The Other Meaning of Special Relativity

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ABSTRACT

Einstein's special theory of relativity postulates that the speed of light is a constant for all inertial observers. This postulate can be used to derive the Lorenz transformations relating length and time measurements by different observers. In this paper it is shown that the Lorentz transformations can be obtained for any type of wave simply by defining distance to be proportional to wave propagation time. The special nature of light is that length and time measured by light propagation correspond exactly with length and time measured by material rulers and clocks. This suggests that material objects consist of waves propagating at the speed of light. Taking this as an alternative postulate for special relativity implies constancy of the measured speed of light without any recourse to non-Euclidean geometry of physical space-time. This alternative postulate is consistent with de Broglie's wave hypothesis, with the Dirac velocity operator of quantum mechanics, and with experimental observations of transformations between matter and light.

I. INTRODUCTION

Einstein's theory of Special Relativity rests on the observation that the laws of physics, in particular the Maxwell-Lorentz equations for electromagnetism or light, are valid in any inertial frame of reference. The most important experimental data for this claim is from experiments first performed by Michelson and Morley, who demonstrated that electromagnetic phenomena are not affected by the translational velocity of the

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earth.[1] Specifically, the speed of light parallel to the direction of earth's motion appears to be identical to the speed of light perpendicular to the direction of earth's motion.

The laws of physics are invariant for different observers if time and distance transform according to the Lorentz transformation. The coordinates (t,x,y,z) and (t',x',y',z') of two observers with relative velocity v in the x-direction are related by:[2]

$$\begin{aligned} ct' &= \frac{ct - vx/c}{\sqrt{1-v^2/c^2}} \\ x' &= \frac{x - vt}{\sqrt{1-v^2/c^2}} \\ y' &= y \\ z' &= z \end{aligned} \tag{1}$$

These transformations explicitly contain the speed of light in vacuum, c, which must be the same for any inertial observer if the physical laws are to be considered as valid. This assumption of the constancy of the speed of light with respect to different inertial reference frames makes a distinction between electromagnetic waves and other types of waves. Sound waves, for example, do not maintain constant speed independent of the velocity of an observer.

It is well-known, however, that any wave equation of the form:

$$\left[\frac{\partial^2}{\partial t^2} - c^2\nabla^2\right] f = 0 \tag{2}$$

is invariant under Lorentz transformations with wave speed c. In other words Lorentz invariance is a general property of waves and not specific to electromagnetic waves. Therefore we should consider what distinguishes light waves from other types of waves.

II. ALTERNATIVE INTERPRETATION

The first part of the following discussion closely follows Einstein's explanation of special relativity[2] but with different rationale. Let us consider what relativity principal would hold in general if distances were always measured by wave propagation times (or conversely if time were defined in terms of wave propagation distance). The defining equation would be:

$$d_s^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 = \sum_{i=1}^{3} (\Delta x_i)^2 = c^2 t_p^2 \tag{3}$$

where d_s is the spatial distance between two points at a fixed time, c is an arbitrary constant, and t_p is the time it would take to propagate a wave from one point to the other if they remained stationary. With this definition of distance, the constant c is simply a scaling factor which relates the units of distance to the units of time. This distance corresponds to the usual definition of distance if c is the speed of the wave used in the measurement.

Now suppose we consider propagation of a wave from point P_1 to point P_2. In a reference frame in which the points are stationary, Eq. 3 holds. An observer in a different inertial reference frame using the same definition of distance would have:

$$\sum_{i=1}^{3} (\Delta x_i')^2 = c^2 t_p'^2 \tag{4}$$

The coordinate transformations which relate the primed and unprimed coordinates are the Lorentz transformations.

For example, suppose a submarine navigator is using sonar to detect a fish in the water. If both the sub and the fish are at rest in the water, a sound wave reflected from the fish at distance ℓ' would return after time $t'=2\ell'/c_s$, where c_s is the sound speed. The distance to the fish is therefore taken to be $\ell' = t' c_s/2$. Suppose now that the sub and fish are moving together in the water with common speed v perpendicular to the original direction of wave propagation. If the navigator doesn't realize that she is moving, she would assume the same relation between distance and time. The navigator of a second submarine sitting still in the water would observe the wave propagate over a distance:

$$d \equiv c_s t = 2\sqrt{\ell'^2 + \left(\frac{vt}{2}\right)^2} \tag{5}$$

Substituting $t'=2\ell'/c_s$ and solving for t' yields:

$$t' = t\sqrt{1 - v^2/c_s^2} \tag{6}$$

Since the stationary navigator sees the fish (and first sub) move a distance $x=vt$ while the wave is propagating, the above equation can be rewritten as:

$$t' = \frac{t\left(1 - v^2/c_s^2\right)}{\sqrt{1 - v^2/c_s^2}} = \frac{t - vx/c_s^2}{\sqrt{1 - v^2/c_s^2}} \tag{7}$$

which is the Lorentz transformation of time. Of course, if the wave propagation were timed with an ordinary clock, the measured propagation times would be equal for the two observers. In order to maintain the proportionality between distance and time, the sailors would have to use special sonar clocks which measure time by cycling sound wave pulses back and forth across a fixed distance in the water. If the sonar clock orientation is perpendicular to the direction of submarine motion then the stationary observer would conclude that the moving clock ticks slowly ($t' < t$) according to Eq. 6 because of the extra propagation distance introduced by the relative motion.

This might seem like an odd sort of clock, but consider the standard definition of a second, which is 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom.[3] If we regard the cesium atom as a kind of optical cavity which resonates at the prescribed frequency, then this is quite similar to our sonar clock.

Consider also the definition of the meter, which is the length of the path traveled by light in vacuum during a time interval of $1/c$ =1/299,792,458 of a second.[3] So we do in fact equate length with wave propagation time just as our hypothetical sailors do, and the quantity c is simply a unit conversion factor.

Now suppose that the first sub and fish are moving relative to the second sub parallel to the direction of wave propagation. As seen by the second sub, the frequency of the sonar clock on the first sub appears to be slow according to Eq. 6:

$$\omega = \omega' \sqrt{1 - v^2 / c_s^2} \tag{8}$$

The wavelengths of the reference waves are therefore related in the two reference frames by:

$$\lambda = \lambda' \frac{\omega'}{\omega} = \frac{\lambda'}{\sqrt{1 - v^2 / c_s^2}} \tag{9}$$

To measure the length of a stationary object we could of course count wavelengths of waves at the frequency of a stationary reference clock (using the wavelength without Doppler shifts). The "true" distance, independent of sound waves, is simply L=$n\lambda$=$n'\lambda'$. Using this equation yields the relation between measured distances:

$$\ell = \ell' \frac{n}{n'} = \ell' \frac{\lambda'}{\lambda} = \ell' \sqrt{1 - v^2 / c_s^2} \tag{10}$$

As noted previously, the coordinate of a fixed point in the moving frame moves with speed v in the stationary frame. Therefore the point at $x' = \ell'$ in the moving frame has coordinate $x = \ell + vt$ in the stationary frame (with proper choice of origin) and the coordinates are therefore related by:

$$x' = \frac{\ell}{\sqrt{1 - v^2 / c_s^2}} = \frac{x - vt}{\sqrt{1 - v^2 / c_s^2}} \tag{11}$$

which is the Lorentz transformation of position along the direction of motion.

Let's see if this relation between lengths yields consistent descriptions of the sound propagation to the fish and back. The first (moving) sub still has $t'=2\ell'/c_s$. The second (stationary) sub observes the propagation time of the wave to the fish ($\ell/(c_s - v)$) and back ($\ell/(c_s + v)$) to be:

$$t = \frac{\ell}{c_s - v} + \frac{\ell}{c_s + v} = \frac{2\ell}{c_s (1 - v^2 / c_s^2)} \tag{12}$$

Substituting the length relation of Eq. 10 yields:

$$t = \frac{2\ell}{c_s(1 - v^2/c_s^2)} = \frac{2\ell'}{c_s\sqrt{1 - v^2/c_s^2}} = \frac{t'}{\sqrt{1 - v^2/c_s^2}} \tag{13}$$

which agrees with Eq. 6. Therefore we do have a consistent set of time and length definitions which is independent of the orientation of the clock.

Thus we see how Lorentz transformations can be obtained by using sonar or any other type of wave to measure time and distance. Lorentz invariance is not a property of time and space *per se*. Rather it results from the methods used to measure time and distance. If the above-mentioned sailors were to rendezvous to share their data and some vodka, they might conclude after a few drinks that absolute time and space in moving underwater reference frames are related by Lorentz transformations using the speed of sound in water. After sobering up, however, they would realize that sonar is not the only way to measure time and distance and that their measurements are not evidence of any non-Euclidean properties of underwater time and space.

We should likewise consider whether the Lorentz transformations of special relativity arise because of innate properties of space-time or are simply due to the techniques which we use to measure time and distance.

III. MATTER WAVES

One limitation of the above discussion is that sound waves in water are too simple to serve as a model of matter. In particular, sound waves are scalar waves, described by a single number (e.g. pressure) at each point. A more interesting medium to consider is an elastic solid, which can support shear waves whose amplitude (displacement or rotation) can have multiple components. Waves which include significant rotations are especially of interest, since rotations of the medium can alter the direction of propagation of the waves. This variability of propagation direction is crucial to understanding how special relativity applies to matter waves.

Let the characteristic wave speed of transverse waves in an elastic medium be c_τ to distinguish it from light and sound waves. Suppose that a wave packet can be formed which propagates in a circular motion so that its energy remains localized. Such a wave packet is called a solitary wave or soliton. Although the local wave speed is the same as for waves propagating in straight lines (c_τ), the velocity of the packet as a whole will always be less than c_τ because of the circulating component of the motion. Assume for the moment that the translational motion is perpendicular to the plane of circulation so that the wave travels in a spiral. Separating the circulating ($\perp$) and translational ($\|$) parts of the wave equation yields:

$$\frac{\partial^2 a}{\partial t^2} = c_\tau^2 \left(\nabla_\perp^2 + \nabla_\|^2 \right) a \tag{14}$$

It is common to use Fourier decomposition so that the wave equation can be written as:

$$\omega^2 A = c_\tau^2 \left(k_\perp^2 + k_\|^2 \right) A \tag{15}$$

where $A(\mathbf{k},\omega)$ is the Fourier transform of the wave amplitude $a(\mathbf{x},t)$. The wave packet must contain a range of $\mathbf{k}$ values in order to be localized. For the moment, however, it is convenient to consider a fixed value of $k_\|$ and fixed magnitude of $\mathbf{k}_\perp$. The translational component of wave velocity in direction $x_\|$ is clearly given by:

$$v_{\|} = c_\tau \frac{k_{\|}}{|\mathbf{k}|} = c_\tau \frac{k_{\|}}{\left(k_\perp^2 + k_{\|}^2\right)^{1/2}} \tag{16}$$

Solving for $k_{\|}$ yields:

$$k_{\|}^2 = \left(1 - \frac{v^2}{c_\tau^2}\right)^{-1} \frac{v^2}{c_\tau^2} k_\perp^2 \equiv \gamma^2 \frac{v^2}{c_\tau^2} k_\perp^2 \tag{17}$$

where we have used the familiar definition of γ to obtain the expression on the right. Substitution into the wave equation yields:

$$\omega^2 A = c_\tau^2 k_\perp^2 \left[1 + \gamma^2 \frac{v^2}{c_\tau^2}\right] A = \gamma^2 c_\tau^2 k_\perp^2 A \tag{18}$$

In quantum mechanics the energy density operator is $E=\hbar\omega$ and the momentum density operator is $p_i=\hbar k_i$. With a probabilistic interpretation of the quantum wave function the integral of the square of the wave amplitude $\int |\psi|^2 d^3x$ is normalized to one and the factor of $\hbar$ is an ad hoc scaling factor. In a soliton model $\hbar$ should be equal to the integral of wave function $\int |\psi|^2 d^3x$. Since $\hbar$ has units of angular momentum, the quantity $|\psi|^2$ should be interpreted as an angular momentum density. This analysis suggests that the quantum wave function describes waves of rotation, or torsion waves. In fact, it has recently been shown that torsion waves in an elastic solid are described by a Dirac equation.[4] Nonetheless we will use the conventional unit normalization below.

Eq. 15 can be rewritten in terms of energy and momentum density as:

$$E^2 = c_\tau^2 \left(p_\perp^2 + p_{\|}^2\right) \tag{19}$$

The rest energy density (or mass density) is proportional to the circulating component of momentum:

$$E_0^2 \equiv m_0^2 c_\tau^4 = c_\tau^2 p_\perp^2 = c_\tau^2 \hbar^2 k_\perp^2 \qquad (20)$$

The linear momentum density of the wave packet is derived from Eqs. 17 and 20 to be:

$$p_\parallel^2 = \hbar^2 k_\parallel^2 = \gamma^2 \frac{\hbar^2 k_\perp^2}{c_\tau^2} v_\parallel^2 = \gamma^2 m_0^2 v_\parallel^2 \qquad (21)$$

The total energy density can thus be written as:

$$E^2 = m_0^2 c_\tau^4 + p_\parallel^2 c_\tau^2 = \gamma^2 m_0^2 c_\tau^4 \qquad (22)$$

Hence Einstein's famous mass-energy relation follows directly from the assumption that matter consists of soliton waves simply by applying the Pythagorean theorem to the wave vector components.

It is worth noting that the definitions of E and p_i lead directly to the equation of motion $k_i E = \omega p_i$, which in spatial coordinates is:

$$F_i \equiv \frac{\partial E}{\partial x_i} = \frac{\partial p_i}{\partial t} \qquad (23)$$

One limitation of the above analysis is that we assumed the bulk soliton motion to be perpendicular to the circulating motion. We should also consider bulk motion in the same plane as the circulation, which results in cycloidal motion. Let $\mathbf{k}_{\text{circ}} = \hat{\mathbf{x}}_\perp k_\perp \cos\theta + \hat{\mathbf{x}}_\parallel k_\perp \sin\theta$, where $\hat{\mathbf{x}}_\perp$ is any direction perpendicular to the bulk propagation direction $\hat{\mathbf{x}}_\parallel$. In this case we cannot separate $k_\perp$ and $k_\parallel$, so we must consider the integral of Eq. 15:

$$\int d^3k\ \omega^2 A = \int d^3k \left[c_\tau^2 k_\perp^2 \cos^2\theta + c_\tau^2 \left| k_\perp \sin\theta + k_{||} \right|^2 \right] A \tag{24}$$

Since the cross term $c_\tau^2 k_\perp k_{||} \sin\theta$ integrates to zero, the resulting energy-momentum relations will be equivalent to Eqs. 19-22. In fact, the integral of $\mathbf{k}_{\text{circ}}$ must be zero if the wave with $k_{||}=0$ is indeed stationary. Therefore it is clear that the energy-momentum relation of special relativity holds whether the wave circulation is parallel or perpendicular to the bulk motion.

We must also consider whether the relations derived above hold when a wider range of **k**-values is present. Let the integral of an arbitrary operator q be denoted by $\langle q \rangle$. Eq. 24 was derived for a fixed $k_{||}$ and fixed magnitude of $\mathbf{k}_{\text{circ}}$. However, it is clear that if this equation holds for each set of ($k_{||}$, $\mathbf{k}_{\text{circ}}$) individually then it must also hold when integrated over all values of ($k_{||}$, $\mathbf{k}_{\text{circ}}$). Therefore:

$$\left\langle \omega^2 \right\rangle = c_\tau^2 \left(\left\langle k_{\text{circ}}^2 \right\rangle + \left\langle k_{||}^2 \right\rangle \right) \tag{25}$$

If the soliton moves without changing shape (as seen by an observer moving along with the wave) then the velocity $\mathbf{v}_{||}$ (and consequently γ) is constant throughout the wave packet. We can therefore write:

$$\left\langle \omega^2 \right\rangle = \gamma^2 c_\tau^2 \left\langle k_{\text{circ}}^2 \right\rangle \tag{26}$$

and all of the relativistic relations hold for the wave packet as a whole and not just individual k-values.

Consider next how two observers undergoing relative motion would interpret soliton waves. Both observers would see the same $\mathbf{k}_\perp' = \mathbf{k}_\perp$. Suppose one observer sees a stationary soliton wave packet with $\omega = c_\tau k_\perp$ and $k_{||}=0$. Then an observer moving with relative speed v_0 would see a soliton moving with speed $-v_0$. With no means to determine his absolute velocity relative to the medium, this observer assumes that the wave moves at the characteristic speed c_τ rather than at the relative speed $\left(c_\tau^2 + v^2\right)^{1/2}$, yielding (from Eq. 17 above with $k_\perp' = k_\perp$):

$$ck_{||}' = -\gamma \frac{v_0}{c_\tau} c_\tau k_\perp = -\gamma \frac{v_0}{c_\tau} \omega \quad \text{(for } k_{||}=0\text{)} \tag{27}$$

The frequency of the soliton seen by the moving observer is given by Eq. 18 (with $k_\perp' = k_\perp$):

$$\omega' = \gamma c_\tau k_\perp = \gamma\omega \quad \text{(for } k_{||}=0\text{)} \tag{28}$$

Therefore the transformation of wave variables between stationary and moving observers must be of the form:

$$\begin{aligned} c_\tau \mathbf{k}_\perp' &= c_\tau \mathbf{k}_\perp \\ c_\tau k_{||}' &= \alpha_1 c_\tau k_{||} - \gamma \frac{v}{c_\tau}\omega \\ \omega' &= \gamma\omega - \alpha_2 c_\tau k_{||} \end{aligned} \tag{29}$$

where the coefficients α_1 and α_2 can be determined by considering the inverse transformations ($v=-v_0$):

$$\begin{aligned} c_\tau k_{||} = 0 &= \alpha_1 c_\tau k_{||}' + \gamma \frac{v_0}{c_\tau}\omega' = -\alpha_1 \gamma \frac{v_0}{c_\tau}\omega + \gamma^2 \frac{v_0}{c_\tau}\omega \\ \omega &= \gamma\omega' - \alpha_2 c_\tau k_{||}' = \gamma^2 \omega + \alpha_2 \gamma \frac{v_0}{c_\tau}\omega \end{aligned} \tag{30}$$

Solving for α_1 and α_2 yields:

$$\alpha_1 = \gamma$$
$$\alpha_2 = -\gamma \frac{v_0}{c_\tau} = \gamma \frac{v}{c_\tau} \tag{31}$$

So that the transformation between observers with relative velocity v is:

$$c_\tau \mathbf{k}'_\perp = c_\tau \mathbf{k}_\perp$$
$$c_\tau k'_\| = \gamma c_\tau k_\| - \gamma \frac{v}{c_\tau} \omega \tag{32}$$
$$\omega' = \gamma\omega - \gamma\left(\frac{v}{c_\tau}\right) c_\tau k_\|$$

These are the Lorentz transformations for the 4-vector (ω,**k**). Hence we have shown that the Lorentz transformations also apply to soliton waves, following directly from the definition of particle velocity as the translational component of soliton wave propagation.

Finally, let us consider whether quantum mechanics can be interpreted as describing soliton waves rather than probabilistic waves. If we neglect spatial derivatives it is straightforward to show that *x*- and *y*-components of the wave 'current' **J** of the Dirac equation evolve as: [4]

$$\frac{d}{dt} J_y = 2MJ_x$$
$$\frac{d}{dt} J_x = -2MJ_y$$

Therefore mass is clearly associated with rotation of the propagation direction, indicating a soliton wave. The factor of two arises because the current **J** is proportional to the square of the wave amplitude.

IV. LIGHT AND MATTER

The above results show that the equations of special relativity are applicable to a wide variety of wave phenomena. The Lorentz transformations relate wave measurements made in different frames of reference and the energy-momentum relation is a general property of soliton waves.

Now we are in a position to appreciate what is special about light. Ordinarily we do not measure distances and times by propagating waves back and forth. Instead we use material clocks and rulers. The amazing thing about material clocks and rulers is that the resulting distance and time measurements transform with exactly the same Lorentz transformations as would be obtained if the measurements had been made by propagating light waves. In other words, matter behaves as if it consists of soliton waves which propagate at the speed of light. Let us take this as an alternative postulate for special relativity: *matter consists of waves which propagate at the speed of light*. This alternative postulate is simply the de Broglie wave hypothesis[5] with the condition that the waves propagate at the speed of light. This physical picture suggests that matter and anti-matter can annihilate into photons and *vice versa* because photons and matter are simply different packets of the same type of wave. Our new hypothesis is also consistent with the Dirac equation for the electron, in which the velocity operator has eigenvalues of magnitude c. The mass term represents rotation of the propagation direction, which explains why the apparent speed is always less than the speed of light. The wave paths are like spirals (or cycloids) rather than straight lines.

IV. DISCUSSION

Since the use of wave propagation to measure distance and time yields the same Lorentz invariance as the postulate that the speed of light is the same in all inertial reference frames, the question arises as to what difference there is between the two interpretations of special relativity. As long as we deal only with waves propagating at the speed of light (and phenomena derived from them), it makes little difference whether we assume that Lorentz invariance is truly a property of time and space or whether it is merely the result of using matter waves to make measurements. However, if the vacuum can support other types of waves then we need to consider the possibility of other means to measure time and distance. Einstein's postulate essentially requires that all waves in vacuum travel at the speed of light, whereas the proposed wave postulate allows the possibility of other types of waves. For example, since gravity waves have never been directly observed it is possible that they might propagate at a different speed than light waves. In that case the wave equation for gravity waves would not satisfy Lorentz invariance using the speed of light. It is also possible that the apparent speed of gravity waves might be direction-dependent, which would indicate that motion can be defined relative to the vacuum.

With respect to Michelson-Morley experiments, it is clear that if matter waves have the same speed as light waves then any effect of earth's propagation through the vacuum would equally affect the light waves and the apparatus used to measure them. It has long been recognized that Lorentz invariance of matter is required to explain the null result of such experiments.[6] What has not been generally recognized is that special relativity is a consequence of the wave nature of matter and is entirely consistent with classical notions of absolute space and time.

V. CONCLUSION

Lorentz invariance is a property of the wave equation and Lorentz transformations relate measurements in different reference frames whenever wave propagation is used to measure length and time. The special nature of light is that time and distance measured by light propagation correspond exactly with time and distance measured by material clocks and rulers in all inertial reference frames. This leads to the inference that material objects consist of soliton waves propagating at the same speed as light. This alternative postulate of special relativity is consistent with the wave nature of matter, with the energy-momentum relation, with the Dirac equation, and with experimental evidence that matter and antimatter can be converted into photons and *vice versa*.

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