Gravitational waves encounter vacuum energy

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Abstract: Analysis of the gravitational wave designated GW150914 shows that this wave encountered spacetime as being a very stiff elastic medium. The source of this stiffness can be determined by analyzing wave amplitude, frequency, intensity, etc. The implication is that GW150914 encountered vacuum energy density of about 4.2 x 10^{31} J/m^3 at 200 Hz. This finding has implications for the "cosmological constant problem" which is a 10^{120} discrepancy between general relativity and quantum field theory concerning the energy density of the universe. The energy density encountered by GW150914 is far above the "critical" energy density from general relativity but agrees with the frequency dependent vacuum energy density expected from quantum field theory at 200 Hz. Equations related to the properties of vacuum energy are developed and a model of vacuum energy is proposed. It is shown that Planck length vacuum fluctuations can generate the vacuum energy density encountered by GW150914, as well as explain zero-point energy and virtual particle formation. The Planck length vacuum fluctuations of the model oscillate between positive and negative spacetime curvature which offset and cancel gravitational effects. Therefore, vacuum energy can strongly interact with gravitational waves but be undetectable to fermion based instruments.

Introduction

The first detection [1] of a gravitational wave (GW) designated GW150914 has been hailed as a very important advance in astronomy. It opens a new way of observing massive objects in the universe and confirms another prediction of general relativity. All of this is true, but this article makes the case that an unanticipated result of the observation of GW150914 is that it gives important new experimental support for the existence of vacuum energy (VE) at a high energy density level. This pertains to the ongoing scientific discussion known as the "cosmological constant problem" [2] or also known as the "vacuum catastrophe" [3, 4]. There is a disagreement between the values of the average energy density of the universe obtained from cosmology ($\sim 10^{-9}$ J/m^3) compared to the much larger value of $\sim 10^{113}$ J/m^3 for zero-point energy suggested by quantum field theory. The large value of VE density is almost universally rejected. For example, the book "General Relativity: An Introduction to Physics" [5] discusses VE and states, "This gives an answer about 120 orders of magnitude higher than the upper limits on (vacuum energy) set by cosmological observations. This is probably the worst theoretical prediction in the history of physics! Nobody knows how to make sense out of this result." Stephan Hawking said this is "the most spectacular failure of any physical theory in history." However, quantum mechanics is the most successful quantitative theory ever produced, so this interpretation of quantum mechanics presents a serious problem for physics. A review article on the cosmological constant problem [2] lists 180 references. Most of these references propose alternatives that attempt to eliminate the large value of VE.

If the universe has density larger than the critical density given by the Friedmann equation [6], then the universe should gravitationally collapse. Observations made by the WMAP [7] and the Planck space mission [8] imply that spacetime is flat to within the 0.4 % observational accuracy. Therefore, energy density of 10^{113} J/m^3 appears to be ridiculous and completely incompatible with observations. For VE to physically exist at energy density vastly larger than 10^{-9} J/m^3, it would be necessary for VE to be a previously unknown energetic property of space that does not exert gravity. This question will be examined later.

A strong theoretical case for the importance of zero-point energy (also designated VE) in quantum field theory is made by Milonni [9] in the book *The Quantum Vacuum*. This book explains how VE permeates all of quantum field theory. Even when VE is eliminated by renormalization in one area, the need for VE reappears in other areas. The great accuracy of quantum electrodynamics and quantum chromodynamics requires the existence of VE. Some examples of quantum mechanical effects requiring VE are: 1) virtual particle formation and annihilation, 2) the uncertainty principle, 3) the Lamb shift, 4) the Unruh effect, 5) spontaneous emission initiation, 6) the Casimir effect, 7) the electron's anomalous magnetic dipole moment and 8) zero-point energy in quantum systems. However, there is no undisputed experimental evidence that VE physically exists. For example, the Casimir effect [10–12] is often cited as experimental proof of VE. There is definitely a force between two closely spaced metalized plates which has been measured and agrees with the QED predictions for VE within a few percent. However, there are alternative explanations involving charges and currents [13] which generate the same magnitude of force between the plates.

Another great advocate for VE was John Archibald Wheeler. For example, he said "Empty space is not empty… The density of field fluctuation energy in the vacuum argues that elementary particles represent percentage-wise almost completely negligible change in the locally violent conditions that characterize the vacuum." [14]. To explain the chaotic properties of space he visualized on the Planck scale, Wheeler proposed the term "quantum foam" [14, 15].

The overall objective of this work is to use the observational data from GW150914 to help resolve the cosmological constant problem. A model of VE will be proposed and tested.

Gravitational wave introduction

GWs were always considered to be a controversial prediction of general relativity. Their existence was debated at conferences and they were not detected even after 50 years of experimental attempts. Then in September 2015 the LIGO experiment started a new series of observations using instruments with increased sensitivity. Within a few days of restarting, the two separate LIGO interferometers detected the GWs emitted by the merging of two black holes at 1.3 billion light years. A second, weaker GW designated GW151226 was also detected in December 2015 [16].

GWs propagate in the medium of spacetime. They are transverse quadrupole waves which slightly distort the "fabric of space". For example, a GW propagating in the "Z" direction would cause a sphere made from baryonic matter such as aluminum to become an oscillating ellipsoid. When the sphere expands in the X direction it contracts in the Y direction and vice versa. The GW produces: 1) no change in the total volume of the oscillating sphere 2) no change in the rate of time, 3) no displacement of the center of mass of the oscillating sphere.

Point #3 addresses an important point. If there are two isolated masses such as two LIGO interferometer mirrors suspended by wires [17], the passage of a GW does not move the mirror's center of mass. Instead of the mirrors physically moving, the GW changes the properties of spacetime producing a redshift and a blue shift on LIGO's laser beams. This difference in wavelength is detected by the interferometer as a fringe shift that we will designate $\Delta\ell$. This measurement is made over the round-trip length of the interferometer that will be designated L. If we assume that L is much smaller than the GW wavelength ($L \ll \lambda$), then the maximum strain (maximum slope of the sinusoidal GW) can be approximated as $\Delta\ell/L$. Knowing the maximum slope, it is possible to calculate the theoretical maximum optical path length displacement ΔL produced by a GW as $\Delta L \approx \bar{\lambda}\Delta\ell/L$ where lambda bar is $\bar{\lambda} = \lambda/2\pi$. The approximation incorporating interferometer length L is eliminated if we state the maximum spacetime displacement as $\Delta L = A_s\bar{\lambda}$ or $A_s = \Delta L/\bar{\lambda}$. The spacetime displacement amplitude ΔL has some analogies to the particle displacement δ of acoustic equations.

One of the most useful GW equations is Eq. (1) below which gives the GW intensity (I with units of $\mathrm{w/m^2}$) in terms of frequency f and amplitude. Eq. (1) has dimensionless strain amplitude of two polarizations designated with the symbol "$\mathscr{h}$" where $\mathscr{h}^2 = \mathscr{h}_+^2 + \mathscr{h}_\times^2$. Eq. (1) is a standard GW equation

[18, 19] but the remainder of this article will use different symbols to avoid confusion with Planck's constant h, and convert to angular frequency ω. Therefore, Eq. (2) is the same as Eq. (1) but uses the symbol A_s for strain amplitude ($\hbar = A_s$) and $\omega = 2\pi f$. The 2π difference between ω and f requires the appropriate adjustment in the numerical constant.

$$I = \frac{\pi}{4}\frac{c^3}{G} f^2 \hbar^2 \tag{1}$$

$$I = \left(\frac{1}{16\pi}\right) A_s^2 \omega^2 \left(\frac{c^3}{G}\right) \tag{2}$$

Eq. (2) should be compared to the generalized intensity equation for the intensity of waves of any kind. This equation is: $I = kA^2\omega^2 Z$. Comparing this to Eq. (2), it is obvious that in Eq. (2) the numerical constant $k = 1/16\pi$, the strain amplitude term $A = A_s$, and the impedance term (Z) is c^3/G. Blair [19] was the first person to identify the impedance of spacetime as $Z_s = c^3/G = 4 \times 10^{35}$ kg/s.

We will next convert Eq. (2) to another more general equation for the energy density (U) of a wave propagating in spacetime at the speed of light because $U = I/c$. The constant $1/16\pi$ is specifically for transverse quadrupole GWs. Therefore, Eq. (3) below will substitute k for the numerical constant to have application to other waves in spacetime including virtual particles. Also we set $c^3/G = Z_s$.

$$U = \frac{I}{c} = kA_s^2\omega^2 Z_s/c \tag{3}$$

Results

Gravitational wave observation verifies vacuum energy

It is possible to use the experimental observation [1, 20] of GW150914 to support the contention that spacetime contains a form of energy density which strongly interacts with GWs but is undetectable to fermion-based instruments. GW150914 was a chirp which went from 30 Hz to about 250 Hz. For analysis, we will use the wave properties at 200 Hz as well as standardizing on angular frequency ω and reduced wavelength: lambda bar = $\lambda\!\!\!^{-} = c/\omega$. Therefore, GW150914 had angular frequency of about $\omega = 1250$ s^{-1} and a reduced wavelength of $\lambda\!\!\!^{-} = 2.4 \times 10^5$ m at 200 Hz. The measured strain amplitude at 200 Hz was about $A_s = \Delta L/\lambda\!\!\!^{-} = 1.25 \times 10^{-21}$, therefore the displacement amplitude of the GW was $\Delta L = \lambda\!\!\!^{-} A_s = 3 \times 10^{-16}$ m. Substituting $A_s = 1.25 \times 10^{-21}$ and $\omega = 1250$ s^{-1} into Eq. (2), we obtain the observed GW intensity was $I = 0.02$ w/m^2. This is a substantial intensity, but the GW encountered spacetime as such a stiff medium that it took an incredibly sensitive instrument to detect the $\Delta L/L \approx 10^{-21}$ spacetime strain.

The maximum GW power emitted by GW150914 was reported [1] to be 3.6×10^{49} w. This approaches Planck power ($c^5/G = 3.6 \times 10^{52}$ w). This emitted power is easily checked because it is the power required to achieve intensity of 0.02 w/m^2 over the area of a sphere with radius of 1.3 billion light years. The mass/energy radiated into GWs was equivalent to 3 solar masses (5×10^{47} J) which is about 4.6 % of the total mass of the two black holes before merging. At a distance of ½ wavelength (7.5×10^5 m) from the merging black holes, the GW power of 3.6×10^{49} w achieves intensity of about $I \approx 5 \times 10^{36}$ w/m^2. The GW is propagating at the speed of light so this intensity converts to energy density of 1.7×10^{28} J/m^3.

Therefore, what is the physical model of spacetime which permits it to possess energy density of 10^{28} J/m^3 in the form of GWs? If the explanation is that a propagating GW is just "curved spacetime" or a high density of gravitons, then there is no physical model that can be analyzed. However, a GW has amplitude, frequency, intensity, momentum, propagation speed and encounters impedance. These are all properties we normally associate with a sound wave propagating in a physical acoustic medium. We will test the possibility that a GW is analogous to a sound wave propagating in an acoustic medium.

Quantum field theory has been telling us that spacetime is filled with zero-point energy (VE), so perhaps this serves as a propagation medium for GWs. The first test will be to solve for the density ρ of the acoustic medium required to propagate the observed properties of GW150914. The acoustic equation that will be used is Eq. (4). In this equation c_a is the acoustic speed of sound and δ the particle's displacement amplitude with units of length. Eq. (5)

rearranges the terms in Eq. (4) to yield the equivalent density ρ encountered by GW150914. We will then substitute the following experimentally determined numbers: $I = 0.02$ w/m^2; $\delta = \Delta L = 3 \times 10^{-16}$ m, $\omega = 1250$ s^{-1}; and $c_a = c$. Eq. (6) converts this to energy density.

$$I = \delta^2 \omega^2 (\rho c_a) \quad (4)$$

$$\rho = \frac{I}{\omega^2 \, \delta^2 c_a} = 4.7 \times 10^{14} \text{ kg/m}^3 \quad (5)$$

$$U = \rho c^2 = \frac{I \, c}{\omega^2 \, \delta^2} = 4.2 \times 10^{31} \text{ J/m}^3 \quad (6)$$

Therefore, GW150914 encountered a medium with equivalent density of $\rho_v = 4.7 \times 10^{14}$ kg/m^3 which is about 10^{10} times the density of osmium. This density converts to vacuum energy density of $U = 4.2 \times 10^{31}$ J/m^3. Is this reasonable? We previously calculated that the 3.6×10^{49} watts emitted by the merging black holes had energy density of 1.7×10^{28} J/m^3 at a distance of ½ wavelength from these black holes. Energy density of 1.7×10^{28} J/m^3 is a reasonable factor of about 2500 less than the energy density of the propagation medium obtained from Eq. (6) at 200 Hz.

We have just calculated the energy density of the propagation medium at a specific angular frequency of $\omega = 1250$ s^{-1}. Next, we will take Eq. (2) from general relativity and calculate the energy density ($U = I/c$) encountered by a wave in spacetime with arbitrary angular frequency ω. We will also set the constant $1/16\pi = k$ to make the equation applicable to other types of waves in spacetime discussed later. The substitution $A_s = 1$ is discussed below.

$$U_V = \frac{1}{16\pi} \frac{A_S^2 \omega^2}{c} \frac{c^3}{G} = \frac{k \, 1^2 \omega^2 c^2}{G} = k \frac{\omega^2 c^2}{G}$$
$$= k \left(\frac{\omega}{\omega_p}\right)^2 U_p = k \left(\omega T_p\right)^2 U_p = k \left(\frac{L_p}{\lambdabar}\right)^2 U_p \quad (7)$$

$$\rho_V = \frac{U_V}{c^2} = k \frac{\omega^2}{G} \quad (8)$$

Eq. (7) contains several equalities which include Planck angular frequency $\omega_p = \sqrt{c^5/\hbar G}$, Planck length $L_p = \sqrt{\hbar G/c^3}$; Planck time $T_p = \sqrt{\hbar G/c^5}$ and Planck energy density $U_p = c^7/\hbar G^2 \approx 10^{113}$ J/m^3. The portion of Eq. (7) containing U_p will be discussed later. For now, the portion of the equation being addressed is: $U_V = k\omega^2 c^2/G$. The key substitution to get this is $A_s = 1$ and this requires some explanation. In Eq. (7) we are calculating the energy density of the propagation medium encountered by a spacetime wave with frequency ω rather than the energy density of a particular wave propagating in the medium. Therefore, the substitution for the strain amplitude term (A_s) must be for the maximum possible strain amplitude which is $A_s = 1$. This strain amplitude results in 100% modulation of the medium which happens when the displacement of spacetime (ΔL) equals the reduced wavelength ($\Delta L = \lambdabar$) and achieves $A_s = \Delta L/\lambdabar = \lambdabar/\lambdabar$.

We can check this reasoning by using Eq. (7) to calculate the energy density of the medium encountered by GW150914. When we substitute $\omega = 1250$ s^{-1} and $k = 1/16\pi$ into Eq. (7) the answer is $U = 4.2 \times 10^{31}$ J/m^3 which matches Eq. (6). Therefore, the substitution of $A_s = 1$ is justified. If we substitute Planck angular frequency $\omega_p = \sqrt{c^5/\hbar G}$ into $U_V = k\omega^2 c^2/G$ we get k times Planck energy density $k(c^7/\hbar G^2) \approx 10^{113}$ J/m^3.

Discussion

The initial reaction to the high energy density of the vacuum implied by Eq, (7) is that this is in conflict with general relativity. However, there is no conflict with general relativity. Eq. (7) is an extension of Eq. (1, 2) which are GW equations from general relativity. Therefore, it is actually general relativity, not quantum field theory that generated Eq. (7). The logical question is: How is it possible for general relativity to generate both the critical energy density of the universe (10^{-9} J/m^3) and energy density which exceeds the critical energy density by more than 10^{40} times? The following discussion section attempts to answer this question. However, the short answer is that these are two different types of energy. It is a testament to the universality of general relativity that it is capable of generating both answers. The critical energy density calculation addresses energy which possess spin and generates gravity. In Eq. (7) we asked for the energy density of spacetime encountered by GWs. This is a different form of energy than the energy in fermions and bosons which possess spin.

Modeling vacuum energy

So far, this article has described the mechanical properties of VE (energy density, impedance, etc.). This was a logical extension of treating the "fabric of space" encountered by GWs as an acoustic medium and calculating its properties. The calculations so far have been based on GW and acoustic equations. The deeper questions deal with the underlying physics of VE. What is the physical model of VE? Why does VE not produce gravity and collapse the universe? Then Eq. (7) will be subjected to several tests to show that it represents a fundamental property of spacetime that has physical implications.

The first step in answering these questions requires that a hypothesis be presented that describes the proposed model of VE. Eq. (7) contains hints as to the composition of VE. We see that this equation for VE density contains L_p, T_p and U_p. This can be interpreted as Planck length and Planck time are key to achieving Planck energy density. We also know that the laws of physics do not permit distance to be measured to an accuracy of Planck length [21 - 25] and time cannot be measured to an accuracy of Planck time [22, 23]. These references show that this is a fundamental limitation that is device independent. Therefore, the same way that the uncertainty principle allows unmeasurable energy fluctuations in the vacuum, so also unmeasurable Planck length and Planck time fluctuations should be occurring in the vacuum. John Archibald Wheeler discussed "field fluctuation energy in the vacuum" as "locally violent conditions that characterize the vacuum." [14]. There is a proposed connection between not being able to make measurements more accurate than Planck length (L_p) and the vacuum having "field fluctuation energy". The proposed model of VE is that spacetime is a sea of vacuum fluctuations which modulate distance between points by Planck length. This modulation of distance would be the background "noise" of the vacuum and explains both the probability characteristics of quantum mechanics and the inability to make distance measurements more accurate than Planck length. When a Planck length vacuum fluctuation occurs, it is distributed at the speed of light and strains a volume of spacetime with radius r much larger than Planck length. The resulting temporary strain amplitude in this volume is $A_s = L_p/r$. There is also a temporal modulation of the rate of time such that a hypothetical perfect point clock would speed up and slow down by Planck time (T_p). This sets a T_p limit to the accuracy of a time measurement. Both the spatial and temporal modulations occur predominantly at Planck angular frequency (ω_p). Lower frequency vacuum fluctuations are also present, but these will be discussed later. Introducing a ± Planck length vacuum fluctuation into a volume of spacetime expands and contracts a volume by Planck length. Adjacent volumes have opposite effects. If one volume expands by Planck length, the adjacent volume contracts by Planck length. There is a similar effect on the rate of time. A volume which has spatially expanded has a slower rate of time and a volume which has spatially contracted has a faster rate of time. The magnitude of the effect on the 3 space dimensions and 1 time dimension are such that the 4-dimensional volume of spacetime (space + time volume) remains constant.

The model of VE that will be tested a sea of closely packed harmonic oscillators producing L_p and T_p modulation at approximately Planck frequency. The radius of each oscillator is fluctuating, but for analysis we can assume a spherical volume with a Planck length radius ($r = c/\omega_\text{p} = L_p$). This is the foundation of zero-point energy, so each harmonic oscillator has an mathematical volume of $V_{zp} = (4\pi/3)L_p{}^3$ and energy of $E_{zp} = \tfrac{1}{2}\hbar\omega_\text{p}$. The energy density of such a volume will be designated U_Z.

$$U_Z = \left(\frac{1}{2}\hbar\omega_p\right)\left(\frac{3}{4\pi}\right)\frac{1}{L_p^3} = \frac{3}{8\pi}\frac{c^7}{\hbar G^2} = k_1 U_p$$
$$= 5.5 \times 10^{112}\ \text{J/m}^3 \qquad (9)$$

In Eq. (9) we use $U_p = c^7/\hbar G^2$ for Planck energy density. We also define $k_1 \equiv 3/8\pi$. Recall that the numerical constant associated with GWs was $k = 1/16\pi$. A GW is a transverse quadrupole wave that apparently does not couple to the full VE density. Eq. (9) generated $k_1 = 3/8\pi$ which is a factor of 6 larger than the GW constant $1/16\pi$.

Gravitation properties of vacuum energy model

There is another important part of this model which results in VE not producing its own gravity. Since the vacuum fluctuations both increased and decreased radius distance, this means that the distortion (curvature) of spacetime being produced both increases and decreases volume. The rate of time also fluctuates by Planck time. Another way of saying this is that the oscillation is between positive and negative curvature. When the volume increases relative to Euclidian geometry, the rate of time decreases. This is analogous to the positive curvature of spacetime produced by gravity. When the opposite happens (decreased volume and increased rate of time) this is analogous to negative curvature or antigravity curvature. There is no matter with antigravity properties, but if there was an antigravity body, the surrounding spacetime would have increased rate of time and decreased volume compared to a distant zero gravity volume. A triangle drawn around a hypothetical antigravity mass would have angles which totaled less than 180 degrees.

Adjacent fluctuating volumes of spacetime are out of phase. A vacuum fluctuation which increases the volume of one region, decreases the volume of an adjacent region. The Planck frequency oscillation is between equal parts positive and negative curvature which can also be stated as equal parts of gravity and antigravity components. Therefore, the gravitational effects cancel and the proposed model of VE is a form of energy which does not produce gravity.

In this model, the distinguishing feature between energy which generates gravity (fermions and bosons) and energy which does not generate gravity (VE) is the presence or absence of quantized angular momentum. This leads to models of fermions and gravity which is beyond the scope of this article. Next we will subject the VE model to tests.

Five tests of the vacuum energy model

Test #1: Stiffness of spacetime: It is well established that GWs encounter spacetime as a very stiff elastic medium. Would a GW propagating through this model of VE interact with this model of VE in a way that energy is exchanged causing the apparent stiffness? The GW has a specific frequency which is far below Planck frequency. The GW will slightly distort these harmonic oscillator volumes and slightly modulate (increase and decrease) the Planck frequency oscillation. This is analogous to the GW introducing redshifts or blue shifts on the laser beams of the LIGO experiment. The GW is redshifting and blue shifting a part of the VE harmonic oscillators with energy density $U_Z = k_1 U_p$ as specified in Eq. (9). Eq. (7) gives an insight into this interaction. The harmonic oscillators are Planck frequency, therefore the much lower frequency GWs experience impedance mismatch. There is a frequency dependent coupling constant of $(\omega/\omega_p)^2$ which can also be expressed as $(T_p\omega)^2$ or $(L_p/\bar{\lambda})^2$. If there was a wave in spacetime with Planck frequency, the coupling constant would be equal to 1 and that wave would experience the full energy density of $U_Z = 5 \times 10^{112}$ J/m^3. Therefore, it is possible to conceptually understand the stiffness of spacetime encountered by GWs.

Test #2: Black hole energy density: Black holes represent the maximum distortion of spacetime for a given radius. Eq. (7) was obtained by assuming the maximum strain of VE ($A_s = 1$) for a given wavelength or frequency. If VE gives spacetime its properties, then maximum distortion of spacetime and maximum strain of VE should be connected. Therefore, we will test whether the energy density of a black hole and the wavelength dependent energy density of VE described by Eq. (7) are related. A black hole with mass m has energy of mc^2 and Schwarzschild radius of $r_s = 2Gm/c^2$. The volume of a black hole, as perceived from the outside, is $V_{bh} = (4\pi/3)r_s^3$. The energy density of a black hole U_{bh} is:

$$U_{bh} = \frac{mc^2}{V_{bh}} = \frac{r_s c^2}{2G}\frac{3c^2}{4\pi r_s^3} = k_1 \frac{c^4}{r_s^2 G} = k_1 \left(\frac{L_p}{r_s}\right)^2 U_p \quad (10)$$

One of the equalities in Eq. (7) was $U_V = k\left(L_p/\bar{\lambda}\right)^2 U_p$. Therefore, the energy density of a black hole exactly matches the VE density U_V of Eq. (7) when $r_s = \bar{\lambda}$ and $k = k_1$. It is true that the space near a black hole is also highly distorted (curved) but the Schwarzschild radius defines the condition of $A_s = 1$. Black holes are the domain of general relativity, but the use of T_p and L_p, in Eq. (7 and 10) appears to be bridging the gap between quantum mechanics and general relativity. This is a successful test supporting VE as the physical basis of spacetime.

Test #3: Impedance comparison: It is informative to compare the impedance encountered by a GW to the impedance encountered by a sound wave. However, there is a problem because there are two different ways of expressing impedance with different units. GWs express amplitude as dimensionless strain (slope), and the impedance of spacetime ($Z_s = c^3/G$) has units of kg/s. Sound waves usually use displacement amplitude with units of length (meters) and impedance with units of kg/m^{2}s. When the impedance of spacetime (Z_s) is converted to a form compatible with amplitude expressed in meters, the impedance conversion is: $Z = Z_s/ƛ^2 = c\omega^2/G$ with units of kg/m^{2}s. Therefore, this is the spacetime impedance that must be used to make a comparison to acoustic impedance.

The largest acoustic impedance is osmium with specific impedance $z_o = \rho c_a = 1.1 \times 10^8$ kg/m^{2}s where ρ is density and c_a is the acoustic speed of sound. A direct comparison to the impedance encountered by GWs can only be made using $Z = c\omega^2/G$ at a specific frequency. For example, a 200 Hz GW ($\omega = 1250$ s^{-1}) would encounter spacetime as having impedance of 7×10^{24} kg/m^{2}s. This enormous impedance is about 10^{17} times greater than the impedance of osmium at 200 Hz. The Compton frequency of an electron (7.8×10^{20} s^{-1}) would encounter spacetime as having impedance about 10^{52} times greater than the impedance of osmium because of the ω^2 term. This is another test implying that GWs encounter VE with a large energy density.

Test #4: Critical energy density: This test will show that Eq. (7) can generate both the VE density which agrees with quantum field theory and the critical energy density of the universe from the Friedmann equation of general relativity. Substituting $\omega = \omega_p = \sqrt{c^5/\hbar G}$ and $k = k_1 = 3/8\pi$ into Eq. (7) gives $U = k_1 U_p = 5.5 \times 10^{112}$ J/m^3. This is the full VE density obtained from the highest possible angular frequency in the universe.

The opposite extreme energy density of the universe (about 10^{-9} J/m^3) should be associated with the lowest possible angular frequency in the universe (designated ω_u). The lowest angular frequency in the universe would be the inverse of the age of the universe expressed in seconds ($\omega_u = 1/t_u$ where t_u is the age of the universe). The expansion of the universe has analogies to the start of an expansion wave with angular frequency $\omega_u = 1/t_u$. The actual age of the universe is about 13.8 billion years old, but this number incorporates nonlinear expansion rates over the age of the universe. To make a connection to the current properties of the universe we need to use the age of the universe implied by the current expansion rate given by the Hubble constant H_o. The best measurement of the current value of H_o is from an analysis of data generated by the Hubble Space Telescope [26]. The value is $H_o = 73.24$ km/s/Mpc which converts to $H_o = 2.37 \times 10^{-18}$ s^{-1} in SI units. Using this value of H_o, the implied age of the universe is $t_u = 1/H_o \approx 4.21 \times 10^{17}$ seconds = 13.4 billion years. This differs slightly from the 13.8 billion year age of the universe because this value is a measurement of the current expansion rate of the universe and excludes past nonlinear expansion rates. Therefore, the calculation will use $\omega = \omega_u = H_o$. The other substitution into Eq. (7) is: $k = k_1 = 3/8\pi$.

$$U = k\frac{\omega^2 c^2}{G} = \frac{k_1 \omega_u^2 c^2}{G} = \frac{k_1 H_o^2 c^2}{G} = 9 \times 10^{-10} \text{ J/m}^3 = U_c \tag{11}$$

$$\rho_c = \frac{U_i}{c^2} = \frac{k_1 H_o^2}{G} = \frac{3H_o^2}{8\pi G} \tag{12}$$

Eq. (12) yields the Friedmann equation for the "critical density of the universe" obtained from the of general relativity [6]. Therefore, this is another successful test. Eq. (11) shows that substituting the angular frequency of the expanding universe ($\omega = \omega_u = H_o$) and $k = k_1$ into Eq. (7) generates 9×10^{-10} J/m^3 which is the exact critical energy of the universe assuming $H_o = 2.37 \times 10^{-18}$ s^{-1}. Also, the zero point VE density of the universe (5.5×10^{112} J/m^3) is generated when $\omega = \omega_p$ and $k = k_1$ is substituted into Eq. (7). The only difference between these two extremes of energy density is the substitution of $\omega = \omega_p$ and $\omega = H_o$. Both of these terms are squared in their respective equations. Therefore, the relationship between the critical energy density of the universe U_c and zero point VE density ($U_Z = k_1 U_p$) is succinctly stated in Eq. (13). This equation also incorporates the Hubble radius of the universe which is $r_h \equiv c/H_o \approx 13.4$ billion light years.

$$\frac{U_Z}{U_c} = \frac{\omega_p^2}{H_o^2} = \frac{1}{T_p^2 H_o^2} = \frac{r_h^2}{L_p^2} = 7 \text{ x } 10^{121} \tag{13}$$

$$U_c = \left(T_p H_o\right)^2 U_Z = \left(L_p / r_h\right)^2 U_Z \tag{14}$$

The ratio of the two vastly different energy densities $U_Z/U_c = 7 \text{ x } 10^{121}$ at the heart of the cosmological constant problem can be expressed very simply as a combination of a quantum mechanical term (ω_p, T_p, or L_p) and a cosmological term (H_o, or r_h). This is reasonable since the cosmological constant problem compares energy density from both branches of physics. Eq. (13 and 14) show that even on the cosmological scale, there is a connection to Planck length and Planck time.

Test #5: Virtual particles: The final test for the proposed model of VE is whether it can give a reasonable explanation to the generation and annihilation of virtual particles. This is a key part of quantum field theory. Quantum electrodynamics and quantum chromodynamics quantify the effects of this process with exquisite accuracy. Something is physically happening in the vacuum but we lack a conceptually understandable model of the underlying physics that generates virtual particles.

It is proposed that Planck length vacuum fluctuation, combined with the properties of VE, creates the virtual particle characteristics described by quantum electrodynamics and quantum chromodynamics. To explain this, we start with the Planck length vacuum fluctuation hypothesis previously described and add two points. 1) Fundamental particles have wave properties at the particle's Compton angular frequency (ω_c) and 2) A Planck length vacuum fluctuation that lasts for a time period of $1/\omega_c$ achieves the energy required of a virtual particle. The expansion of these two points follows.

First: Moving fundamental particles exhibit de Broglie waves with wavelength $\lambda_d = h/mv$ and phase velocity $w_d = c^2/v$. This combination implies an underlying frequency generating these waves which can be calculated from: $w_d/\lambda_d = mc^2/h = \omega_c/2\pi$ where ω_c is the fundamental particle's Compton angular frequency. This is the frequency interacting with VE. The connection between the particle's Compton angular frequency and its de Broglie frequency has been analyzed [27] in more detail.

Second: Inserting a fundamental particle's Compton angular frequency into Eq. (7) gives the VE density encountered by the particle. For example, an electron has $\omega_c = 7.8 \text{ x } 10^{20}$ s^{-1}, therefore this frequency encounters $U_V = k_1 \omega_c^2 c^2/G \approx 10^{68}$ J/m^3. This is such a large energy density that even a Planck length stretch or compression of a spherical volume with radius $r = c/\omega_c = \lambdabar_c$ for a time period of $1/\omega_c$ will represent a substantial amount of energy. Next, we will calculate this energy and show it equals the energy of the corresponding virtual particle.

Introducing a Planck length distortion into a volume of VE can significantly affect (strain) a volume with dimensions much larger than Planck length. For example, introducing a Planck length vacuum fluctuation for a time of $1/\omega_c$ would be distributed over a spherical volume with radius of $r = c/\omega_c = \lambdabar_c$. An electron's reduced Compton wavelength is $\lambdabar_c = 3.86 \text{ x } 10^{-13}$ m. This is the radius of the spherical volume of VE affected by a Planck length vacuum fluctuation that lasts for a time period of $1/\omega_c$. Stretching or compressing VE by Planck length L_p over a distance of $\lambdabar_c$ for this time period introduces strain with amplitude of $A_s = L_p/\lambdabar_c = 3.86 \text{ x } 10^{-23}$. Using Eq. (3), we can calculate how much energy this Planck length vacuum fluctuation has temporarily introduced to a virtual particle spherical volume $V_{vp} = (4\pi/3)\,\lambdabar_c^3$. Eq. (15) will also use $k_1 = 3/8\pi$ and $\omega_c = c/\lambdabar_c$.

$$E_{vp} = U_v V_{vp} = \frac{k_1 A_s^2 \omega_c^2 Z_s V_{vp}}{c} = \frac{3}{8\pi} \frac{L_p^2}{\lambdabar_c^2} \frac{c^2}{\lambdabar_c^2} \frac{c^3}{G} \frac{4\pi \lambdabar_c^3}{3c}$$
$$= \frac{1}{2} \hbar \omega_c \tag{15}$$

$$\Delta E \Delta \omega_c^{-1} = \frac{1}{2} \hbar \tag{16}$$

This is another successful test. Eq. (15) has generated the equation for the energy of a virtual particle ($E_v = ½\, \hbar\omega_c$) such as a virtual electron. Also Eq. (16) has rearranged the terms in Eq. (15) to the form of the uncertainty principle $\Delta E \Delta T = ½\, \hbar$ where $\Delta\omega_c^{-1} = \Delta T$. Therefore, Eq. (15, 16) are important because they show how a Planck length vacuum fluctuation can generate zero-point energy, the uncertainty principle and the energy of a virtual particle such as a virtual electron. For an electron, $\omega_c = 7.76 \times 10^{20}$ s^{-1} so $E_v = \frac{1}{2} \hbar\omega_c = 4.1 \times 10^{-14}$ J. A Planck length vacuum fluctuation of this duration affects a spherical volume with radius $r = ƛ_c = c/\omega_c = 3.86 \times 10^{-13}$ m. This volume would contain about 10^{67} of the Planck frequency harmonic oscillators previously described. To create the energy of a virtual electron, all that must happen is for the 10^{67} harmonic oscillators in this volume to interact in such a way that a single Planck length strain of spacetime extends over a volume with radius $ƛ_c$.

The energy of any virtual particle in the standard model can be generated this way. For example, a Planck length vacuum fluctuation that lasts for 9×10^{-27} second encounters interactive energy density of about 10^{79} J/m^3 and momentarily generates the energy of a virtual top quark.

Since only a few frequencies correspond to the Compton frequency of fundamental particles, why are these frequencies special? It is proposed that these few frequencies correspond to resonances in VE. A resonance of any kind occurs when energy is fed back to an oscillation, thereby reducing or eliminating energy loss. It was previously proposed that VE is predominantly a vacuum fluctuation at Planck frequency. Lower frequencies also occur as a lower frequency beat of these higher frequency components. The favored Compton frequencies achieve a resonance and other not resonant frequencies are minimized.

The missing component of the universe

The standard model is a field theory that has 17 named particles which are considered to be "excitations" of their respective fields [28]. For example, an electron is an excitation of the electron field and the Higgs boson is an excitation of the Higgs field. Therefore, the standard model implies that space is filled with many overlapping fields. The proposed model of VE gives a physical structure to these fields. However, rather than many overlapping fields in spacetime, it is proposed that there is only 1 universal field – which will be called the "spacetime field". This is another name for VE. The multiple discrete fields of the standard model are proposed to be unified into a single spacetime field with a fundamental frequency of ω_p and multiple resonances at frequencies corresponding to the Compton frequencies of fundamental particles. There is a more complete development of this idea including a particle model that generates forces [27].

Einstein intuitively knew there was a physical component of space. From 1916 until his death he used the terms: "relativistic ether", "physical space" and "total field" to express this concept. [29] Here are three representative quotes. In 1934 he said "Physical space and the ether are different terms for the same thing; fields are physical states of space". [30] "There is no such thing as an empty space, i.e., a space without field. Space-time does not claim existence on its own, but only as a structural quality of the field" (1954) [31]. "According to general relativity, the concept of space detached from any physical content does not exist." (1950) [32].

Today, most physicists hold the opposite view and believe space has no "physical content". However, it is proposed that failure to recognize the physical presence of VE ignores the largest component of the universe and removes a key element required to explain the cause of many of the laws of physics. An analogy would be a fish that lives at the bottom of the ocean but the fish fails to recognize the existence of water. This hypothetical fish would be able to designate laws of physics applicable to its world, but the underlying cause of these laws would be a mystery. For example, to this fish a bubble would be a mysterious particle with properties which can be mathematically described but not conceptually understood. Similarly, an electron appears to us to be a mysterious particle with zero volume but somehow possess energy, spin, charge, gravity, wave properties and probabilistic characteristics. To make progress in conceptually understanding how an electron acquires these properties, it is necessary to realize that an electron has a Compton frequency ($\omega_c = 7.8 \times 10^{20}$ s^{-1}) which is interacting with VE.

Conclusion

The first detection of a gravitational wave (GW) has important implications beyond cosmology. The experimentally observed characteristics of GW150914 confirm that this GW encountered spacetime as a very stiff elastic medium. The impedance encountered by GWs is obtained from general relativity to be $Z_s = c^3/G \approx 10^{35}$ kg/s. This enormous impedance can be interpreted as implying GWs encounter a vastly larger energy density than the critical energy density of the universe (10^{-9} J/m^3) obtained from cosmology.

The observational data from the GW designated GW150914 was analyzed by treating this GW like an acoustic wave propagating in a medium. The observed amplitude, frequency, intensity and propagation speed permits the energy density of the propagation medium to be calculated. The result implies that the 200 Hz portion of GW150914 was encountering spacetime as a propagation medium with energy density of 4.2×10^{31} J/m^3. This is about 10 billion times the energy density of osmium and 10^{40} times larger than the critical energy density of the universe. While this seems incompatible with the critical energy density of the universe, it fits with quantum field theory which predicts that the vacuum has a large zero-point energy density. At a frequency of 200 Hz a GW should be coupling into a portion of this VE density.

The energy density of the vacuum that would be encountered by GWs at other frequencies has been calculated. This energy density encountered by a wave in spacetime scales with ω^2 and reaches Planck energy density if extrapolated to Planck frequency. The ω^2 term is proposed to be due to impedance mismatch caused by the frequency difference when a GW interacts with Planck frequency vacuum fluctuations.

A model of VE has been proposed that is consistent with the calculated properties of VE. In this model, spacetime is a sea of Planck length and Planck time vacuum fluctuations associated with the uncertainty principle. These Planck length vacuum fluctuations oscillate between positive and negative curvature of spacetime (gravity and antigravity curvature). This would cancel all the gravitational effects of the vacuum fluctuations and explain how the vacuum can have large energy density without causing gravitational collapse.

The famous cosmological constant problem is a 10^{120} discrepancy between the critical energy density of the universe (10^{-9} J/m^3) confirmed by observation and general relativity compared to the zero-point energy density of the vacuum (10^{112} J/m^3) derived from quantum field theory. This discrepancy is one of the major mysteries in physics. The conclusion of this analysis is that if GWs are treated like sound waves propagating in a physical medium, then the vacuum of spacetime appears to have the large energy density predicted by quantum field theory. In other words, gravitational waves encounter vacuum energy.

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