Vacuum energy connects gravitational waves to quantum field theory

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[1]

Abstract: Gravitational waves (GWs) have some characteristics of acoustic waves. For example, GWs have amplitude, frequency, intensity, propagation speed and encounter spacetime as having a quantifiable impedance. These characteristics permit GWs to be analyzed to obtain the apparent "acoustic" properties of spacetime. The result is that GWs encounter spacetime as if it is an extremely stiff elastic medium with a large energy density. The energy density encountered by GWs scales with frequency squared and equals Planck energy density ($\sim 10^{113}$ J/m^3) at Planck frequency. This matches the vacuum energy density predicted by quantum field theory at this frequency. This finding makes a new contribution to one of the major mysteries of physics known as the cosmological constant problem. An analysis of the GW designated GW150914 is also given as a numerical example. A model of vacuum energy is proposed to be Planck length vacuum fluctuations at Planck frequency.

Keywords: gravitational waves; vacuum energy; cosmological constant problem; impedance of spacetime; Planck length

Abbreviations: GWs = gravitational waves; QFT = quantum field theory; VE = vacuum energy

1. Introduction

There is no single generally accepted model of how gravitational waves (GWs) propagate. One of the leading candidates is that GWs are waves of gravitons propagating at the speed of light ***through*** the empty void of the vacuum. The leading competing model is an extension of the idea that gravity is a geometric effect produced by mass/energy. Therefore, in this competing model, a GW is a curvature ripple propagating ***in*** the medium of spacetime. This model implies that spacetime has quantifiable properties which can be deduced from an analysis of general relativity and GWs.

For example, two books on GWs [1, 2] describe spacetime as an "extremely stiff elastic medium … with impedance of c^3/G." The terms "elastic medium" and "impedance" are normally associated with an acoustic medium. A GW also has wave amplitude, frequency, intensity and a speed of propagation. Again, these are terms normally associate with an acoustic wave propagating in a physical medium. Can GWs reveal properties of spacetime that have previously gone unnoticed? This article will address the following question: Since a GW has similarities to an acoustic wave propagating in a physical medium, what are the properties of spacetime implied by this analogy? This question might sound like heresy to physicists that believe GWs are propagated by gravitons or propagated as a purely geometrical distortion of space. However, it is a valid question which deserves a thoughtful answer.

The impedance of spacetime $Z_s = c^3/G \approx 4 \times 10^{35}$ kg/s was derived from general relativity and is associated with the large coupling coefficient in Einstein's field equation. [1, 2]. This coefficient is Planck force c^4/G divided by 8π. This enormous force ($\approx 5 \times 10^{42}$ N) quantifies the stiffness of spacetime and results in GWs being very difficult to detect. Even a relatively large intensity GW produces an extremely small distortion of space (small wave amplitude).

The finite stiffness of spacetime encountered by GWs seems to indicate that spacetime has physical properties that are interacting with GWs. The standard model has multiple fields existing in space. Do these fields interact with GWs? Also, quantum field theory (QFT) indicates that the vacuum has a large finite energy density associated with zero-point energy [3]. If this energy density exists and interacts with GWs, then this analysis should yield a quantifiable vacuum energy (VE)

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density. If space is an empty void, the analysis should yield answers of either zero or infinity.

A finding of a quantifiable VE density would have implications for the famous debate in physics known as either the "cosmological constant problem" [4, 5] or the "vacuum catastrophe" [6]. This is the approximately 10^{120} discrepancy between the critical density of the universe ($\sim 10^{-26}$ kg/m^3) required to achieve the observed flat space [7, 8] and the enormous vacuum density ($\sim 10^{96}$ kg/m^3 or 10^{113} J/m^3) required to achieve the highly accurate results of quantum electrodynamics and quantum chromodynamics.

2. Conversion of a gravitational wave equation to an acoustic equation

$$I = \frac{\pi}{4}\frac{c^3}{G} f^2 \left(\frac{\Delta L}{L}\right)^2 \tag{1}$$

$$I = \left(\frac{1}{16\pi}\right)\left(\frac{\delta}{\lambda\!\!\!^{-}}\right)^2 \omega^2 \left(\frac{c^3}{G}\right) \tag{2}$$

$$I = \left(\frac{1}{16\pi}\right)\delta^2 \omega^2 \left(\frac{c^3}{G\lambda\!\!\!^{-2}}\right) \tag{3}$$

While equations associated with GWs can be complex, (1) is a very simple equation that gives the intensity (I) of a GW in the limit of a weak plane wave. This equation is derived from general relativity and quoted in standard texts on GWs [1, 2]. In (1), $\Delta L/L$ is the GWs strain amplitude (maximum slope). When interferometers are used to detect GWs, ΔL is interpreted as the measured fringe shift in an interferometer and L is the round-trip path length of the interferometer. Also, f is frequency, c is the speed of light and G is the gravitational constant. If we assume that the interferometer's round-trip path length, L, is less than about 10% of the GW wavelength, then the maximum strain (maximum slope of the sinusoidal GW) is approximated by $h = \Delta L/L$ where $h^2 = h_+^2 + h_\times^2$. The subscripts + and × represent the well-known GW polarizations.

When (1) is adapted for use with interferometers, then fringe shift divided by round trip path length approximates the maximum slope of the sinusoidal GW. Equation (2) is a partial conversion to the form of an acoustic equation. We have converted to angular frequency ω and made the appropriate conversion of the numerical constant term. Most important, the strain amplitude (maximum slope) interferometer approximation $\Delta L/L$ has been replaced with the exact strain amplitude (exact slope) $\delta/\lambda\!\!\!^{-}$ where δ is the magnitude of the maximum displacement produced by the sinusoidal GW over an entire wavelength and lambda bar is: $\lambda\!\!\!^{-} = \lambda/2\pi$ (wavelength/2π). In acoustics, the wave amplitude is usually defined as the maximum particle displacement δ from the center position. A GW does not physically displace the center of mass of an isolated object such as an interferometer mirror suspended by wires. Instead, the space between the mirrors is affected such that the distance between mirrors as measured by a laser beam can change without physically displacing the center of mass of the mirrors. There is no concise wording in English to express this concept, so hereafter we will refer to "displacement amplitude" of a GW and the reader must accommodate this imprecise simplification to imply a distortion of the properties of space.

Equation (2) has also rearranged terms to correspond to the universal equation for intensity: $I = k A^2\omega^2 Z$. In this equation, k is a numerical constant, A is amplitude, ω is angular frequency and Z is the impedance of the acoustic medium. Comparing $I = k A^2\omega^2 Z$ to (2), it is obvious that the impedance of spacetime encountered by GWs is: $Z_s = c^3/G \approx 4 \times 10^{35}$ kg/s. This impedance of spacetime has previously been derived from general relativity [1, 2]. This impedance (c^3/G) is a very impressive large number, but it is not in a form that allows easy comparison to the impedance of acoustic materials. The problem is that (2) expresses wave amplitude as dimensionless strain amplitude (maximum slope) rather than displacement amplitude with dimensions of meters as is common with acoustic equations. When amplitude is expressed as displacement (metes), the impedance term must have units of kg/m^{2}s.

Equation (3) takes the next step to convert (1) into the format of an acoustic equation. In (3) the amplitude term is δ with dimensions of length (meters). This is the maximum displacement produced over the entire wavelength of the GW. This is analogous to the particle displacement amplitude term of an acoustic wave. Making this change in the amplitude designation means that we must introduce another form of the impedance of spacetime with compatible units. This will be designated the "displacement impedance of spacetime, Z_d". This form of impedance must be used when amplitude has units of length (meters).

$$Z_d = \left(\frac{c^3}{G\lambda\!\!\!^{-}{}^2}\right) = \frac{c\omega^2}{G} \text{ kg/m}^2\text{s} \qquad (4)$$

$$\rho_a = k\frac{\omega^2}{G} \text{ kg/m}^3 \qquad (5)$$

$$U_v = k\frac{c^2\omega^2}{G} = k\frac{c^4}{G\lambda\!\!\!^{-}{}^2} \text{ J/m}^3 \qquad (6)$$

The displacement impedance term contained in (3) is restated in (4) and designated with the symbol Z_d. This form of the impedance of spacetime is frequency or wavelength dependent but can still be traced back to general relativity. To put this in perspective, at 200 Hz ($\omega \approx 1250$ s^{-1}) a GW encounters spacetime as having displacement impedance of about 10^{25} kg/m^{2}s which is about 10^{17} times larger than the impedance of osmium, the highest impedance solid. This enormous impedance is required for an acoustic medium to propagate at the speed of light while simultaneously exhibiting the large stiffness of space encountered by GWs. The ω^2 term in (4) means that higher frequency waves encounter even larger displacement impedance up to Planck impedance $c^6/\hbar G^2 \approx 10^{105}$ kg/m^{2}s at Planck frequency. At the opposite extreme, when $\omega = 0$, then the displacement impedance of spacetime also equals zero ($Z_d = 0$).

The specific impedance of an acoustic medium is $Z_o \equiv \rho c_a$ kg/m^{2}s where ρ is the density of the acoustic medium and c_a is the acoustic speed of propagation. For GWs, $c_a = c$. In (5) we make use of the analogy between Z_d and Z_o to solve for the "acoustic density of the vacuum" (ρ_a) encountered by GWs: ($Z_d = k\rho_a c = c\omega^2/G$). Equation (5) is the key equation of this article and the implications of this equation are discussed later.

Spacetime obviously does not have conventional density created by the presence of a medium with rest mass. The term "acoustic density" will be used to make a distinction from a medium exhibiting rest mass density. Perhaps the more accurate term is that GWs encounter the vacuum of spacetime to have energy density which exhibits the properties of a medium with density. Equation (6) converts (5) to vacuum energy (VE) density designated (U_v). Both (5 and 6) have substituted the symbol k to represent any numerical constant. Quadrupole GWs require $k = 1/16\pi$, but another numerical constant will be calculated later requiring a different constant.

3. Numerical example using GW150914 data

The implications of (5, 6) can be illustrated using the observed characteristics of GW150914 [9, 10]. This GW was a chirp that went from about 30 Hz to 250 Hz. We will analyze the 200 Hz portion of this wave which had $\omega \approx 1250$ s^{-1}, $\lambda\!\!\!^{-} = 2.4 \times 10^5$ m, peak strain amplitude $\hbar \approx 1.25 \times 10^{-21}$, propagation speed c and intensity calculated using (1) of $I = 0.02$ w/m^2. The maximum "displacement amplitude" (δ) is calculated from $\delta = \hbar\lambda\!\!\!^{-} \approx 3 \times 10^{-16}$ m. There are two ways of calculating the acoustic density of spacetime encountered by this GW at 200 Hz. One way is to make the appropriate substitutions into the acoustic equation $\rho = I/\omega^2\delta^2 c$. The other way is to use (5) setting $k = 1/16\pi$ and $\omega = 1250$ s^{-1}. Both give the same answer which is $\rho_a = 4.7 \times 10^{14}$ kg/m^3 at 200 Hz. This acoustic density of the vacuum converts to VE density of about 4×10^{31} J/m^3. This is the energy density required to propagate a 200 Hz GW at the speed of light with intensity of 0.02 w/m^2 and strain amplitude of only $\Delta L/L \approx 10^{-21}$.

A GW is a transverse wave. It is sometimes stated that transverse sound waves can only propagate in a solid. However, in this case our experience with sound does not translate to waves propagating at the speed of light because the speed of light is an absolute speed limit which limits the type of waves that can propagate. A longitudinal wave cannot propagate at the speed of light because the displacement is in the direction of propagation. One part of a longitudinal wave would have to propagate faster than the speed of light and another part of the longitudinal wave would have to propagate slower than the speed of light. An electromagnetic wave or GW wave does not produce an actual displacement as previously explained. Both are merely transverse "fields". All parts of this type of transverse wave propagate at the same speed and are proposed to be favored for speed of light propagation.

Is there any other evidence to support the large VE density encountered by GW150914? To answer this question, we will look at the emission process. The maximum GW power emitted by the merging of two black holes was reported [9] to be 3.6×10^{49} w. This approaches Planck power ($c^5/G = 3.6 \times 10^{52}$ w). The emitted power of 3.6×10^{49} w is easily checked because it is the power required to achieve intensity of 0.02 w/m^2 over the area of a sphere with radius of 1.3 billion light years. The mass/energy radiated into GWs was equivalent to 3 solar masses (5×10^{47} J). Most of this enormous energy was radiated in about 0.15 seconds. At a distance of ½ wavelength (7.5×10^5 m) from the merging black holes, the GW power of 3.6×10^{49} w achieves intensity of about $I \approx 5 \times 10^{36}$ w/m^2 which converts to energy density of $U = I/c = 1.7 \times 10^{28}$ J/m^3.

If this GW is assumed to be the result of gravitons, there is no further obvious analysis. However, this article is examining the implications of treating a GW as an acoustic wave propagating in a physical acoustic medium. In this case, the acoustic propagation medium must have a larger energy density than the energy density of the GW being propagated. In other words, the propagation medium must have energy density substantially larger than energy density of GW150914 at ½ wavelength from the merging black holes (greater than 1.7×10^{28} J/m^3). The previously calculated energy density of the medium (4×10^{31} J/m^3) encountered by GW150914 at 200 Hz is a reasonable factor of about 2,400 times larger than the energy density of the GW at ½ wavelength. Therefore, the model of the vacuum having a large energy density is compatible with the vacuum being able to propagate a GW intensity of about 10^{37} w/m^2.

4. Discussion

4.1 The debate about vacuum energy

This article started by quoting books on GWs [1, 2] which state that GWs encounter spacetime as "an extremely stiff elastic medium … with impedance of c^3/G". Since GWs also have amplitude, frequency, intensity and propagation speed, this raises the following question: Since a GW has similarities to an acoustic wave propagating in a physical medium, what are the properties of spacetime implied by this analogy? This is an interesting question with even more interesting answers. The answers discussed in this section will extend to QFT, black holes, the Freidman equation, the cosmological constant problem, the physical structure of fields and fundamental particles.

We will start with QFT which requires the vacuum to have a vast VE density. Calculations incorporating the required VE give answers correct to 10 significant figures. [3] Some examples of quantum mechanical effects requiring large VE are: 1) the Lamb shift, 2) the Unruh effect, 3) spontaneous emission initiation, 4) the Casimir effect, 5) the electron's anomalous magnetic dipole moment, 6) the uncertainty principle and 7) zero-point energy in quantum systems. However, there is no undisputed experimental evidence that VE physically exists. For example, the Casimir effect [11 - 13] is often cited as experimental proof of VE. There is definitely a force between two closely spaced metalized plates which has been measured and agrees with the QED predictions for VE within a few percent. However, there are alternative explanations involving charges and currents [14] which generate the same magnitude of force between the plates.

QFT says that the vacuum has zero-point energy associated with vacuum fluctuations [3, 15]. However, if this VE physically exists, it must be a form of energy that does not exert gravity. Most physicists today believe there must be some unknown cancelation which eliminates this large VE density but carefully preserves the one part in 10^{120} that is the observable universe. Also, the cancelation must somehow leave the quantum mechanical effects requiring the calculated large VE.

However, this is a real debate with some famous experts in general relativity supporting the concept that the vacuum can have a large undetectable energy density. For example, Charles Misner, Kip Thorne and John Archibald Wheeler are the authors of the 1279 page famous textbook on general relativity titled "Gravitation" [16]. In the last chapter of this book they specifically address the subject of the enormous VE density associated with QFT. Here is an extended quote from this book. "No point is more central than this: empty space is not empty. It is the seat of the most violent physics…. The density of field fluctuation energy in the vacuum ~ 10^{94} g/cm^3, argues that elementary particles represent a percentage-wise almost completely negligible change in the locally violent conditions that characterize the vacuum…The vacuum has to be described properly before one has a foundational starting point for a proper perturbation-theoretical analysis." [16] This chapter also suggests how this undetectable energy density is achieved. This book says, "The geometry of space is subject to quantum fluctuations in metric coefficients of the order of:
$\delta_g \approx {}^{\text{Planck length}}/_{\text{length extension of the region under study}}$".

Another supporter of a large VE density is Andrei Sakharov. In the article, *Vacuum Quantum Fluctuations in Curved Space and the Theory of Gravitation*, [17] Sakharov proposes that field fluctuations have energy of 10^{28} eV (Planck energy) and exist on the scale of 10^{-33} cm (Planck length). This combination implies vacuum field fluctuations achieve Planck energy density which is equivalent to the 10^{96} kg/m^3 described in the previous reference. Sakharov extends this concept to form a connection between quantum fluctuations and the metric elasticity of space. He then progresses from the elasticity of space to the gravitational constant and gravitational curvature.

These four famous experts in general relativity are cited to refute the contention that a large value of VE is forbidden by the equations of general relativity. A distinction is going to be made between the subjects directly addressed by these equations compared to physical interpretations of general relativity terms. For example, a disagreement about the gravitational effect on the rate of time would be a disagreement about a subject directly addressed by the equations. In comparison, the stress-energy-momentum tensor enumerates the known sources of gravity, but it does not specifically forbid the existence of a form of energy which does not exert gravity. In fact, we have already rationalized that the energy in virtual particle pairs does not create gravity. For example, if electron/positron pairs created gravity, a volume of space with a radius smaller than the earth – moon distance would form a black hole.

Reference [5] is a review article titled "Categorizing different approaches to the cosmological constant problem". It analyzes over 250 papers and divides them into various categories according to the proposed mechanism of VE cancelation. The conclusion of this article states that "none of the approaches described above is a real outstanding candidate for a solution" to the cosmological constant problem.

4.2 "Fields are physical states of space" *(Einstein)*

Einstein is generally credited with eliminating the need for the aether. However, as documented in the book "Einstein and the Ether" [18], from 1916 until his death in 1955, he believed the various fields were physically present in space. In these years, he used the terms "relativistic ether" and "physical space" to convey this idea. For example, in 1934 he wrote, *"Physical space and the ether are different terms for the same thing; fields are physical states of space."* [19]

The standard model is a field theory where all fundamental particles are considered "excitations" of their respective fields. [20] For example, there is an electron field, a muon field and a Higgs field. Do these fields physically exist in spacetime as Einstein believed? If so, it should be possible to quantify the internal structure of fields and unify this with QFT. The enormous energy density of VE greatly exceeds the energy density required to build the fundamental particles.

4.3 The connection to quantum mechanics

We can test whether (5, 6) are compatible with the VE described by Misner, Thorne, Wheeler and Sakharov [16, 17]. These authors reference vacuum density of roughly 10^{96} kg/m^3. This is referring to Planck density $\rho_p = c^5/\hbar G^2$ and Planck energy density $U_p = c^7/\hbar G^2$. The "Gravitation" book [16] specifically mentions Planck length $L_p = (\hbar G/c^3)^{1/2}$ vacuum fluctuations. These fluctuations are occurring at Planck angular frequency $\omega_p = (c^5/\hbar G)^{1/2}$. Therefore, the test will be whether (5 and 6) can be shown to be based on Planck length, L_p, Planck angular frequency ω_p, Planck energy density U_p and Planck density ρ_p. Equations (7 and 8) converts (5 and 6) to incorporate these quantum mechanical terms.

$$\rho_a = k\frac{\omega^2}{G} = k\left(\frac{\omega}{\omega_p}\right)^2 \rho_p = k\left(\frac{L_p}{\lambda\!\!\!^-}\right)^2 \rho_p \qquad (7)$$

$$U_v = k\frac{c^2\omega^2}{G} = k\left(\frac{\omega}{\omega_p}\right)^2 U_p = k\left(\frac{L_p}{\lambda\!\!\!^-}\right)^2 U_p \qquad (8)$$

Equation (7) shows a fundamental connection between the acoustic density of the vacuum ($k\omega^2/G$) and Planck density (ρ_p). When $\omega = \omega_p$, then the acoustic density of the vacuum equals Planck density times a constant ($\rho_a = k\rho_p$). At lower frequencies, the connection to ρ_p is reduced by the coupling constant $(\omega/\omega_p)^2$. For quadrupole GWs, $k = 1/16\pi$. However, a different value of k might be required to describe the fundamental density and energy density of VE. Equation (8) is merely a conversion from acoustic density of the vacuum to vacuum energy density U_v.

4.4 Zero-point energy density

Next, we will attempt to calculate the value of k for zero-point energy at the extreme where the oscillation is at Planck angular frequency (at Planck scale). This will be an estimate that will then be checked to see if it is reasonable. The energy of zero-point harmonic oscillators is $E_z = ½\,\hbar\omega$. When ω equals Planck angular frequency $\omega_p = (c^5/\hbar G)^{1/2}$, then the energy of individual Planck zero-point oscillators is ½ Planck energy $E_p/2 = (\hbar c^5/4G)^{1/2} \approx 10^9$ J. The volume of these Planck length vacuum fluctuations is not exactly agreed upon, partly because they imply a volume oscillation. However, an assumption that will be tested later is that the average volume is the volume of a sphere that is Planck length in radius $V_p = (4\pi/3)L_p^3$. The zero-point energy density will be designated U_z and the equivalent zero-point density will be designated ρ_z

$$U_z = \left(\tfrac{1}{2}\hbar\omega_p\right)\left(\frac{3}{4\pi}\right)\frac{1}{L_p^3} = \frac{3}{8\pi}\frac{c^7}{\hbar G^2} = k_z U_p \qquad (9)$$

$$U_z = 5.5 \times 10^{112} \text{ J/m}^3$$

$$\rho_z = \left(\frac{3}{8\pi}\right)\frac{c^5}{\hbar G^2} = k_z \rho_p = 6.6\text{x}10^{95} \text{ kg/m}^3 \qquad (10)$$

Therefore, the value of k obtained from this combination of energy and volume as shown in (9, 10) is defined as: $k = k_z \equiv 3/8\pi$. This numerical constant associated with zero-point energy is 6 times larger than the value of k obtained from GW equations where $k_{gw} = 1/16\pi$. It is reasonable that quadrupole GWs would not couple into the full VE available at a particular frequency. While this value of k was calculated for Planck density, it should apply to lower frequency conditions which encounter reduced density because of the coupling constant $(\omega/\omega_p)^2$. We will assume that $k = k_z$ in (7, 8) in further tests which pertain to the properties of VE.

4.5 Black hole test

The first of these tests is to compare the density of a black hole with Schwarzschild radius $r_s = 2Gm/c^2$ to zero-point density associated with reduced wavelength $\bar{\lambda}$. Black holes represent the maximum possible distortion of spacetime for a given radius. Equation (7) represents the maximum vacuum density accessible in spacetime at a given frequency or wavelength. If VE gives spacetime its properties, then maximum distortion of spacetime produced by a black hole with a given Schwarzschild radius and maximum vacuum density at a given wavelength should be connected. Therefore, we will test whether the density of a black hole and the wavelength dependent density of VE described by (8) are related. A black hole with mass m has a Schwarzschild radius of $r_s = 2Gm/c^2$. The volume of a black hole, as perceived from the outside, is $V_{bh} = (4\pi/3)r_s^3$. The density of a black hole ρ_{bh} is:

$$\rho_{bh} = \frac{m}{V_{bh}} = \frac{r_s c^2}{2G}\frac{3}{4\pi r_s^3} = k_z \frac{c^2}{r_s^2 G} = k_z\left(\frac{L_p}{r_s}\right)^2 \rho_p \qquad (11)$$

When we set $k = k_z$, then one of the equalities in (8) is $\rho_a = k_z(L_p/\bar{\lambda})^2\rho_p$ and (11) is: $\rho_{bh} = k_z(L_p/r_s)^2\rho_p$. Therefore, the density of a black hole exactly matches the zero-point density of the vacuum ($\rho_{bh} = \rho_a$) when: $r_s = \bar{\lambda}$. This exact match gives new insights into the role VE plays in determining the curvature of spacetime when mass is present.

As previously explained, a GW couples into only 1/6 of the zero-point energy density at a given reduced wavelength. Therefore, a GW does not encounter this exact match. Instead, a GW with wavelength λ encounters the vacuum as having a density approximately equal to the externally perceived density of a black hole with diameter $d \approx \lambda$.

4.6 Critical density of the universe

Setting $\omega = \omega_p$ into (5) has been shown to generate Planck density ($\rho_p = c^5/\hbar G^2$) times numerical constant k_z. The opposite extreme for the lowest angular frequency present in spacetime is not zero because the expansion of the universe is clearly a distortion of spacetime that that can be associated with a finite angular frequency. The Hubble parameter H_o is a measurement of the expansion rate of space. The units associated with the Hubble parameter is usually expressed as km/s/Mpc which converts to s^{-1} in SI units. Angular frequency has units of s^{-1}.. What happens when we equate the Hubble parameter H_o with units of s^{-1} to the lowest angular frequency present in spacetime? Equation (12) below answers this question. In this equation we set $\omega = H_o$ and $k = k_z = 3/8\pi$ into (5).

$$\rho_c = k\frac{\omega^2}{G} = \frac{k_z H_o^2}{G} = \frac{3H_o^2}{8\pi G} \approx 10^{-26} \text{ kg/m}^3 \qquad (12)$$

This is a surprising result! Equation (12) shows that substituting the angular frequency of the expansion of spacetime ($\omega = \omega_u = H_o$) and the constant associated with zero point energy $k = k_z$ into (5) generates the Friedmann equation [21]. This is the equation from general relativity which designates the "critical density of the universe" which is: $\rho_c = (3/8\pi)(H_o^2/G$.. This is the density of mass/energy required to achieve flat spacetime. Since we are making a distinction between observable matter which generates gravity and VE, we will clarify that the Friedmann equation designates the critical density of observable matter in the universe.

The best measurement of the current value of H_o is from an analysis of data generated by the Hubble Space Telescope [22]. This value is H_o = 73.24 km/s/Mpc which converts to $H_o = 2.37 \times 10^{-18}$ s^{-1} in SI units. Setting

$H_o = 2.37 \times 10^{-18}$ s^{-1} in (12) yields about 10^{-26} kg/m^3 which is equivalent to about 10^{-9} J/m^3. Experimental observations [7, 8] have determined that space is flat to within 1% experimental accuracy. The density of observable matter in the universe also appears to confirm the Friedmann equation.

The critical density of the universe from the Friedmann equation is sometimes used to refute the large VE density. Now we discover that VE, as quantified by (5, 7), can generate the Friedmann equation for the critical density of the universe. Therefore, the Friedmann equation is a special case of (5).

Next, we are going to calculate the exact value of the cosmological constant problem using (10, 12). Until now we have been stating that the cosmological constant problem is about a 10^{120} difference between the zero-point energy density and the critical density of the universe. However, we can use (10) and the Friedmann equation $\rho_c = (3/8\pi)(H_o^2/G$ to calculate the exact value of the density ratio ρ_z/ρ_c. Equation (13) below shows the result of this calculation. It includes Planck time $T_p = \sqrt{\hbar G/c^5}$ and sets $H_o = 2.37 \times 10^{-18}$ s^{-1}.

$$\frac{\rho_z}{\rho_c} = \left(H_o T_p\right)^{-2} = 6\text{x}10^{121} \qquad (13)$$

4.7 Proposed model of vacuum energy

A common rationalization of a solution of the cosmological constant problem is to assume that some unknown effect cancels the large VE density while leaving the quantum mechanical effects and leaving the one part in 10^{120} that is the observable universe. However, an equally valid interpretation is that VE actually exists but in a form that cancels gravitational effects but gives the vacuum constants of: G, c, and ħ. To elevate this alternative and stimulate further discussion, a hypothesis will be presented. This proposed model of VE attempts to be compatible with (7, 8) and suggest a way that energy in the form of Planck length and Planck time vacuum fluctuations can cancel gravitational effects.

In this model, the quantum vacuum is a sea of closely packed harmonic oscillators which lack spin. These oscillations are Planck length and Planck time vacuum fluctuations at approximately Planck frequency. This is consistent with the fundamental limitation that distance cannot be measured to an accuracy of Planck length and time cannot be measured to an accuracy of Planck time [23 – 27]. These L_p and T_p fluctuations represent the background "noise" of the vacuum. The radius of each harmonic oscillator is fluctuating but for analysis we will assume a spherical volume with a Planck length radius ($r = c/\omega_p = L_p$). Lower frequency harmonic oscillators are created by combinations of these approximately Planck frequency components creating beats and a few resonances.

This model is compatible with (7, 8) because the basic VE density is Planck energy density (U_p) times a numerical constant k near 1. If a hypothetical wave at Planck angular frequency encountered these harmonic oscillators, then substituting $\omega_p = (c^5/\hbar G)^{1/2}$ (Planck frequency) into (4) results in the wave encountering Planck impedance $c^6/\hbar G^2 \approx 10^{105}$ kg/m^{2}s. However, substituting a lower frequency into (4) results in a lower impedance. This can be interpreted as an impedance mismatch with a coupling constant of $(\omega/\omega_p)^2$. This leads to the lower frequencies encountering the energy density and the "acoustic density of the vacuum" shown in (7, 8).

There is another important part of this model which suggests a mechanism by which VE does not produce its own gravity. Since the vacuum fluctuations both increase and decrease radius distance, this means that the distortion (curvature) of spacetime being produced both increases and decreases volume. The rate of time also fluctuates by Planck time. Another way of saying this is that the oscillation is between positive and negative spacetime curvature. When the volume increases relative to Euclidian geometry, the rate of time decreases. This is analogous to the positive curvature of spacetime produced by gravity. When the opposite happens (decreased volume and increased rate of time) this is analogous to negative curvature or antigravity curvature. There is no matter with antigravity properties, but if there was an antigravity body, the surrounding spacetime would have increased rate of time and decreased volume compared to a distant zero gravity volume. The Planck frequency oscillation is between equal parts positive and negative curvature which can also be stated as equal parts of gravity and antigravity components. These opposite curvatures are proposed to cancel gravity.

For those physicists that believe in gravitons, the point can be made that Planck length vacuum oscillations are undetectable as waves and lack spin. Therefore, it is possible that these Planck length vacuum fluctuations which oscillate between positive and negative curvature are a form of energy that is incapable of emitting gravitons.

4.8 Expansion of the universe

Another problem with a large VE is that the expansion of the universe seems to require new VE added to the universe every second to maintain a constant VE density. This problem is outside the scope of this article which addresses treating GWs like acoustic waves. However, there is an answer even to this problem. The expansion of the universe is indicative of a much more complex transformation of spacetime which results in the covariance of the laws of physics.

An analogy will be made to the changes required to maintain the laws of physics (covariance) in different gravitational potentials with different rates of time. For example, part of Neptune has the same gravitational acceleration as earth, but this part of Neptune has a slower rate of time. When the rate of time is different between two locations but the laws of physics are the same, there must also be changes to the value of several units of physics (on an absolute scale) to offset the difference in the rate of time.

For example, momentum scales proportional to $1/t$, force scales proportional to $1/t^2$, power scales proportional to $1/t^3$ and the fine structure constant is independent of time ($1/t^0$). This is time raised to four different powers, yet the laws of physics are constant. Many non-obvious changes are required to offset the change in the rate of time and preserve the covariance of the laws of physics. Similar covariant changes are taking place in the universe to maintain a constant perceived VE density and preserve the equations of QFT.

5. Imagining a universe based on vacuum energy

If VE really does exist at the levels indicated by QFT, then VE would be the biggest component of the universe by a factor of 10^{120}. This is a very disruptive concept because it causes us to introduce a dominant new component into our model of the universe. Beyond this initial disruption, it is helpful to also imagine some of the positive effects this change would introduce to physics.

Our model of the universe currently has many mysteries. We acknowledge the existence of many fields, but we treat the physical structure of these fields as unknowable. The introduction of VE into a model of the universe would be the introduction of a single universal field which can be quantified and conceptually understood. All other fields would become distortions and resonances within this universal field.

The energy density of VE is vastly larger than is required to create any fundamental particle. All that is required is a way to organize the Planck length fluctuations into a cohesive unit that can interact with the observable components of the universe. For example, fermions might be modeled as ½ ħ "excitations" of this universal field. This quantized angular momentum unit introduces the necessary cohesion to create observable excitations (particles) with wave-particle properties and probabilistic characteristics. For example, the Compton frequencies of electrons and muons would be modeled as resonances which stabilize ½ ħ quantized rotations at those frequencies.

If the structure of the vacuum is Planck length fluctuations at Planck frequency defining a volume Planck length in radius, then this introduces the constants c, G and ħ as fundamental properties of the vacuum. If fundamental particles are modeled as quantized rotating wave distortions of this universal field, then this explains why fermions and bosons encounter the same speed of light limitation. It also hints at the underlying physics that achieves the special relativity effects.

The point is that once we imagine the possibilities of introducing VE into our model of the universe, the result is a model which can explain many of the current mysteries of physics. For example, the need to renormalize equations is a mathematical proof that the starting assumptions used to generate the equations contain a flaw. The act of renormalization means the starting assumptions are being changed for the remainder of the mathematical analysis. Incorporating the VE-based model of the universe into calculations are predicted to have a beneficial effect.

Also, this model change would give a quantum mechanical bases to the properties of spacetime described by general relativity. Einstein's field equations would be viewed as a description of the bulk properties of VE. Another set of equations should be possible which start with the quantum mechanical properties of VE and transition to Einstein's field equations in the limit of the bulk properties.

6. Summary and conclusion

Einstein's field equations contain the large force constant $c^4/8\pi G$. This translates into GWs being very difficult to detect because they encounter space as a very stiff medium. This article has noted the similarities between GWs and acoustic waves. The GWs have been analyzed as if they are acoustic waves propagating in a physical medium. The main findings of this analysis are:

1) the displacement impedance of spacetime is $Z_d = c\omega^2/G$, 2) the acoustic density of spacetime encountered by GWs is: $\rho_a = k\omega^2/G$, 3) the VE density of spacetime encountered by GWs is: $U_v = kc^2\omega^2/G$

This acoustic analysis of GWs implies that spacetime has a VE density equal to Planck energy density times a numerical constant near 1 ($\sim 10^{113}$ J/m^3). This is in substantial agreement with the zero-point energy density predicted to exist in the vacuum by QFT. This is an important finding since it is a new piece of information relevant to the ongoing debate known as the cosmological constant problem. This problem is one of the major mysteries in physics.

The proposed model of the vacuum is Planck length vacuum fluctuations which oscillate at Planck frequency. This proposed model opens a new area of discussion into the internal structure of both fields and fundamental particles with wave-particle duality. The enormous VE density would allow fermions to be modeled as ½ ħ excitations of this universal field. A quote from the famous general relativity textbook "Gravitation" [16] is pertinent, "The density of field fluctuation energy in the vacuum $\sim 10^{94}$ g/cm^3, argues that elementary particles represent a percentage-wise almost completely negligible change in the locally violent conditions that characterize the vacuum."

7. References

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