# On a model of the electron and the other leptons

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**Abstract:** The central idea for a model of the electron and the other leptons is presented. In this model, the electron and the charged leptons are coreless spiral electric field vortices derived from the Lituus spiral. The central region of the vortices is free of field. Such a vortex structure makes the lepton electric field self-supporting (no dependence on a central charge). The electric field configuration at large distances is the same for all charged leptons, assigning them the same charge. Their mass and stability would depend on their winding numbers. Some considerations are presented on a structure of the neutrinos and the massive gauge bosons. In this model, neutrinos turn out to be different from their corresponding antineutrinos (Dirac neutrinos). The above spiral structure could imply nonzero spin, and might thus explain the observed scarcity (or absence) of scalar elementary particles. In addition, it will mean that the heavier leptons and the massive gauge bosons decay through vortex fission; therefore the muon decay to an electron and a photon would not exist. It could also help to give some insight into some branching ratios in the massive gauge boson decays, which are presently determined only experimentally. © *2012 Physics Essays Publication*. [DOI: 10.4006/0836-1398-25.4.547]

**Résumé:** L'idée centrale d'un modèle concernant l'électron et les autres leptons est présentée. Dans ce modèle, l'électron et les leptons chargés sont des vortex sans noyau d'un champ électrique spiral dérivé de la spirale lituus. La région centrale des vortex est sans champ. Une telle structure de vortex permet d'avoir un champ électrique leptonique *self-supporting* (pas de dépendance à une charge centrale). La configuration du champ électrique à grande distance est la même pour tous les leptons chargés, en leur attribuant la même charge. Leur masse et stabilité dépendraient de leur nombre de tours. Quelques considérations sont présentées concernant une structure des neutrinos et des bosons de jauge massifs. Dans ce modèle, les neutrinos se révèlent être différents de leurs antineutrinos (neutrinos de Dirac). La structure en spirale ci-dessus pourrait impliquer un spin différent de zéro et pourrait ainsi expliquer la rareté observée (ou l'absence) de particules scalaires élémentaire. De plus, cela signifierait que les leptons lourds et les bosons massifs de jauge se désintègreraient à travers une fission de vortex. Ainsi, la désintégration du muon en un électron et un photon n'existerait pas. Cela pourrait également contribuer à donner un aperçu des rapports d'embranchement intervenant dans les désintégrations des bosons massifs de jauge, qui ne sont actuellement déterminés que de manière expérimentale.

Key words: Vortices; Vortex Fission; Electron Model; Lepton Structure; Lepton Masses; Scalar Elementary Particles.

## I. INTRODUCTION

According to our present understanding, leptons and quarks are pointlike particles. The term *pointlike* bears in itself a hint that leptons and quarks may be extended. In fact, search for quark and lepton substructure is an active field of research even in the most modern particle accelerators. Furthermore, in string theory elementary particles are extended. In this situation, it is not surprising that models of the (extended) electron are still being constructed.[1–5]

The main problem of electron models is the difficulty in identifying the mechanism that keeps the charge together. This mechanism is usually given by an ad hoc hypothesis. In the present work an attempt is made to address this problem in a new way: We avoid considering the charge as confined to the core of the particle.

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## II. ON THE STRUCTURE OF THE CHARGED LEPTONS

We start by considering that for an elementary particle the basic concept is the field and not the particle.[6–10] Further, we keep in mind that if we want to include the heavier leptons, we need a model that gives the same charge but larger energy.

To find what a field configuration should look like in order to describe a charged particle at very small radii, we believe that one has to look at a process that generates the particle, and therefore we are guided by the pair photoproduction reaction $\gamma p \rightarrow pe^{+}e^{-}$. In this reaction, close to threshold, the resulting final-state electric field consists of the two familiar static radial electric field configurations of the electron and the positron.

In superfluidity, according to experimental observations and numerical simulations with the use of the

 

Gross–Pitaevski equation, pairs of vortices are created on an obstacle.[11,12] In analogy to this, particle pairs may be produced in photoproduction as follows: When the photon approaches the proton, this obstacle induces a turbulence and forces the electromagnetic field to stop oscillating and to transform itself to a vortex–antivortex pair (the $e^+e^-$ system), with two (macroscopically) radial electric fields. Thus, the proton, besides fulfilling conservation laws, has an active role in the process of pair photoproduction. Vortex-pair creation occurs in superconductivity too, and some analogies between superfluidity and electrodynamics exist, with vortices playing the role of charged particles and phonons the role of photons.[13] In our case, the result may be two stable spiral vortices, as this is happening in vortex–antivortex pair production.[14]

Such spiral fields should tend in the large radius region to coincide with the electron radial field, and in the very small radius region to approach the point at $r = 0$. Looking for a spiral with such characteristics, we found the Lituus spiral (Fig. 1),

$$r^2\theta = K^2, \tag{1}$$

a spiral that was studied in 1704 by Roger Cotes. The Lituus spiral has two very interesting features:

(a) It has, as asymptotes, lines going through $r = 0$, as required if the spiral is to reproduce the radial electron field at large distances.
(b) As $\theta$ increases, the curve approaches the pole without ever reaching it.[15]

From point b it follows that if the electron would be described as a Lituus-spiral vortex (comprised of many Lituus spirals, rotated relative to each other in order to reproduce the radial field of the electron at large distances), this vortex would be coreless, so that the central region would be void of an electric field. It is interesting to point out that the same holds in the Dirac extensible electron model and in several other models. Furthermore, the electron radius would not be zero, helping to avoid energy infinities. The electron radius would obtain a new meaning, and the term *electron cavity radius* may be more appropriate. The positive and negative charges would be distinguished by the direction of the electric field toward the vortex core or away from it.

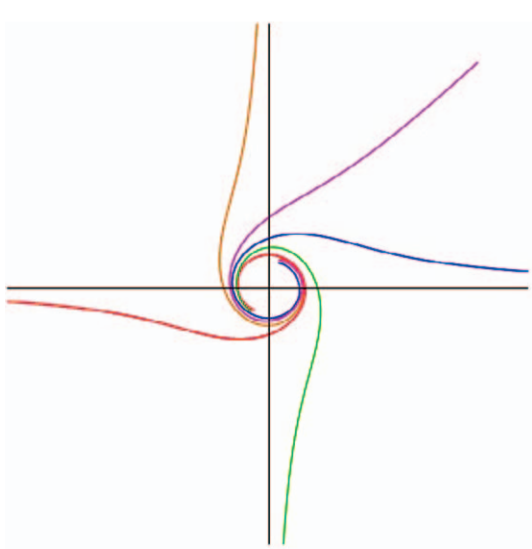

FIG. 1. (Color online) The Lituus spiral vortex (a charged lepton) in two dimensions. A discussion on the 3D case is given at the end of Sections III and IV.

## III. THE SPIRAL FIELD ENERGY AND THE LEPTON MASSES

In what follows, a calculation of the energy of the Lituus spiral electric field in two dimensions is presented. In the laboratory, spiral electric fields can be produced by an electric charge running on a circular trajectory.[16] As the velocity of the charge increases toward $c = 1$, the spiral electric field lines are winding up more and more, as more energy is stored in the field. Bearing in mind that there are analogies between electrodynamics and elasticity, which even inspired J. C. Maxwell to construct a mechanical model of electromagnetic phenomena, we may consider the above spiral electric field as an analogue to a mechanical torsion spring. Therefore, in the calculations that follow, we may expect that the energy $W$ stored in such a spiral field would follow a dependence similar to

$$1/2\kappa\theta^2, \tag{2}$$

which holds for a torsion spring ($\theta$ is the winding angle measured from the equilibrium position). Because in this model we avoid using the charge of the particle, we will calculate the energy $W$ from the field, integrating

$$dW = E\,ds \tag{3}$$

along the Lituus spiral, with $E$ being the field strength, taken to be along $ds$ at each point. Further, we regard the spiral field as having been constructed from a radial field through winding of the field, and assume that the magnitude of the field is not changed during winding, so that it is still proportional to $1/r^2$

$$E = k/4\varepsilon\pi r^2 = k\theta/4\varepsilon\pi K^2, \tag{4}$$

with $k$ a proportionality factor, used instead of charge, because we consider that charge cannot be used in this context. In this model, charge is not concentrated in the (pointlike) central region, but is rather a macroscopic quantity reflecting the density of field lines far away.

Because for the Lituus spiral in the central region the spiral arms are almost circular, we could use the approximation $ds = r\,d\theta$ in that region. Then, integration of (3) from 0 to an angle $\theta$ gives the approximation

$$W = k\theta^{3/2}/4\varepsilon\pi K, \tag{5}$$

which we consider as not being too far from the dependence [Eq. (2)].

The correct expression for $ds$ in polar coordinates is $(ds)^2 = (dr)^2 + r^2(d\theta)^2$, and after some calculations it turns out that integration of (3) means evaluation of the integral $(k/4\varepsilon\pi K)\int\sqrt{1/4\theta^{-1}+\theta\, d\theta}$.

This evaluation was done with the use of the Mathematica Integrator. The result is a complicated expression containing a hypergeometric function. For the central region $\theta > 1$ radians, this expression is given as

$$W = \frac{k}{4\varepsilon\pi K}\left(\frac{2}{3\theta^{3/2}} - \frac{1}{4\theta^{-1/2}} + O\theta^{3/2}\right). \tag{6}$$

From the above analysis, we conclude that in this model the energy stored in the Lituus spiral field in two dimensions (which in three dimensions would give the mass of the electron) is proportional roughly to $\theta^{3/2}$. The masses of the other charged leptons would be larger than the electron mass, corresponding to larger values of $\theta$ (smaller values of radii).

Because this model of the charged leptons is based on the Lituus spiral (of the Archimedean family of spirals) having inherent chirality, the charged leptons would be chiral, as expected from the Standard Model. They could have a spin $\neq 0$, simply because of their structure, rather than because of any spinning or rotating motion, in agreement with quantum mechanics. It has been stated[17–19] that string sources of gravitation would also possess a spin because of their structure.

Stability of (coreless) vortices depends on their winding number.[20] Because the winding number is a measure of how many times the field is wound around the central region, and therefore this number depends on the angle $\theta$, mass and stability of leptons would be interrelated in this model. Stability of the lepton spiral vortices should be investigated, but it has been shown that stable vortex solitons exist.[21–24] In principle, from the present experimental limit of the electron radius one could calculate a limit for the constant $k$ in a 3-D version of (4) and then limits on the heavier lepton radii.

In three dimensions, because of the hairy ball theorem of algebraic topology, the above vortices would not be spherically symmetric but would have two poles and would be axially symmetric. Axially symmetric solutions for the electron have been found long ago in the framework of the Dirac extensible electron model[25] and recently too.[3]

As in other cases,[26] the Ginzburg–Landau equation with complex coefficients or other nonlinear equations could be used to study the spiral vortices proposed here and observe if they are stable. Another possible approach may be through bifurcation theory applied to the pair production reaction mentioned in the beginning. Because quantized vortices exist in other fields (as in superfluidity), it may be possible to quantize such vortices.

## IV. ON THE STRUCTURE OF THE NEUTRINOS AND THE MASSIVE GAUGE BOSONS

The initial aim of this work was the development of a model for the charged leptons, but it turned out later that some statements can/should be made about the structure of the neutrinos and the massive gauge bosons. We think that a successful model for the charged leptons should also consider the structure of those bosons and the neutrinos, because all these particles are involved in the decays of the heavier charged leptons.

Because of their involvement in the lepton decays, we expect that these bosons and the neutrinos have a spiral electric-field structure similar to the charged leptons. In the case of the (neutral) neutrinos, the field cannot be radial and extend to infinity. Thus, for the neutrinos we need vortices that are derived from curves like the Archimedes or the Fermat spirals. Because we believe that we need to chose an associated curve to the Lituus, and the Fermat spiral is the inverse curve[a] of the Lituus spiral, we favor the Fermat spiral as a possible basis for the emitted neutrino vortices. There may exist topological arguments favoring the emission/absorption of energy by spiral electric field vortices in form of the corresponding Inverse-spiral vortices. The Fermat spiral has another favorable characteristic in comparison to the Archimedes spiral: The distance between its successive turnings decreases going outside, thus limiting the neutrino field radius and charge radius.

In the present model, muon decay would take place through vortex fission, so that the emission of at least a second vortex (besides the electron) would be required. Therefore we would not expect the muon to decay to an electron and a photon. Loss of energy would occur mainly through the emission of a fraction of the spiral electric field, in the form of (spiral) neutrinos. This could happen near the points of maximum acceleration, namely, at the beginning and at the end of the unwinding of the spiral field.

Along the same lines of thinking, we make the following considerations regarding a possible structure of the massive gauge bosons: According to the description of muon decay in the present model given above, the W boson is an intermediary state between the muon and the electron, both of the latter being Lituus-spiral vortices. This may indicate that, like these charged leptons, the W boson would have a spiral structure as well. We think that a spiral produced by the unwinding of the Lituus that could correspond to a W boson should again be an associated curve to the Lituus (as is the Fermat spiral chosen above). A good candidate may be the Lituus evolute spiral shown in Fig. 2 (*evolute* comes from the Latin *evolvere,* meaning unroll or unwind). Because this spiral's arm extends far away from the origin, a vortex derived from this spiral would exhibit an electric charge. As mentioned above, at the end of unwinding (decelerating phase), a fraction of the spiral field would be emitted in the form of an antineutrino, forcing the evolute spiral to wind back (involute) to the original Lituus spiral shape (an electron).

If the W were be a spiral vortex, we may expect that the Z boson would also have spiral structure. Because the

[a] The inverse, evolute, and involute curves are defined in differential geometry.

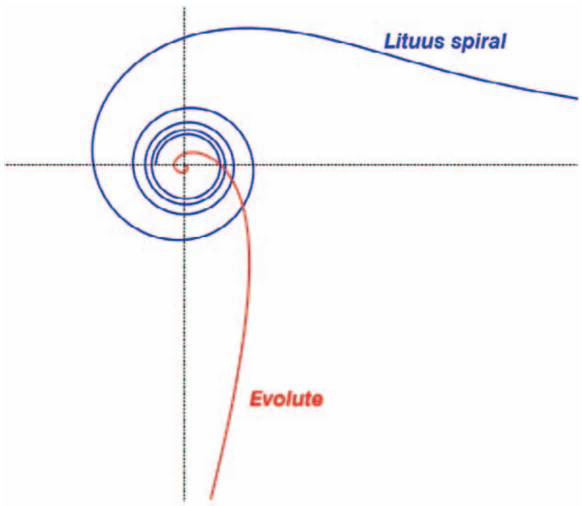


FIG. 2. (Color online) The Lituus and its evolute, which could result from the unwinding of the Lituus and might form a corresponding vortex for the W boson.

Z decays to a neutrino–antineutrino pair, it may have the structure of a double-arm Fermat spiral vortex (Fig. 3). Such a Z could simply split[27] to two single-arm (Fermat) spirals (Fig. 4), which would practically exhibit no charge (neutrinos).

As in the case of the charged leptons above, and because of the hairy ball theorem, in three dimensions, the Z and the neutrino vortices would not be spherically symmetric but would have two poles and would exhibit axial symmetry.

## V. DISCUSSION

With the above structure (Fig. 4), neutrinos would be different from their antiparticles (Dirac neutrinos). The Z would be the same as its antiparticle (as in the Standard Model).

The Z decay to Lituus spiral vortices (charged leptons), would require transformation of vortices and could therefore be expected to be less frequent than the Z-vortex fission to neutrino–antineutrino pairs (which, as said above, is simply a split of the Z in two parts). It should be noted that in the Standard Model, the Z branching ratios to charged/neutral leptons are not resulting from theory, but are determined from experiment. Further, for a W boson with the above spiral-vortex structure, we could expect that it will decay through vortex fission and form the two vortices of a charged particle and a neutrino. For some fraction of these decays a photon could be emitted in addition, in the sudden rearrangement of the field during the vortex fission.

In the model presented here, the large mass difference between the Z and the neutrinos (despite their similar structure), could eventually be understood as follows: Again following the mechanical analogue, the neutrino single-arm spiral vortices, contrary to the Z spiral vortex, are more like a spring without anchor, and thus they cannot store much energy (mass).

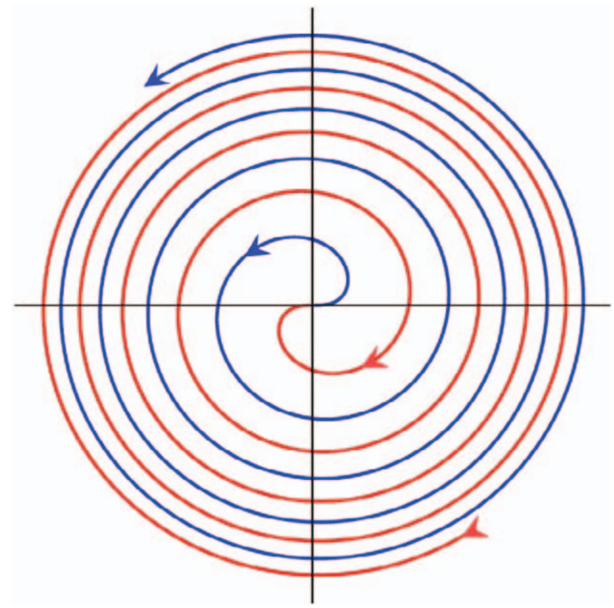

FIG. 3. (Color online) A double-arm Fermat spiral vortex (the Z boson) in two dimensions.

If this model proves to be successful, lepton flavor/number conservation will root on the structure of elementary particles. Further, because leptons and quarks are grouped into generations, a similar model may also be possible for the quarks: As for the leptons, the transformation of quark flavors would take place through the production of a virtual W with spiral structure. This may indicate that the quarks have a spiral structure similar to that of the charged leptons.

In lepton (and quark) compositeness search experiments at very high energies (in the International Linear Collider or eventually in the Large Hadron Collider), an electron or a quark with the structure described in the present model (no field in the central region), could exhibit a different signature as compared to a composite or a point like particle: The winding electric field lines in the central region, show that the electron may have some similarity to a charged shell (with a sharp border on the inner side and a diffuse boundary on the outer side). Therefore the lepton (and quark) form factor, instead of the linear dependence on the squared momentum transfer expected in the case of a charged sphere, may show the exponential dependence present in shell-model form factor calculations for light nuclei, which reproduce the experimental data.[28,29] This could be a test of the present model.

In this model, the radius of the charged lepton is smaller, the heavier the lepton is. This is consistent with the definition of the electron and muon classical radius and with some theoretical calculations,[30,31] although it disagrees with others.[32]

Because of the hairy ball theorem, the elementary particles of this model are axially symmetric and not spherically symmetric. Axially symmetric solutions for the electron have been found in the Dirac extensible electron model and in other models.[3,25]

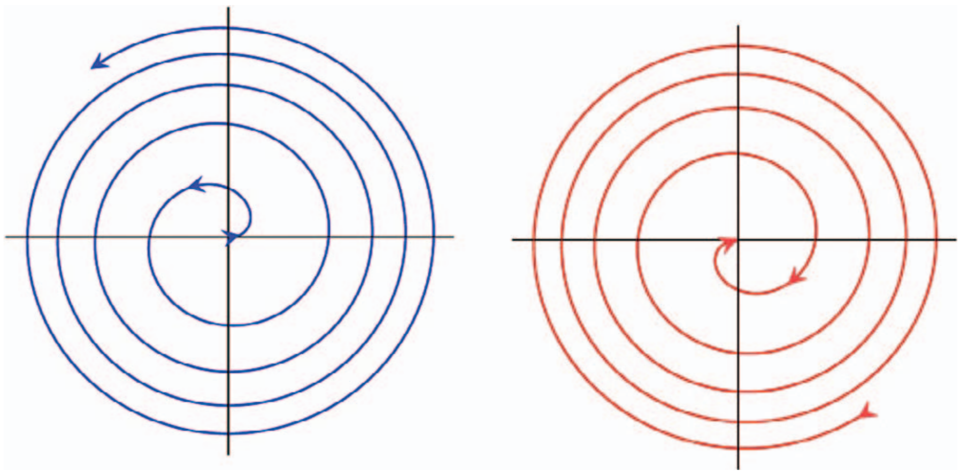

FIG. 4. (Color online) The double-arm Fermat spiral vortex (the Z boson, Fig. 3) could split into the two single-arm vortices shown here (a neutrino and an antineutrino).

As mentioned earlier, the above (spiral) structure of the leptons and quarks and not any motion or spinning of these particles (see also Refs. 17–19), would result in nonzero spin and may thus help us to understand the reason for the observed scarcity (if not absence) of scalar elementary particles in nature.

It may be interesting to note that, whereas in two dimensions (Fig. 1) the spiral field lines are planar curves, in three dimensions because of the hairy ball theorem the field lines are no longer planar and a torsion is introduced to the spiral field.

The medium in which the lepton vortices are created is the quantum vacuum, which can be considered as a fluid medium with friction.[33,34] Quantum vacuum fluctuations are widely believed nowadays to be the source of the Casimir effect, which can provide attractive or repulsive forces depending on the boundary conditions. The boundary conditions need not be due to material boundaries only, but can also be the result of space–time curvature[35] or some background field.[36] Therefore we argue that the spiral electric field in the here-presented model of electrons and other particles can provide the boundary conditions for the existence of Casimir forces which hold the vortices together. In Ref. 36, it is described how a helical "quantum spring" can be formed with the help of the Casimir effect and is stated that this spring behaves like a mechanical spring. Eventually a similar analysis could be applied to the spiral vortices presented here. These vortices have been considered as analogues to mechanical torsion springs already at early stages of this work (Section III).

It could be assumed that the existence of friction in the quantum vacuum fluid may not allow vortex motion for a very long time, questioning the compatibility of this model with the electron stability limits (present lower limit for the electron lifetime given by the Particle Data Group is $4.6 \times 10^{26}$ yr). As said in Section III and in the Abstract, the stability of the vortices is based on the (invariant) winding number and is thus of topological origin, but for the following reasons too we believe that quantum vacuum friction would not represent an obstacle to this model: A body does not experience quantum vacuum friction when it moves with constant velocity,[37,38] or when it has a uniformly accelerated motion.[39] If the present model would successfully describe the electron, the central region of the vortex (where the electric field is strong) would define the electron radius (present limit of the radius is of the order of $10^{-26}$ m). Such a very small radius will help to reduce the effects of the friction on vortex motion during a period of nonuniform acceleration.

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1. A. Davidson and S. Rubin, Classical Quantum Gravity **26**, 235006 (2009).
2. E. P. Likhtman, arXiv:hep-th/0611052.
3. B. Lehnert, Phys. Scr. **T113**, 41 (2004).
4. S. J. G. Gift, Phys. Essays **17**, 117 (2004).
5. G. Rosen, Eur. Lett. **62**, 473 (2003).
6. A. Einstein and N. Rosen, Phys. Rev. **48**, 73 (1935).
7. A. Einstein, J. Franklin Inst. 221, 349 (1936).
8. S. A. Fulling, Phys. Rev. D **7**, 2850 (1973).
9. H. D. Zeh, Phys. Lett. A 309, 329 (2003).
10. H. Nikolic, Eur. Phys. J. C **47**, 525 (2006).
11. T. Frisch, V. Pomeau, and S. Rica, Phys. Rev. Lett. **69**, 1644 (1992).
12. T. W. Neely, Phys. Rev. Lett. **104**, 160401 (2010).
13. P. Fedichev, U. Fischer, and A. Recati, Phys. Rev. A **68**, 011602 (2003).
14. D. M. Whittaker, Superlattices Microstruct. **41**, 297 (2007).
15. G. A. Wentworth, *Elements of Analytic Geometry* (BiblioLife, Charleston, SC, 2010), p. 223.
16. R. Y. Tsien, Am. J. Phys. **40**, 46 (1972).
17. R. Hammond, Gen. Relat. Gravit. **31**, 233 (1999).
18. R. Hammond, Gen. Relat. Gravit. **32**, 347 (2000).
19. R. Hammond, Rep. Prog. Phys. **65**, 599 (2002).

[20]M. Takahashi, V. Pietilä, M. Möttönen, T. Mizushima, and K. Machida, J. Phys. Conf. Ser. **150**, 032103 (2009).
[21]L. C. Crasovan, B. A. Malomed, and D. Mihalache, Phys. Rev. E **63**, 016605 (2000).
[22]H. Leblond, B. A. Malomed, and D. Mihalache, Phys. Rev. A **80**, 033835 (2009).
[23]C. Wong and Y. Tserkovnyak, Phys. Rev. B **80**, 184411 (2009).
[24]K. J. H. Law, D. Song. P. G. Kevrekidis, J. Xu, and Z. Chen, Phys. Rev. A **80**, 063817 (2009).
[25]N. K. Sharma, Nuovo Cimento A **48**, 971 (1967).
[26]O. Tornkvist and E. Schroeder, Phys. Rev. Lett. **78**, 1908 (1997).
[27]G. T. Gurija and M. A. Livshits, Phys. Lett. A **87**, 175 (1983).
[28]H. R. Collard and R. Hofstadter, Phys. Rev. **131**, 416 (1963).
[29]F. C. Khanna, Nucl. Phys. A **165**, 475 (1971).
[30]M. Goldhaber, Proc. Nat. Acad. Sci. U. S. A. **99**, 33 (2002).
[31]G. Koepp, D. Schaile, M. Spira, and P. Zerwas, Z. Phys. C **65**, 545 (1995).
[32]G. Rosen, Int. J. Theor. Phys. **17**, 1 (1978).
[33]M. Kardar and A. Golestanian, Rev. Mod. Phys. **71**, 1233 (1999).
[34]P. C. W. Davies, Chaos **11**, 539 (2001).
[35]A. D. Sakharov, Dokl. Akad. Nauk SSSR **177**, 70 (1967) [Sov. Phys. Dokl. **12**, 1040 (1968)].
[36]C. J. Feng and X. Z. Li, Phys. Lett. B **691**, 167 (2010).
[37]T. G. Philbin and U. Leonhardt, New J. Phys. **11**, 033035 (2009).
[38]N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, United Kingdom, 1984).
[39]M. T. Jaekel and S. Reynaud, Quantum Semiclassical Opt. **7**, 499 (1997).