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Heisenberg's Microscope—A Misleading Illustration

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According to the Rayleigh criterion of classical optics, the finite resolving power of a microscope is due to the width of the central peak of the Fraunhofer diffraction pattern produced by the microscope's finite lens aperture. During the last few decades, theories and techniques for superresolution beyond the Rayleigh criterion have been developed in classical optics. Thus, Heisenberg's microscope could also in principle be made to give superresolution and thereby appear to violate the uncertainty relation. We believe that this paradox is due to the inappropriate use of a definition, based purely on experimental convenience, to support a quantum mechanical theorem.

Heisenberg's uncertainty relation $\Delta x \, \Delta p \geqslant \hbar/2$ can be derived in its most generalized form using the quantum mechanical commutation relation, the definition of standard deviation, and quantum averaging.[1-3] However, in his original paper Heisenberg[4] derived the relation in a somewhat restricted form because of his use of a coordinate-space wave function of Gaussian form which gives a momentum wave function of the same form. Apparently to illustrate the physical meaning of the uncertainty relation, Heisenberg analyzed the physical situation of a possible gamma-ray microscope to measure the position and momentum of an electron through gamma–electron scattering.[5,6] But the phenomenon of diffraction due to the finite aperture of the microscope objective limits the precision in the measurement of the position and momentum of the electron. The finite angle of acceptance of the microscope objective for the scattered gamma ray reduces the precision in knowledge of the angle of scattering and hence the momentum of the electron, and the finite width of the image of the point source (gamma–electron scattering center) due to diffraction reduces the precision in know-

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ledge of the position of the electron. Using simple algebra and the diffraction theory along with the de Broglie hypothesis ($\lambda = h/p$), one can easily derive the uncertainty relation.[5–8] Diffraction theory of image formation states that the image of a point source (the point spread function) is the Fourier transform of the finite imaging aperture, or equivalently, the Fraunhofer diffraction pattern of the aperture.[9] Thus, the "derivation" of the uncertainty relation using the Fraunhofer diffraction pattern due to a single aperture is completely equivalent to Heisenberg's microscope. Yet, many textbooks[7,8] present them separately to illustrate the uncertainty relation without mentioning that they are optically equivalent experiments.

It is interesting to note that in the original version of his paper on the uncertainty relation, Heisenberg[4] did not consider the diffraction spread due to the finite aperture of the microscope as the root of imprecision in the measurement. This was pointed out by Bohr when he first saw Heisenberg's manuscript.[4,6] Heisenberg's original reasoning was that the gamma–electron Compton scattering produces an uncontrollable, discontinuous change in the momentum of the electron. This change becomes greater, the higher the energy of the gamma ray. But a higher energy gamma ray should be prefered because it is more localized (smaller wavelength) and can provide a more precise position of the electron. Thus, one obtains the reciprocal relation between Δx and Δp. Heisenberg's original thinking may have been that it is the undefined but finite size of an elementary particle which is at the root of the uncertainty. Such an interpretation has recently been promoted by Bunge.[3]

Let us come back to the diffraction theory of image formation and the classical definition of resolving power for an imaging device.[9,10] As mentioned before, each point of the object plane produces a Fraunhofer diffraction pattern (point spread function) due to the finite aperture of the imaging device at the image plane, instead of a point image. For practical visual work one defines the resolving power, following Rayleigh, as the width Δx of the first zero from the central maximum of the Fraunhofer pattern. But this is only a semiobjective criterion. This is due to the fact that this criterion is only useful when the object consists of a multitude of close points, instead of a single point, such that partial overlapping of the many point spread functions still remains identifiable by visual observation as due to many separate point objects. For a single point object such a criterion has no objective meaning because, no matter how wide the point spread function is, the center of symmetry of this function is the precise location of the image point; there is no loss of resolving power. Further, even for complicated objects, a finite width of the point spread function does not imply any theoretical limit to the resolving power of the imaging device. Using mathematical tools like deconvolution, analytic continuation, and/or experimental techniques, one can achieve superresolution beyond the Rayleigh limit.[9–12] As a simple example of an experimental technique for achieving a higher resolution, we could mention that the diameter of the central fringe of the point-spread function due to a very narrow aperture of outer diameter d is about 0.6 times smaller than that due to the completely open aperture of diameter d.[13]

Here one might argue that either in the case of precisely locating the center of the point spread function from its symmetry or using a superresolution method requires the formation of the complete diffraction pattern. Formation of such a complete pattern requires waves of sufficient energy in classical physics or a multitude of similarly prepared particles in quantum physics. Heisenberg wanted to refer to the measurement of the position of only one quantum, to which the above-mentioned techniques cannot be applied. But we mention that from the purely practical point of view, one cannot distinguish the single point "blackening" of the detector due to the desired quantum out of the everpresent multitude of "background blackening" due to various terrestrial and cosmic radiations. Of course, here one can say that this not being a theoretical limit, Heisenberg's microscope as a "*gedanken* experiment" still stands. To this we can raise the following question: What is the physical or philosophical importance of a "*gedanken* experiment" which can never be proved or disproved either experimentally or theoretically? Further, the quantum formalism and its experimental verifications are based on the statistical interpretation; quantum measurements never refer literally to an isolated single particle but to an ensemble.[14] In the general mathematical derivation of the uncertainty relation, Δx and Δp are square roots of quantum mechanical averages of mean square deviations like $\langle(p - \langle p\rangle)^2\rangle^{1/2}$. Thus Δx and Δp are no more than the statistical scatter or spread of the observables x and p for an ensemble and do not refer to a single measurement with a single particle.

Finally, reference should be made to the work of Beck and Nussenzveig,[15] who presented a rigorous derivation and analysis of the single-slit diffraction pattern. Their major finding is that the uncertainty product diverges if one uses for Δx and Δp the definition of root-mean-square deviation that is used in the derivation of the uncertainty relation. An intuitive interpretation of the divergence can be presented as follows: Since theoretically the diffraction pattern extends infinitely far on both the sides of the central maximum, the imprecision in the measurement can also, in principle, be infinitely extended. Techniques have been developed[16] to measure more than 3000 fringes on either side of the central maximum. Thus the uncertainty due to diffraction spread should be at least 10^3 times larger than is customarily accepted. Beck and Nussenzveig[15] have also found that the product $\Delta x\ \Delta p$ is nondivergent only if one adopts the width of the central peak for the root-mean-square deviation. But we have just mentioned that neither from the theoretical nor from the experimental standpoint does the width of the central peak of a diffraction pattern imply a limit of resolution in a measurement. (This is also true for the product $\Delta\nu\,\Delta t$ in classical spectroscopy.[17])

Thus, it can be seen that the use of the classical definition of resolving power due to diffraction broadening was a misleading attempt to "explain" the concept behind a purely quantum mechanical theorem, the uncertainty relation. We think that numerous debates and misunderstandings could have been avoided during the last five decades if the analogy of the classical diffraction pattern and the semiobjective definition of resolving power had not been used by Heisenberg,[4] under the influence of Bohr,[4,6] to illustrate the uncertainty relation.

Lamb[18] has criticized the gamma-ray microscope as unsuitable for position measurement, but his reasoning is different from ours. Bunge has also expressed the view that the use of such classical optical experiments to explain the quantum concepts is mistaken, because they "employ propositions belonging to optics, not to quantum mechanics."[3] This is more along the line of our thoughts. The other optical experiment that is extensively used for illustrating the concept of quantum mechanics is the double-slit interference pattern. This illustration is also full of contradictory claims, as can be found in the general literature.[6] As a specific example, we mention that many quantum scientists explicity state that the phenomena of interference and diffraction are not local effect.[20] Yet classical experiments reveal exactly the opposite. For example, a double slit can be illuminated by a space-limited narrow laser beam to illuminate only one slit and to produce only a single-slit pattern. The two slits cannot act in unison unless an electromagnetic field passes through both the slits and arrives simultaneously at the space and time point under observation. Further, using holography (to record both amplitude and phase), one can record the patterns due to one slit at a time and then reproduce the double-slit pattern, proving thereby that the phenomena of interference and diffraction are local effects due to the real physical superposition of more than one signal at the plane of observation.[19,17]

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