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Inhomogeneous Vacuum: An Alternative Interpretation of Curved Spacetime *

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The strong similarities between the light propagation in a curved spacetime and that in a medium with graded refractive index are found. It is pointed out that a curved spacetime is equivalent to an inhomogeneous vacuum for light propagation. The corresponding graded refractive index of the vacuum in a static spherically symmetrical gravitational field is derived. This result provides a simple and convenient way to analyse the gravitational lensing in astrophysics.

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Gravity is still not unified with other three fundamental forces.[1−3] The problem probably arises from the geometrization of the gravitation, which may lose some important physical details. Since vacuum is the only medium between the gravitational matters, physicists are more interested in the investigation of the relation between the vacuum and the gravitation.[4−15] Through such investigations the true mechanism of gravitation will be hopefully found and a workable theory of quantum gravity may finally be established.[13−15]

Some recent theoretical and experimental progresses have shown that the vacuum can be influenced by electromagnetic field. Ahmadi and Nouri-Zonoz,[16] Rikken and Rizzo,[17] Dupays *et al.*[18] have pointed out that the light propagation in vacuum can be modified by applying electromagnetic fields to the vacuum. This indicates that vacuum is actually a special kind of optical medium[16,18] and may also have its inner structure. Actually, the structure of quantum vacuum has been investigated in quite a number of papers recently.[19−21] Besides the electromagnetic field, the existence of matter can also influence the vacuum. For example, the vacuum inside a microcavity is modified due to the existence of the cavity mirrors, which will alter the zero-point energy inside the cavity and cause an attractive force between the two mirrors known as the Casimir effect,[22,23] which has been verified experimentally.[24,25]

The refractive index of vacuum, as a special optical medium, may be changed under the influence of gravitational matter. In fact, there has been a long history of such an idea. In 1920, Eddington[26] suggested that the light deflection in solar gravitational field can be conceived as a refraction effect of the space (actually the vacuum) in a flat spacetime. The idea was further studied by Wilson,[27] Dicke,[28] Felice,[29] and Nandi *et al.*[30−32] Recently, this thought of vacuum has been investigated further by Puthoff[13,14] and Vlokh.[33] In Puthoff's paper, the influence of gravitational field on the vacuum refractive index is analysed through the vacuum polarization. Vlokh discussed the value of this refractive index.

In our recent paper,[15] we analysed some simple cases of gravitational lensing by using an approximated expression of the refractive index for the vacuum outside the gravitational matter system. In this Letter, we emphasize the strong similarities between the light propagation in a curved spacetime and that in a medium with graded refractive index. These similarities suggest that an inhomogeneous vacuum may be the physical reality of the curved spacetime. We provide a general method to derive exactly the corresponding graded refractive index of the vacuum in a static spherically symmetrical gravitational field both for outside and inside the gravitational matter system, and point out that the refractive index profile is simply a unified exponential function of the gravitational potential for a weak gravitational field. We show that even the long puzzling central image missing problem in gravitational lensing[34] can now be solved clearly with the use of the obtained refractive index profile.

One of the strong similarities between the light propagation in a curved spacetime and that in an inhomogeneous medium locates in Fermat's principle. Landau and Lifshitz have derived from the general relativity Fermat's principle for the propagation of light in a static gravitational field as $\delta\int g_{00}^{-1/2}dl = 0$,[35] which can be rewritten as follows[15]

$$\delta\int\frac{dt}{d\tau}\frac{dl}{ds}ds = 0, \tag{1}$$

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where dl is the length element of the passing light $\overset{\frown}{P_0P}$ measured by the local observer, while ds is that measured by the observer at infinity; $d\tau$ represents the time interval measured by the local observer for a light ray passing through the length dl, while dt is the corresponding time measured by the observer at infinity (Fig. 1).

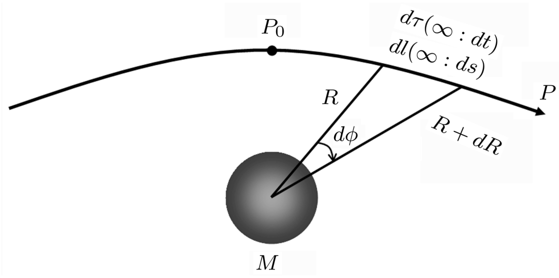

Fig. 1. Light deflection caused by a curved spacetime.

On the other hand, the light propagation in a medium with graded refractive index n satisfying the conventional Fermat's principle:

$$\delta \int nds = 0, \tag{2}$$

where ds is the length element of the passing light measured by the observer in a flat spacetime.

It is well-known that the light deflection in a medium is caused by the inhomogeneous refractive index n, while the light deflection in a gravitational field is caused by the curved spacetime, i.e., the $dt/d\tau$ (related to the curved time) and dl/ds (related to the curved space) as shown in Eq. (1). The similarity between the relativistic Fermat's principle and the conventional Fermat's principle gives us such an idea: a special inhomogeneous optical medium with graded refractive index may be the physical reality of the curved spacetime. Since only vacuum exists between gravitational matters, we suppose that vacuum is just this special optical medium.

An inhomogeneous vacuum means its refractive index is not constantly 1 as one considers usually. To obtain this special refractive index of vacuum, let us see another strong similarity between the curved spacetime and the inhomogeneous medium. The general relativity gives the angular displacement $d\phi$ of the coordinate radius R in a curved spacetime as[36–38]

$$d\phi = \frac{dR}{R/\sqrt{A(R)}\sqrt{\Big[\dfrac{R/\sqrt{B(R)}}{R_0/\sqrt{B(R_0)}}\Big]^2 - 1}}, \tag{3}$$

where R_0 is the coordinate radius at the point P_0 of the light ray nearest to the gravitational matter M, $A(R)$ and $B(R)$ come from a static and spherically symmetric metric of the standard form

$$d\mathcal{T}^2 = B(R)c^2dt^2 - A(R)dR^2 - R^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{4}$$

For the light propagation in a medium of a spherically symmetric refractive index n, the angular displacement $d\phi$ of the corresponding radius r in a flat spacetime can be derived from the Fermat's principle as[39,15]

$$d\phi = \frac{dr}{r\sqrt{\Big(\dfrac{nr}{n_0r_0}\Big)^2 - 1}}, \tag{5}$$

where r_0 and n_0 represent the radial distance and refractive index at the point closest to the centre, respectively.

The strong similarity between Eqs. (3) and (5) indicates again that an inhomogeneous vacuum may be the physical reality of the curved spacetime. From these two equations and the boundary conditions at infinity, the refractive index of this inhomogeneous vacuum can be derived as follows:

$$n = \frac{R}{r\sqrt{B(R)}}, \tag{6}$$

where R/r can be figured out through the integration of the following equation:

$$\frac{dR}{R/\sqrt{A(R)}} = \frac{dr}{r}. \tag{7}$$

Equations (6) and (7) provide a general method for finding the vacuum refractive index profile of a static spherically symmetric gravitational field, where the coefficients $A(R)$ and $B(R)$ can be obtained from the Schwarzschild solutions. The Schwarzschild exterior solution ($R \geq R_M, R_M$ is the radial coordinate at the surface of the gravitational matter system) gives[38]

$$A(R) = \Big(1 - \frac{2GM}{Rc^2}\Big)^{-1}, \tag{8}$$

$$B(R) = 1 - \frac{2GM}{Rc^2}, \tag{9}$$

where G is the gravitational constant, c is the velocity of light in vacuum without the influence of gravitational field. The Schwarzschild interior solution ($R \leq R_M$) gives[38]

$$A(R) = \Big[1 - \frac{2GM(R)}{Rc^2}\Big]^{-1}, \tag{10}$$

$$B(R) = \exp\Big\{ - \int_R^\infty \frac{2G}{R^2c^2}\Big[M(R) + \frac{4\pi R^3 p(R)}{c^2}\Big] \cdot \Big[1 - \frac{2GM(R)}{Rc^2}\Big]^{-1} dR\Big\}, \tag{11}$$

where $M(R) = \int_0^R 4\pi R^2 \rho(R) dR$, $\rho(R)$ is the mass density, and for an ordinary gravitational matter system, the pressure $p(R) = 0$.

Combining Eqs. (8)–(11) with Eqs. (6) and (7) will give the exact expressions of the exterior and interior refractive index, respectively.[40] It is interesting to see that if the gravitational field is not extremely strong, i.e., GM/Rc^2 or $GM/rc^2 \ll 1$, the two expressions approach a unified form as follows:

$$n = \exp\Big(-\frac{2P_r}{c^2}\Big), \tag{12}$$

where P_r is the gravitational potential at position r, that is, outside the gravitational matter system $P_r = -GM/r$, and inside the gravitational matter system $P_r = -\{GM(r_M)/r_M + \int_r^{r_M}[GM(r)/r^2]dr\}$ (r_M is the radial coordinate in flat spacetime corresponding to R_M in curved spacetime).

The above result is derived from a single static spherically symmetric gravitational matter system. For a multi-body system, the total gravitational potential will be the superposition of each potential; therefore, the refractive index of vacuum can be expressed as

$$\begin{aligned} n &= \exp\Big(-\frac{2P_r}{c^2}\Big) \\ &= \exp\Big[-\frac{2}{c^2}(P_{r1}+P_{r2}+P_{r3}+\cdots)\Big] = n_1 n_2 n_3 \cdots, \end{aligned} \tag{13}$$

where $P_{r1}, P_{r2}, P_{r3}, \cdots$ and $n_1, n_2, n_3, \cdots$ are the gravitational potential and the corresponding refractive index caused by each gravitational body respectively. This expression may be extended to arbitrary distributed matter systems. Figure 2 shows such an example, where the brighter grey around the three celestial bodies represents the higher value of refractive index, and the closed curves are the isolines, i.e. the denser the lines, the quicker the change of vacuum refractive index.

The result given by Eq. (12) or Eq. (13) provides a convenient optical way to describe the effect of gravitational lensing. In Ref. [15], we simulated the light path and the image shape of gravitational lensing by using Eq. (12). We found that a source star can give rise to two lensing images located at both sides of the lens body. The two images are both elongated tangentially. If the source star, the lens body and the observer are in a line, the two images will be interconnected to form a ring-like image known as the Einstein ring. These results are in agreement with the known facts.

By using the vacuum refractive index, other problems of the gravitational lensing such as the calculation of the time delay between the two lensing images, the estimation of the lens mass, the determination of the Hubble constant and so on, can also be treated in a simple optical way.[15]

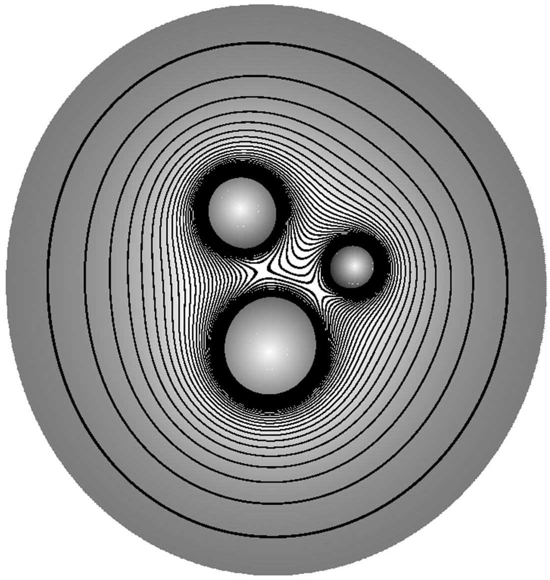

Fig. 2. Refractive index profile of a gravitational matter system composed of three celestial bodies of different masses.

In addition, the refractive index given by Eq. (12) or Eq. (13) makes it possible to study optically the formation of the central image, which is predicted by the general relativity but not observed in almost all known cases of gravitational lensing. This problem has puzzled people for many years.[34] Our computer simulations show that the larger the distance from the observer (or from the source) to the lens body, or the larger the mass of the lens body, the closer the central imaging ray to the lens centre, where the probably denser blocking matter leads to little chance of finding the central image of a gravitational lensing.[40] This accounts for the central image missing in almost all observations of gravitational lensing.

In summary, we have shown the two strong similarities between the light propagation in a curved spacetime and that in a medium with graded refractive index: one is in Fermat's principle; the other is in angular displacement formula. The similarities indicate that the vacuum around the gravitational matter is probably a special optical medium with graded refractive index, which may be the physical interpretation of the curved spacetime in general relativity. The similarities provide a general method for calculating the corresponding refractive index of the vacuum. Together with the Schwarzschild exterior and interior solutions, this refractive index can be exactly figured out. For a weak gravitational field, the refractive index profile is simply a unified exponential function of the gravitational potential for the vacuum both outside and inside the gravitational matter system. The result provides a simple optical way to analyse the gravitational lensing. We anticipate our work to be a stimulus to the quantum vacuum based investigation of the gravitational force.

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