meissner effect, expulsion of the gravitational and EM fields from the interior of the superconducting source.

3. HIGGS CONDENSATE AND THE MASS OF THE DIRAC FIELD

The Higgs symmetry breaking mechanism used for regularization of the KN solution relates the source of the KN solution to many other extended particle-like models of the electroweak sector of the Standard Model. In particular, we note the superconducting string model of Nielsen and Olesen [26, 27], Coleman's Q-ball models [28–32], and the famous MIT and SLAC bag models. In this paper, we pay especial attention to the fermionic sector of the KN source and obtain a close similarity between the Higgs mechanism of mass generation in the KN soliton model and that in the SLAC bag model [5].

The Hamiltonian of the SLAC model for coupling the Higgs field to the Dirac field ψ has the form

$$\mathcal{H} = \int d^3x \left\{ \psi^\dagger(-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + g\beta\sigma)\psi + \right. \\ \left. + \frac{1}{2}(\dot{\sigma}^2 + |\boldsymbol{\nabla}\sigma|^2) + V(\sigma) \right\}, \quad (10)$$

where g is a dimensionless coupling parameter, and self-interaction of the nonlinear Higgs field Φ is described by the quartic potential

$$V(|\Phi|) = g(\bar{\sigma}\sigma - f^2)^2, \quad (11)$$

where $\sigma = \langle|\Phi|\rangle$ is the vev of the Higgs field. The true vacuum of the Higgs field $\sigma = 0$ is not the lowest-energy state, and the Higgs field is triggered in the false-vacuum state $\sigma = f$, which breaks the gauge symmetry of the spinor field ψ. As a result, the fermion acquires the mass $m = g\eta$, which is used in the confinement mechanism of bag models. However, the false-vacuum state of the Higgs field $\sigma = f$ also breaks the gauge symmetry of the EM fields. In the known bag models, it turns the external EM fields into short-range one, which distorts the external KN solution.

For example, in the MIT bag model, the Higgs vev vanishes inside the bag, $r < R$, and takes a nonvanishing value $\sigma = f$ in outer region $r > R$ (see Fig. 2).

The Dirac equation in the presence of the σ field takes the form

$$(i\gamma^\mu\partial_\mu - g\sigma)\psi = 0, \quad (12)$$

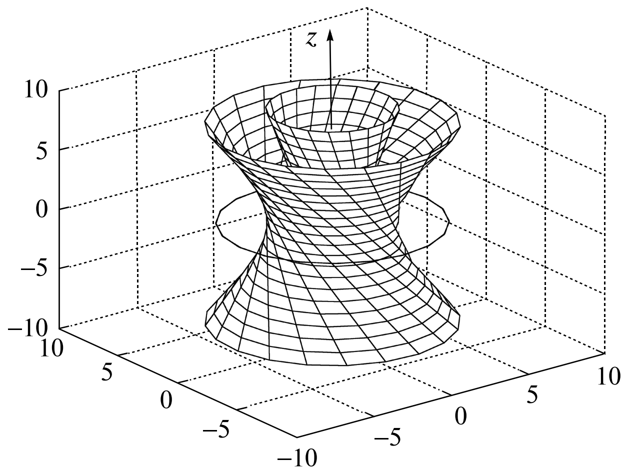

Fig. 2. Kerr's principal congruence of null lines (twistors) is focused on the Kerr singular ring, forming a branch line of the Kerr space into two sheets

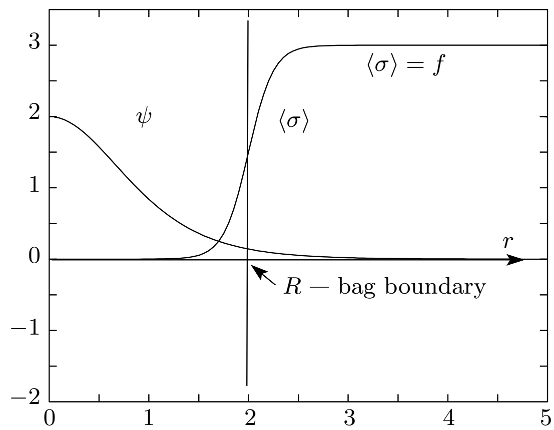

Fig. 3. Positions of the vev of the Higgs field σ and the confined spinor wave function Ψ (quark) in the MIT bag model

and the Dirac wave function ψ turns out to be massless inside the bag and acquires a large effective mass $m = gf$ outside. The quarks are confined inside the bag, where they occupy the most energetically favorable position.

Geometry of the Higgs vacuum state is different in the SLAC bag models (see Fig. 3). The vev σ gives the mass to the Dirac field outside the bag as well as inside. The mass vanishes only in a very narrow region near the surface of the bag, $r \approx R$. Such geometry of the broken vacuum state creates a sharp localization of the Dirac wave function at the border of the bag.

In the bag models, we are faced with several very important novelties.

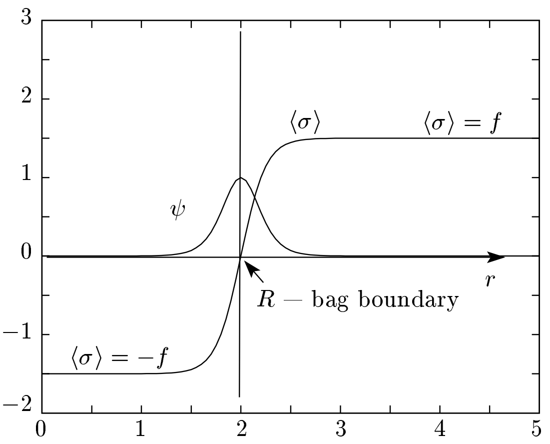

Fig. 4. Classical solutions of the SLAC bag model. The vacuum field σ and the localized spinor (quark) wave function confined to the thin shell, the boundary of the bag

(A) The statement on the impossibility of localization of the Dirac wave function beyond the distances comparable with the Compton wave length $\hbar/mc$ is violated, and quarks can localize within a very thin region at the bag shell. The reason of that is the scalar nature of the confinement potential, for which " ... there is no Klein paradox of the familiar type encountered in the presence of strong, sharp vector potential" [5].

(B) A semiclassical approach to the one-particle Dirac theory is effectively used. Solving the Dirac equation for a quark in a scalar potential assumes that all the negative-energy states are filled, and the treatment is focused on the lowest positive-energy eigenvalues. Therefore, " ... there is no ambiguity in identifying and interpreting the desired positive energy "one-particle" solutions" [5, 33].

(C) The mass term of Dirac equation (12) is determined by the vev of the Higgs field $\sigma(x) = \langle|\Phi(x)|\rangle$, and therefore turns out to be a function in the configuration space.

(D) Bag models are presumed to be very soft, compliable, and extensible. They are easily deformed, and under rotations and deformations they may acquire extended stringy structures accompanied by vibrations.

All these peculiarities of the bag models are compatible with the soliton-bubble source of the KN solution. However, there is one important difference: the typical bag model represents a bubble or cavity in a superconducting media, the Higgs condensate, while in the gravitating bubble-source of the KN solution, the Higgs condensate is enclosed within the bubble, leaving the true vacuum outside the bag unbroken.

In the MIT and SLAC bag models, the Higgs condensate is placed outside the source, and the external vacuum represents a superconducting false-vacuum state (see Fig. 4), leading to the short-range external EM field.

A dual geometry (turned inside out) was suggested in the Coleman Q-ball model [28]. The self-interacting Higgs field of a Q-ball is confined inside a ball-like source, $r < R$, leaving the external vacuum unbroken. Most of the Q-ball models led to a coherent oscillating state of the Higgs vacuum inside the bag (oscillons [30–32])[5]. The KN soliton source [13, 14] also exhibits this peculiarity. We can summarize that confinement of the Higgs condensate inside the bag is a necessary requirement for the correct gravitating properties of the bag models. However, formation of the corresponding potential turns out to be a very nontrivial problem, which cannot be solved by the usual quartic potential (11).

4. FIELD MODEL OF BROKEN SYMMETRY AND PHASE TRANSITION FOR THE GRAVITATING BAG MODEL

Among theories with spontaneous symmetry breaking, an important place is taken by the field model of a vortex in condensed matter, which was considered by Abrikosov in connection with the theory of type-II superconductors. Nielsen and Olesen (NO) used this solution for a semiclassical relativistic string model [26]. The NO string model, representing a magnetic flux tube in a superconductor, was generalized to many other semiclassical field models of the solitonic strings and has found wide application in the electroweak sector of the standard Glashow–Salam–Weinberg model [27, 34].

The NO model [26] contains a complex scalar field Φ and the gauge EM field A^μ, which becomes massive via the Higgs mechanism. The Lagrangian has the form

$$\mathcal{L}_{NO} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(\mathcal{D}_\mu\Phi)(\mathcal{D}^\mu\Phi)^* - V(|\Phi|), \quad (13)$$

where $\mathcal{D}_\mu = \nabla_\mu + ieA_\mu$ are the $U(1)$ covariant derivatives and $F_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu}$ is the field strength. The potential V has the same quartic form as in (11),

$$V = \lambda(\Phi^\dagger\Phi - f^2)^2, \quad (14)$$

where σ is replaced by the complex field $\Phi = |\Phi|e^{i\chi}$.

The Lagrangian $\mathcal{L}_{NO} \equiv \mathcal{L}^{mat}$ describes a vortex string embedded in the superconducting Higgs condensate in flat space–time. Similarly to the bag models, this model cannot be generalized to gravity because

[5] Such a model was first considered by Rosen [29].

the Higgs condensate gives mass to the external EM and gravitational fields, turning them into nonphysical short-range fields conflicting with the real gravitational and EM properties of strings and particles.

An improvement of this flaw was suggested by Witten in his $U(1)\times\tilde{U}(1)$ field model of a cosmic superconducting string [25], in which he used two Higgs-like fields, Φ^1 and Φ^2. One of them, say Φ^1, had the required behavior, being concentrated inside the source, while the other, Φ^2, played an auxiliary role and took the external complementary domain extending up to infinity. These two Higgs field are charged and adjoined to two different gauge fields A^1 and A^2, such that when one of them is long-distant in some region Ω, the other is long-distant in the complementary region $\overline{\Omega}=U_\infty/\Omega$. This model is suitable for any localized gravitating source, but for the superconducting source of the KN solution we used in [13], a supersymmetric generalization of the Witten model was suggested by Morris [35].

4.1. Supersymmetric phase transition

The supersymmetric scheme of a phase transition is based on three chiral fields $\Phi^{(i)}$, $i=1,2,3$ [20]. One of this fields, say $\Phi^{(1)}$, has the required radial dependence, and we chose it as the Higgs field $\mathcal{H}$, setting the additional notation as $(\mathcal{H},Z,\Sigma)\equiv(\Phi^0,\Phi^1,\Phi^2)$.

The action coupled to gravity is given by

$$S=\int\sqrt{-g}\,d^4x\left(\frac{R}{16\pi G}+\mathcal{L}^{mat}\right), \qquad (15)$$

where the full matter Lagrangian takes the form

$$\mathcal{L}^{mat}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\\ -\frac{1}{2}\sum_i(\mathcal{D}^{(i)}_\mu\Phi^{(i)})(\mathcal{D}^{(i)\mu}\Phi^{(i)})^*-V, \qquad (16)$$

which contains a contribution from the triplet of the chiral field $\Phi^{(i)}$.

The potential V required for our model is obtained by a standard supersymmetric scheme of broken symmetry [20], which determines it via a superpotential $W(\Phi^{(i)},\,\Phi^{(i)*})$,

$$V(r)=\sum_i|\partial_i W|^2. \qquad (17)$$

The superpotential leading to the required geometry of broken symmetry was suggested by Morris [35]:

$$W(\Phi^i,\bar{\Phi}^i)=Z(\Sigma\bar{\Sigma}-\eta^2)+(Z+\mu)\mathcal{H}\bar{\mathcal{H}}, \qquad (18)$$

where μ and η are real constants. This yields

$$V=(Z+\mu)^2|\mathcal{H}|^2+(Z)^2|\Sigma|^2+(\Sigma\bar{\Sigma}+\mathcal{H}\bar{\mathcal{H}}-\eta^2)^2, \qquad (19)$$

and the equation

$$\partial_i W=0 \qquad (20)$$

determines two vacuum states separated by a spike of the potential V at $r\approx R$:

EXT: the external vacuum, $r>R+\delta$, $V(r)=0$, with the vanishing Higgs field $\mathcal{H}=0$, and $Z=0,\Sigma=\eta$, and

INT: an internal state of the false vacuum, $r< R-\delta$, $V(r)=0$, with broken symmetry, $|\mathcal{H}|=\eta$, and $Z=-\mu$, $\Sigma=0$.

4.2. Application to the KN source

Choosing López's boundary for regularization of the KN source allows us to neglect gravity inside the source and at the boundary, and we can hence neglect the gravitational field in the zone of the phase transition and consider the space–time as flat. At the same time, outside the source, we have the exact Einstein–Maxwell gravity, because the gauge symmetry is unbroken and all the terms

$$\frac{1}{2}(\mathcal{D}_\mu\Phi^{(i)})(\mathcal{D}^\mu\Phi^{(i)})^*$$

vanish together with the potential $V(|\Phi|)$. Therefore, outside the source, we have only the matter term

$$\mathcal{L}^{mat}\equiv-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

leading to the external KN solution.

Hence, inside the source (zone **INT**) and on the boundary, we have only the part of Lagrangian that corresponds to self-interaction of the complex Higgs field and its interaction with the vector potential of the KN electromagnetic field A^μ in flat space–time.

The field model is reduced to the model considered by Nielsen and Olesen for a vortex string in superconducting media [26],

$$\mathcal{L}_{NO}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{2}(\mathcal{D}_\mu\mathcal{H})(\mathcal{D}^\mu\mathcal{H})^*+V(|\mathcal{H}|), \qquad (21)$$

where $\mathcal{D}_\mu=\nabla_\mu+ieA_\mu$ is the covariant derivative, $F_{\mu\nu}=A_{\mu,\nu}-A_{\nu,\mu}$, and $\nabla_\mu\equiv\partial_\mu$ reduces to a derivative in flat space with the flat D'Alembertian $\partial_\nu\partial^\nu=\Box$. For the interaction of the complex Higgs field

$$\mathcal{H}(x)=|\mathcal{H}(x)|e^{i\chi(x)} \qquad (22)$$

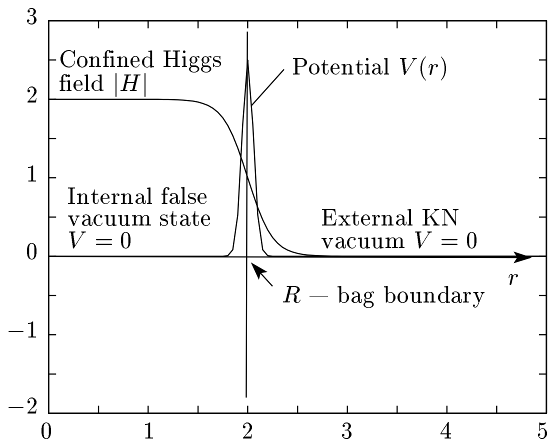

Fig. 5. Region of broken symmetry in the KN soliton bag model. The potential $V(R)$ forms the inner and outer vacuum states $V=0$ with a narrow spike at the boundary of the bag. The Higgs field H is confined inside the bag, $r<R$, forming a false vacuum state, which gives mass to the Dirac equation

with the Maxwell field, we obtain the following complicated systems of nonlinear differential equations:

$$D_\nu D^\nu \mathcal{H} = \partial_{\bar{\mathcal{H}}} V, \tag{23}$$

$$\Box A_\mu = I_\mu = e|\mathcal{H}|^2(\chi,_\mu + eA_\mu). \tag{24}$$

The obtained vacuum states **EXT** and **INT** show that $|\mathcal{H}(r)|$ should be a step-like function

$$|\mathcal{H}(r)| = \begin{cases} \eta, & r \le R-\delta, \\ 0, & r \ge R+\delta, \end{cases} \tag{25}$$

with a transition region $R-\delta<r<R+\delta$, where its behavior is determined by the impact of the electromagnetic field.

Outside the source, $r>R+\delta$, we have $\mathcal{H}=0$ and obtain $I_\mu=0$. Inside the source, with $r\le R-\delta$, we have also $I_\mu=0$, which is provided there by the compensation of the vector potential by the gradient of the phase χ of the Higgs field, $\chi,_\mu + eA_\mu = 0$. Hence, a nonzero current exists only in the narrow transitional region $R-\delta<r<R$, where this compensation is only partial, and (24) describes the "region of penetration" of the EM field inside the Higgs condensate (see Fig. 5).

4.3. Important consequences

The analysis of Eq. (24) in [13, 14] showed two remarkable properties of the KN rotating soliton:

(I) the vortex of the KN vector potential A_μ forms a quantum Wilson loop placed along the border of the disk-like source, which leads to quantization of the angular momentum of the soliton,

(II) the Higgs condensate should oscillate inside the source with the frequency $\omega = 2m$.

The KN vector potential has the form [22]

$$A_\mu dx^\mu = -\operatorname{Re}\left[\frac{e}{r+ia\cos\theta}(dr-dt-a\sin^2\theta\, d\phi)\right]. \tag{26}$$

The maximum of the potential is reached in the equatorial plane, $\cos\theta=0$, at the López's boundary of the disk-like source (9), $r_e = e^2/2m$, which plays the role of a cut-off parameter,

$$A_\mu^{max} dx^\mu = -\frac{e}{r_e}(dr-dt-a\,d\phi). \tag{27}$$

The ϕ component of the vector potential, $A_\phi^{max} = ea/r_e$, shows that the potential forms a circular flow (Wilson loop) near the source boundary. According to (24), this flow is compensated inside the soliton by the gradient of the Higgs phase $\chi,_\phi$, and does not penetrate inside the source beyond a transition region $r<r_e-\delta$. Integrating this relation along the closed loop $\phi=[0,2\pi]$ under the condition $I_\phi = 0$ yields the result (I).

Similarly, using (24) and the condition $I_\phi=0$ for the time component of the vector potential

$$A_0^{max} = \frac{e}{2r_e} = \frac{m}{e},$$

we obtain the result (II).

5. FERMIONIC SECTOR OF THE KN BAG MODEL

Now we have to consider matching the solutions of the Dirac equation with the interior of the regular solitonic source and with the external KN solution. We start from the region inside the KN source and the adjacent δ-narrow layer of phase transition, $r<R+\delta$. In accordance with the used scheme of regularization, these regions are to be flat, and we can use the usual Dirac equation $\gamma^\mu\partial_\mu\Psi = m\Psi$, which in the Weyl representation splits into two equations

$$\sigma^\mu_{\alpha\dot\alpha} i\partial_\mu \bar\chi^{\dot\alpha} = m\phi_\alpha, \quad \bar\sigma^{\mu\dot\alpha\alpha} i\partial_\mu\phi_\alpha = m\bar\chi^{\dot\alpha}, \tag{28}$$

where the Dirac bispinor

$$\Psi = \begin{pmatrix} \phi_\alpha \\ \bar\chi^{\dot\alpha} \end{pmatrix}$$

is presented by two Weyl spinors ϕ_α and $\bar\chi^{\dot\alpha}$.

In the concept of bag models, fermions acquire mass via a Yukawa coupling to the Higgs field, Eq. (12), and because the Higgs condensate in the KN source is concentrated inside the bag, Eq. (25), the mass term of the Dirac equation takes the maximal value

$$m = g\eta \tag{29}$$

in the internal region while the Dirac equation outside the bag turns out to be massless and splits into two independent massless equations

$$\sigma^{\mu}_{\alpha\dot{\alpha}} i\partial_\mu \bar{\chi}^{\dot{\alpha}} = 0, \quad \bar{\sigma}^{\mu\dot{\alpha}\alpha} i\partial_\mu \phi_\alpha = 0, \tag{30}$$

corresponding to the left-handed and right-handed "electron-type leptons" of the Glashow–Salam–Weinberg model [21].

Outside the bag, we have external gravitational and EM fields of the KN solution, and we should use the Dirac equation in the covariant form

$$\gamma^\mu_{KS} \mathcal{D}_\mu \Psi = 0, \tag{31}$$

where γ^μ_{KS} are γ-matrixes adapted to the KS form of metric (2), and

$$\mathcal{D}_\mu = \partial_\mu - \frac{1}{2}\Gamma_{\nu\lambda\mu}\Sigma^{\nu\lambda} - i\frac{k}{2\sqrt{2}}\gamma_\mu F_{\nu\lambda}\Sigma^{\nu\lambda} \tag{32}$$

are covariant derivatives.

The exact solutions on the KS background were previously considered by Einstein and Finkelstein in [36], and following them we can choose the γ^μ_{KS} matrixes in the form

$$\gamma^\mu_{KS} = \gamma^\mu_W + \sqrt{2H} k^\mu \gamma^5_W, \tag{33}$$

where γ^μ_W are matrices of the Weyl representation for the Minkowski space $\eta^{\mu\nu}$. They satisfy the usual anticommuting relations

$$\begin{gathered} \{\gamma^\mu_W, \gamma^\nu_W\} = 2\eta^{\mu\nu}, \quad \{\gamma^\mu_W, \gamma^5_W\} = 0, \\ (\gamma^5_W)^2 = -1, \end{gathered} \tag{34}$$

while γ^μ_{KS} satisfy the anticommuting relations

$$\frac{1}{2}\{\gamma^\mu_{KS}, \gamma^\nu_{KS}\} = \eta^{\mu\nu} - 2Hk^\mu k^\nu = g^{\mu\nu}_{KS}, \tag{35}$$

adapted to the KS metric. It is known that the exact KS solutions belong to the class of algebraically special solutions, for which all the tensor quantities are to be aligned with the Kerr null congruence [22], and the general relations (31), (33), (32) become much simpler when the Dirac field $\Psi(x)$ is "aligned" with the Kerr congruence $k^\mu(x)$,

$$k_\mu \gamma^\mu \Psi = 0. \tag{36}$$

For the aligned Dirac field, the nonlinear terms of the electromagnetic and gravitational interactions cancel, and the Dirac equation linearizes [36], taking the form of a free Dirac equation in flat space–time (30).

The alignment condition (36) can be rewritten in the form

$$(\mathbf{k}\cdot\boldsymbol{\sigma})\phi = \phi, \quad (\mathbf{k}\cdot\boldsymbol{\sigma})\bar{\chi} = -\bar{\chi}, \tag{37}$$

which shows that the left-handed and the right-handed fields $\bar{\chi}$ and ϕ are to be oppositely polarized with respect to the spatial direction of the Kerr congruence $\mathbf{k}$. We obtain that only one of these two "half-leptons", the left-handed ϕ, is indeed consistent with the Kerr congruence $k^+ = (1, \mathbf{k})$, selected for the physical sheet of the KN solution. The consistent solution takes the form $\Psi^T_L = (\phi, 0)$, which shows explicitly that only the left-handed field ϕ is aligned with k^+ and survives on the physical sheet of the KN geometry. This solution is exact, because the left- and right-handed spinors are independent for the massless Dirac equation. Similarly, we obtain the solution $\Psi^T_R = (0, \bar{\chi})$, which is not aligned with k^+ and with the selected physical sheet of the KN solution. However, it is aligned with the congruence k^- and "lives" on the negative sheet of advanced fields. Thus, the massive Dirac solution

$$\Psi = \begin{pmatrix} \phi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}$$

splits into the left and right massless parts Ψ_L and Ψ_R, which outside the bag can live only on the different sheets of the two-sheeted Kerr geometry.

This important peculiarity of the Dirac solutions on the Kerr background was also mentioned in [36], where authors noted that the Dirac equations on the KS background "... are not consistent unless the mass vanishes ... ". Meanwhile, this obstacle disappears inside the bag-like source of the Kerr geometry, where the space is flat by construction of the solitonic source (see Sec. 2). When the massless Weyl spinors pass from two different external sheets on a common flat space inside the bag, they are combined into a Dirac bispinor, which acquires mass from the Higgs condensate via Yukawa coupling (see Fig. 6). Removing the two-sheeted structure that was associated with the problem the source of KN solution, we meet its appearance from another side, by analysis of the consistent solutions of the Dirac equation on the KS background. We obtain that the two-sheeted structure of KS geometry agrees with elementary constituents of the standard model, the massless "left-handed" and "right-handed" electron fields [21, 33], providing the consistency of the external Dirac field with KN gravity.

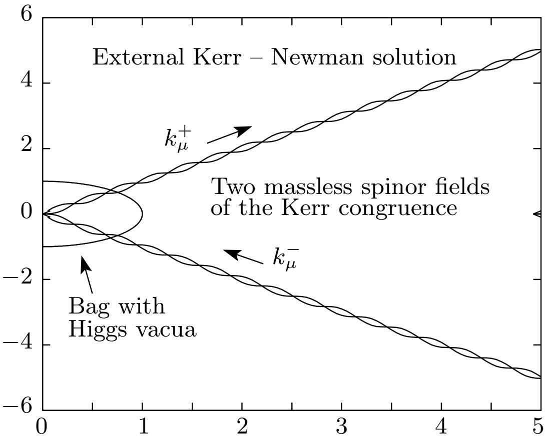

Fig. 6. Two sheets of the external KN solution are matched with flat space inside the bag. The massless spinor fields ϕ_α and $\bar\chi^{\dot\alpha}$ live on different KN sheets, aligned with k_μ^+ and k_μ^- null directions. Inside the bag, they join into a Dirac bispinor, which obtains mass from the Higgs condensate confined inside the bag

The Kerr congruences are determined by the Kerr theorem [22, 37], which is formulated in twistor terms on the Minkowski space $\eta_{\mu\nu}$ auxiliary to KS metric (2). The first twistor component Y also plays the role of a projective spinor coordinate (see details in the Appendix and [16, 37]). The Kerr theorem gives two solutions $Y^\pm(x)$ for the KN particle, which are connected by the antipodal relation $Y^+ = -1/\bar Y^-$ and determine two antipodal congruences $k^+_{\mu\nu}(x)$ and $k^-_{\mu\nu}(x)$. The Weyl spinors corresponding to solutions $Y^\pm(x)$ are exactly the Weyl spinor components ϕ and $\bar\psi$ of the aligned Dirac solutions considered above. Because the Kerr theorem is formulated in flat space–time, the solutions $Y^\pm(x)$ are extended unambiguously from the external KN space to the flat space inside the bag, which determines the Dirac bispinor

$$\tilde\Psi = \begin{pmatrix} f_1(x)\phi_\alpha \\ f_2(x)\bar\chi^{\dot\alpha} \end{pmatrix}, \tag{38}$$

which is aligned to both external congruences and represents a constraint, selecting the Dirac solution with the required polarization in the flat space inside the bag.

Another very specific peculiarity of the bag models is the emergence of the variable mass term in Dirac equation (12). The mass term is determined by the vev of the Higgs condensate σ, which depends on the regions of space–time, and in the region of the maximum of the Higgs condensate $\sigma = \eta$, is called the bare mass $m = g\eta$. The Dirac wave function, a solution of the Dirac equation with a variable mass term, avoids the region with a large bare mass, and tends to occupy a more energetically favorable position, which is the principal idea of quark confinement.

In the SLAC bag model [5], the resulting wave function is determined by the variational approach. The Hamiltonian is

$$H(x) = \Psi^\dagger\left(\frac{1}{i}\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + g\beta\sigma\right)\Psi, \tag{39}$$

and the energetically favorable wave function is determined by minimizing the averaged Hamiltonian $\mathcal{H} = \int d^3x H(x)$ under the normalization condition

$$\int d^3x\, \Psi^\dagger(x)\Psi(x) = 1.$$

This yields

$$\left(\frac{1}{i}\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + g\beta\sigma\right)\Psi = \mathcal{E}\Psi, \tag{40}$$

where $\mathcal{E}$ appears as the Lagrangian multiplier enforcing the normalization condition. Similarly to the results of the SLAC bag model, we expect that the Dirac wave function does not penetrate deep in the region of a large bare mass $m = g\eta$, and concentrates in a very narrow transition zone at the bag boundary $R-\delta < r < R+\delta$. As was argued in [5], the narrow concentration of the Dirac wave function is admissible in bag models because there is no Klein paradox for the scalar potential. The exact solutions of this kind are known in the two-dimensional case, and the corresponding variational problem should apparently be solved numerically by using ansatz (38), where $f_1(x)$ and $f_2(x)$ are variable factors.

The use of classical solutions of the Dirac equation in a given scalar potential leads also to the problem of negative-energy states. In the bag models, this problem is treated semiclassically by using the assumption [5] that " … all the negative-energy states in the presence of this potential are filled … ", and as a result, it is necessary to consider only the lowest positive-energy eigenvalues[6].

[6] This is an approximation to rigorous treatment based on the normal ordering. The negative-energy states correspond to charge-conjugate solutions $\Psi^c(x) = C\bar\Psi(x)$, $\bar\Psi^c(x) = C^{-1}\bar\Psi(x)$ of the charge-conjugate Dirac equations (with the replacement $e \to -e$). In particular, the current density is determined by the commutation relations

$$j_\mu(x) = \frac{ie}{2}[\bar\Psi(x), \gamma_\mu\Psi(x)] = \frac{ie}{2}(\bar\Psi(x)\gamma_\mu\Psi(x) - \frac{ie}{2}\bar\Psi^c(x)\gamma_\mu\Psi^c(x)),$$

and similarly for the expectation value of energy or any other operator bilinear in the fermion field.

The splitting of the KS space–time outside the source of the KN solution looks strange from the standpoint of standard gravitation, but it appears more natural by comparison with electromagnetism, which is sensitive to the difference between retarded and advanced fields.

It is known [38] that the Kerr solution can be represented in the KS form via both Kerr congruences k_μ^+ or k_μ^-, but not via the both simultaneously. For the KN solution with an EM field, the situation is more complicated. Although both representations are admissible, the representation via retarded fields is physically preferable because the asymptotic advanced EM field of the KN solution would contradict its experimental behavior in flat space. The vector potential A_μ of the KN solution must also be aligned with the Kerr congruence, and should be retarded (A_{ret}) on the physical sheet determined by the outgoing Kerr congruence k_μ^+. The appearance of advanced EM fields (A_{adv}) is important in nonstationary problems. In particular, in the Dirac theory of radiation reaction, the retarded potentials A_{ret} are split into a half-sum and half-difference with advanced ones

$$A_{ret} = \frac{1}{2}[A_{ret} + A_{adv}] + \frac{1}{2}[A_{ret} - A_{adv}],$$

where

$$A_{ret}^+ = \frac{1}{2}[A_{ret} + A_{adv}] \qquad (41)$$

is connected with radiation reaction, and

$$A_{ret}^- = \frac{1}{2}[A_{ret} - A_{adv}] \qquad (42)$$

forms a self-interaction of the source. A similar structure is also present in the Feynman propagator.

The fields A_{ret} and A_{adv} cannot reside on the same physical sheet of the Kerr geometry, because each of them should be aligned with the corresponding Kerr congruence. Considering the retarded sheet as a basic physical sheet, we fix the congruence k_μ^+ and the corresponding metric $g_{\mu\nu}^+$, which are not allowed for the advanced field A_{adv} and must be positioned on the separate sheet with a different metric $g_{\mu\nu}^-$.

6. DISCUSSION

Taking the bag model concept, we should also accept the dynamical properties of the bags, which are soft and easily deformable [5, 17], forming a stringy structure. Typically, these are radial and rotational excitations accompanied by the formation of the open tube-shaped string ending with quarks. Another type of deformation was considered in the Dirac model of an "extensible" electron (1962) [39], which can also be regarded as a prototype bag model with radial excitations[7]. The bag-like source of the KN solution without rotation, $a = 0$, coincides with this "extensible" model of the Dirac electron, leading to the "classical electron radius" $R = r_e = e^2/2m$. As we discussed in the Introduction, the disk-like bag of the rotating KN source can be viewed as the stretching of the spherical bag by rotations. For the parameters of an electron, the spinning bag stretched into a disk of the radius $a = \hbar/2mc$, covering the Compton area of the "dressed" electron. The disk is very thin with the degree of flattening $\alpha = 137^{-1}$. The boundary of the disk appears to be very close to the former position of the Kerr singular ring, and the EM field near the boundary may be seen as a regularization of the KN singular EM field. Similarly to other singular lines, the Kerr singular ring was considered as a string in [11]. The structure of the EM field near this string was analyzed first in [10, 11], and much later in [24]. It appeared to be similar to the structure of the fundamental string solution, obtained by Sen in the low-energy heterotic string theory [24]. It is a typical light-like pp-wave string solution [19, 43, 44], which in the Kerr geometry takes a ring-like form.

Regularization of the KN source does not remove this ring-string, but gives it a cut-off parameter (9), $R = r_e$. It was shown in [10, 11] and later specified in [19, 37, 45] that the EM excitations of the KN solution lead to the appearance of traveling waves propagating along this ring-string. However, the light-like ring-string cannot be closed [46], since the points different by the angular period, $x^\mu(\phi, t)$ and $x^\mu(\phi + 2\pi, t)$, should not coincide, and a peculiar point on the ring-string should make it open, forming a single quark-like endpoint.

The string traveling waves deform the bag boundary, creating a singular pole [47]. We do not discuss it here in detail, leaving the treatment to a separate paper. We only note that the exact solutions for the EM excitations on the Kerr background were obtained in [22], and using conditions I and II considered in the introduction, we can unambiguously determine the back-reaction of the local EM field on the metric and obtain the corresponding deformations of the bag boundary. The origin of singular pole is caused by a circulating node in the EM string excitation. This node

[7] This view was also suggested in [40]. An interpretation of the black holes and AdS geometries as a sort of bag was also noted in [41, 42].

yields the zero cut-off parameter R, creating contact of the bag boundary with the singular ring.

This singular pole circulates along the sharp border of the disk with the speed of light and may be considered in three ways: a) as a light-like quark enclosed inside the bag, b) as a single end-point of the light-like ring-string (as show in [46], the light-like fundamental string cannot be closed), and c) as a naked point-like electron enclosed in a circular "zitterbewegung". It leads to an integrated model for the dressed and bare electron as a single coherent system similar to the hadronic bag models.

7. CONCLUSION

Starting from the old problem of the source of the KN geometry, we first obtained a bubble-core model of the spinning particle, the false vacuum of which is formed by the Higgs mechanism of symmetry breaking. Contrary to the most other known models of particle-like objects, the KN bubble forms a gravitating soliton creating the external gravitation and EM field of an electron. This compatibility with gravity has required the use of a supersymmetric field model of phase transition, leading to a supersymmetric false-vacuum state in the core of the particle and leaving the external gravitational and EM fields unbroken.

The resulting soliton model has much in common with the famous MIT and SLAC bag models, but acquires the "dual bag geometry", in which the Higgs condensate is embedded "inside out " compared to the previous bag models.

In this model, the two-sheeted structure of the Kerr geometry is given by a natural space–time (coordinate) implementation, forming a background for the initially massless leptons of the Glashow–Salam–Weinberg model [21].

Without attempting a detailed description, we can note that the described dressed electron may be turned into a positron if we change the role of the advanced and retarded sheets of the Kerr geometry. The higher excitation of the ring-string may generate the muon state, while switching off the scalar and longitudinal components of the EM field corresponding to the charge of the KN solution [48] and preserving only the transversal traveling waves, gives a neutral particle, which has the features of a neutrino. Therefore, some variations of the KN bag model can give the space–time structure for some other spinning particles of the electroweak sector of the Standard Model.

Note added. After this paper finished and submitted for publication I learned from Jim Bogan on the paper [49]. In this paper, which is a development of the previous paper [50], authors consider a geometrical model of the electron and other particles on the base of Taub–NUT solution, the self-dual properties and twistorial structure of which provide connections with the Dirac theory. Most part of their mathematical treatment on the Dirac equation is also related to the Kerr–Newman solution, twistorial structure of which is based on the Kerr theorem. Indeed, the known Kerr–NUT solution represents a common basis for these both lines of investigation. I believe that the structure of electron is rather related with the Kerr rotation parameter, while the monopole parameter of the NUT solution may be important for the structure of hadrons. I am grateful to J. Bogan for pointing this work.

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APPENDIX

The Kerr theorem

The Kerr theorem determines all the geodesic and shear free congruences as analytic solutions of the equation

$$F(T^A) = 0, \tag{43}$$

where F is an arbitrary holomorphic function of the projective twistor variables

$$T^A = \{Y, \zeta - Yv,\ u + Y\bar{\zeta}\}, \quad A = 1, 2, 3, \tag{44}$$

where

$$\zeta = \frac{x+iy}{\sqrt{2}}, \quad \bar{\zeta} = \frac{x-iy}{\sqrt{2}}, \quad u = \frac{z+t}{\sqrt{2}}, \quad v = \frac{z-t}{\sqrt{2}}$$

are null Cartesian coordinates of the auxiliary Minkowski space.

We note that the first twistor coordinate Y is also a projective spinor coordinate

$$Y = \phi_1/\phi_0, \tag{45}$$

and it is equivalent to the two-component Weyl spinor ϕ_α, which defines the null direction[8] $k_\mu = \bar{\phi}_{\dot{\alpha}} \sigma_\mu^{\dot{\alpha}\alpha} \phi_\alpha$.

[8] We use the spinor notation of book [20], where the σ-matrices have the form $\sigma^\mu = (1, \sigma^i)$, $\bar{\sigma}^\mu = (1, -\sigma^i)$, $i = 1, 2, 3$ and $\sigma^\mu = \sigma^\mu_{\alpha\dot{\alpha}}$, $\bar{\sigma}^\mu = \bar{\sigma}^{\mu\dot{\alpha}\alpha}$.

It is known [22, 37] that the function F for the Kerr and KN solutions can be represented in the form quadratic in Y,

$$F(Y, x^\mu) = A(x^\mu)Y^2 + B(x^\mu)Y + C(x^\mu). \tag{46}$$

In this case, Eq. (43) can be solved explicitly, leading to two solutions

$$Y^\pm(x^\mu) = \frac{-B \mp \tilde{r}}{2A}, \tag{47}$$

where $\tilde{r} = (B^2 - 4AC)^{1/2}$. It has been shown in [37] that these solutions are antipodally conjugate,

$$Y^+ = -1/\bar{Y}^-. \tag{48}$$

Therefore, solutions (47) determine two Weyl spinor fields ϕ_α and $\bar{\chi}_{\dot{\alpha}}$, which in agreement with (48) are related with two antipodal congruences

$$Y^+ = \phi_1/\phi_0, \tag{49}$$

$$Y^- = \bar{\chi}_{\dot{1}}/\bar{\chi}_{\dot{0}}. \tag{50}$$

In the Debney–Kerr–Schild formalism [22], the function Y is also a projective angular coordinate

$$Y^+ = e^{i\phi} \tan\frac{\theta}{2}.$$

It gives an explicit dependence on the Kerr angular coordinates ϕ and θ to spinor fields ϕ_α and $\bar{\chi}_{\dot{\alpha}}$.

For the congruence Y^+, this dependence takes the form

$$\phi_\alpha = \begin{pmatrix} e^{i\phi/2}\sin\dfrac{\theta}{2} \\ e^{-i\phi/2}\cos\dfrac{\theta}{2} \end{pmatrix}. \tag{51}$$

In agreement with (48), we have

$$\bar{Y}^- = -e^{-i\phi}\cot\frac{\theta}{2},$$

and from the invariant normalization $\phi_\alpha\chi^\alpha = 1$, we obtain

$$\chi_\alpha = \begin{pmatrix} -e^{i\phi/2}\cos\dfrac{\theta}{2} \\ e^{-i\phi/2}\sin\dfrac{\theta}{2} \end{pmatrix},$$

which yields

$$\bar{\chi}^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}\bar{\chi}_{\dot{\beta}} = \begin{pmatrix} e^{i\phi/2}\sin\dfrac{\theta}{2} \\ e^{-i\phi/2}\cos\dfrac{\theta}{2} \end{pmatrix}. \tag{52}$$

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