*Foundations of Physics, Vol. 1, No. 1, 1970*

# The Reinterpretation of Wave Mechanics

**Louis de Broglie**

Institut de France
Académie des Sciences, Paris, France

*Received July 15, 1969*

*The author begins by recalling how he was led in 1923–24 to the ideas of wave mechanics in generalizing the ideas of Einstein's theory of light quanta. He made himself at that time a concrete physical picture of the coexistence of waves and particles and, in 1927, attempted to give them precise form in his "theory of the double solution." As other ideas prevailed at the time, he abandoned the development of his conception. But for the past twenty years, once again convinced, like Einstein, that present-day quantum mechanics is only a statistical theory and does not give a true picture of physical reality, he has again taken up his old ideas and developed them considerably. He has in particular introduced an element of randomness into the theory and has thus attained to a "hidden thermodynamics of particles," the results of which appear to be very interesting.*

When I conceived the first basic ideas of wave mechanics in 1923–24,[1] I was guided by the aim to perform a real physical synthesis, valid for all particles, of the coexistence of the wave and of the corpuscular aspects that Einstein had introduced for photons in his theory of light quanta in 1905. I did not have any doubts at that time about the physical reality of the wave and the localization of the particle in the wave.

At that time, one remark made a deep impression on me. The phase of the plane monochromatic wave, written as

$$2\pi\left(\nu t - \frac{\alpha x + \beta y + \gamma z}{\lambda}\right) = 2\pi\nu\left(t - \frac{\alpha x + \beta y + \gamma z}{V}\right)$$

permits the definition of a 4-vector "wave" having components $\nu/c$, $\alpha\nu/V$, $\beta\nu/V$, $\gamma\nu/V$; and this shows that the frequency of the wave is transformed, when we pass from the proper system where the frequency has a value $\nu_0$ to a reference system moving with respect to the proper system with a velocity $\beta c$, by the formula $\nu = \nu_0/(1 - \beta^2)^{1/2}$, that is, it transforms like an energy. But, and this is remarkable, the frequency of a clock is transformed according to the different formula $\nu = \nu_0(1 - \beta^2)^{1/2}$, as results from the relativistic theory of the retardation of clocks in motion.

I then noticed that it was possible to establish a relation between the 4-vector defined by the gradient of the phase of a monochromatic wave and the momentum–energy 4-vector by writing:

$$W = h\nu, \qquad p = h/\lambda \tag{1}$$

($h$ is Planck's constant), $W$ and $p$ being the energy of the particle and its momentum in the direction of wave propagation. I was thus induced to imagine that the particle, localized in one point of the plane monochromatic wave, possessed an energy $W$ and a momentum $p$ and that it described one of the rectilinear rays of the plane wave. But I also noticed that, if the particle is considered as containing, at rest, an internal energy $M_0c^2 = h\nu_0$, it could be compared to a small clock of proper frequency $\nu_0$, so that, when it is in motion with a velocity $\beta c$, its frequency is different from that of the wave, namely, $\nu = \nu_0(1 - \beta^2)^{1/2}$. I thus easily demonstrated that, during the motion of the particle in the wave, the internal vibration of the particle was constantly in phase with that of the wave, and this seemed natural if it is considered as a local accident incorporated in the wave.

Now, in relativistic thermodynamics, it is generally accepted, since the classical work of Planck and Laue, that the formula of heat transformation is $Q = Q_0(1 - \beta^2)^{1/2}$. Although this formula has recently been challenged by several authors, I am convinced today after intensive thought[11] that it is exact. We see therefore that the difference between the relativistic transformation formulas for energy $W = W_0/(1 - \beta^2)^{1/2}$ and for heat $Q = Q_0(1 - \beta^2)^{1/2}$ is totally analogous to the difference, that had impressed me so much formerly, between the formula of transformation of the frequency of a wave $\nu = \nu_0/(1 - \beta^2)^{1/2}$ and that of the frequency of a clock $\nu = \nu_0(1 - \beta^2)^{1/2}$. This observation reveals the existence of a very close link between relativistic thermodynamics and the physical ideas at the origin of the discovery of wave mechanics.

But the account which I had elaborated in my thesis had the disadvantage of being only applicable to the particular case of the plane monochromatic wave, which is never completely realized because of the inevitable existence of a spectral width. Sometime after my thesis, I was induced to go further and to generalize the ideas that had guided me in this work, on the one hand, by considering the case of a wave that is not plane monochromatic and, on the other hand, by distinguishing between the real physical wave of my theory and the fictitious wave $\psi$, arbitrarily normalized, introduced by Schrödinger and interpreted by Born as having a purely statistical significance. That is how I was led to expound, in 1927 in an article[2] entitled "The wave mechanics and the atomic structure of matter and radiation," a new interpretation of wave mechanics and to generalize, for any given wave, the law of motion of the particle that I had considered in the particular case of the plane monochromatic wave.

I do not want to develop here in detail the theory of the double solution as it stands now. I refer those who wish to study it thoroughly to the accounts published on this subject since I reconsidered, after having forsaken them for a long time, the ideas originally outlined in my 1927 article.[3–7]

I shall begin by indicating the two main ideas on which this theory rests.

1. The wave, which, in my view, must be a physical wave of very small amplitude which can evidently not be arbitrarily normalized, has to be distinct from the wave $\psi$, normalized in accordance with its statistical significance, in the usual formalism of quantum mechanics. I designate the physical wave by $v$ and I link the wave $\psi$ to the wave $v$ by the relation $\psi = Cv$, where $C$ is a normalization factor so that $\int |\psi|^2 \, d\tau = 1$. It is this distinction, an essential one in my opinion, between the two solutions $v$ and $\psi$ of the wave equation that had caused me to name this theory the "theory of the double solution." For a more thorough study of this question, I refer to the publications mentioned above.

2. For me, the particle, always localized in space in the course of time, constitutes in the wave $v$ a very small region of high-energy concentration that can be represented in first approximation as a kind of moving singularity. If the complex solution of the wave equation that represents the wave $v$ (or, if one wishes, the wave $\psi$, which amounts to the same thing because of the relation $\psi = Cv$) is written in the form

$$v = a(x, y, z, t)\, e^{(i/\hbar)\varphi(xy,z,t)} \qquad \hbar = h/2\pi \tag{2}$$

where $a$ and $\varphi$ are real functions, the introduction of this expression in the wave equation, followed by separation of real and the imaginary parts, leads us to conclude that the movement of the particle in its wave should be described by the

$$W = \partial\varphi/\partial t, \qquad \mathbf{p} = -\mathrm{grad}\ \varphi \tag{3}$$

which, in the case of the plane monochromatic wave, on writing

$$\varphi = h\nu\{t - [(\alpha x + \beta y + \gamma z)/V]\}$$

enables us easily to recover formulas (1). Designating now by $M_0$ the proper mass of the particle, we write

$$W = M_0c^2/(1 - \beta^2)^{1/2}, \qquad \mathbf{p} = M_0\mathbf{v}/(1 - \beta^2)^{1/2} \tag{4}$$

for a particle in motion with the velocity $\mathbf{v} = \beta\mathbf{c}$. The formulas thus give us

$$\mathbf{v} = \frac{c^2\mathbf{p}}{W} = -c^2\frac{\mathrm{grad}\ \varphi}{\partial\varphi/\partial t} \tag{5}$$

We can call this formula, which determines the motion of the particle at each point of its trajectory in the wave, "the guidance formula" of the particle by its wave. It is easily generalized if the particle is subjected to an external field.

The equations of the theory then easily show that the proper mass $M_0$ that appears in the equations (4) is not equal to the usual mass $m_0$ of the particle. It is found to be given by $M_0 = m_0 + (q_0/c^2)$, where $q_0$ corresponds in the proper system of the particle to a growth of its mass, which we will soon be led to associate with the increase of an internal heat hidden in the particle.

In the case where the wave propagation is described by the relativistic Klein–Gordon equation, one finds

$$M_0 = \left(m_0{}^2 + \frac{\hbar^2}{c^2}\frac{\Box a}{a}\right)^{1/2}, \qquad \hbar = \frac{h}{2\pi} \tag{6}$$

This permits one to calculate $M_0$ at each point and at each moment. In the Newtonian approximation represented by the Schrödinger equation, we have

$$q = q_0 = -\frac{\hbar^2}{2m_0}\frac{\Delta a}{a} \tag{7}$$

This is the "quantum potential" in the theory of the nonrelativistic double solution.

It is easy to extend the guidance theory to the case of an electron, which obeys Dirac's equations, and to the photon, which follows Maxwell's equations augmented by some very small mass terms.[7]

When the particle moves in its wave following the guidance law and the wave is not plane monochromatic, the proper mass $M_0$ changes constantly in a way that is measurable if one knows the shape of the wave: Therefore, it obeys the dynamics of a body of variable proper mass. Now, when one carefully studies relativistic thermodynamics, one finds that it is intimately linked to this type of dynamics. We can therefore already surmise that the theory of the double solution should naturally lead to the introduction of some thermodynamic considerations in wave mechanics. We will see this idea becoming more precise in what follows.

Before discussing the hidden thermodynamics of particles, I must point out that there are ways of confirming the exactness of the guidance formula. In one of my books[3] (pp. 101, 287), I have shown that if there exists in a wave a very small region where the wave amplitude grows very rapidly, this small region must remain confined inside a very small tube, limited by some guidance trajectories. This seems to justify the guidance formula. Moreover, in some interesting recent work, Mumm Thiounn[8] has shown that all the equations of ordinary wave mechanics (Schrödinger, Klein–Gordon, Dirac, Maxwell) admit of solutions of singularities moving in the course of time in accordance with the guidance law, the singularity giving here a kind of schematic representation of the particle.

I now turn to the concepts which I have developed after 1960 under the name of "thermodynamics of the isolated particle" or "hidden thermodynamics of particles." Here again, I will limit myself to a summary of the main results, referring to my principal works on this subject[9–11] for a more detailed exposition.

Let us first return to the idea developed and adopted in my Doctoral thesis, according to which a particle rest mass $M_0$ can be associated with a small clock having an internal vibration equal to $M_0c^2/h$. According to the relativistic formula of the retardation of clocks in motion, an observer who sees the particle moving in its wave with a velocity $\beta c$ assumes that it has an internal frequency $\nu = \nu_0(1 - \beta^2)^{1/2}$. This allows us to demonstrate easily that, even in the general case of a wave which is not plane monochromatic, the internal vibration of the particle remains in phase with that of the wave which is carrying it. This result, to which we shall return in a moment, includes as a particular case the one which had been obtained for the plane monochromatic wave, and it can be considered as the essential result of the guidance formula.

But we have already noticed the analogy between the formulas $W = W_0/(1 - \beta^2)^{1/2}$ and $\nu = \nu_0/(1 - \beta^2)^{1/2}$, where $\nu$ is the frequency of the wave, on the one hand, and the formulas $Q = Q_0(1 - \beta^2)^{1/2}$ and $\nu = \nu_0(1 - \beta^2)^{1/2}$, on the other hand, where $\nu$ is the frequency of a clock. In the same way as shown previously (see especially ref. 11), this analogy leads one naturally to regard the particle as a very small body containing a hidden heat equal to $Q_0 = M_0c^2$, so that, for an observer who sees the particle moving at a velocity $\beta c$, it contains an internal heat $Q = M_0c^2(1 - \beta^2)^{1/2}$. Now, when one studies relativistic wave mechanics, one is led to write the relation

$$\frac{Q_0}{(1 - \beta^2)^{1/2}} = Q_0(1 - \beta^2)^{1/2} + \mathbf{v} \cdot \mathbf{p} \tag{8}$$

which asserts that the total energy of the small warm body in motion is equal to the sum of the internal heat it contains and its total translation energy, equal to $\mathbf{v} \cdot \mathbf{p}$. I have studied this question at length in the article[11] to which I have just referred. If one now accepts the relation $Q_0 = M_0c^2$, the relation (8) can be written as

$$M_0c^2(1 - \beta^2)^{1/2} = \frac{M_0c^2}{(1 - \beta^2)^{1/2}} - \mathbf{v} \cdot \mathbf{p} \tag{9}$$

which can be verified immediately, since

$$p = M_0\mathbf{v}/(1 - \beta^2)^{1/2}$$

We will show that the formula (9) expresses the phase agreement between the particle and its wave. Indeed, the guidance theory has taught us that, if $\varphi$ is the phase of the wave written as $ae^{i\varphi/\hbar}$, we have

$$\frac{\partial \varphi}{\partial t} = \frac{M_0c^2}{(1 - \beta^2)^{1/2}}, \qquad -\operatorname{grad} \varphi = \frac{M_0\mathbf{v}}{(1 - \beta^2)^{1/2}} \tag{10}$$

so that, according to (9), one can write

$$M_0c^2(1 - \beta^2)^{1/2} = \frac{\partial \varphi}{\partial t} + \mathbf{v} \cdot \operatorname{grad} \varphi = \frac{d\varphi}{dt} \tag{11}$$

If the particle conforms to a clock of internal proper frequency $M_0c^2/h$, the phase of this internal vibration written in the form $a_ie^{i\varphi/\hbar}$ will be equal to $\varphi_i = h\nu_0(1 - \beta^2)^{1/2}t = M_0c^2(1 - \beta^2)^{1/2}t$, and we have, according to (11),

$$d(\varphi - \varphi_i) = 0 \tag{12}$$

Since in the system where the particle is motionless, we have $\varphi = \varphi_i$, it is evident that $\varphi = \varphi_i$ continually during the movement of the particle.

The principle of the guidance theory can thus be found contained in the formula (8) of relativistic thermodynamics, and this is very remarkable.

We know that, for a particle controlled in its wave by the guidance movement, the proper mass $M_0$ varies in general because of the variation of the wave amplitude along the trajectory, as shown in formula (7). The dynamics of the particle is thus a dynamics of variable rest mass, and one can see then that the particle is subjected to the action of a force due to the variation of its rest mass. This force is the one represented in the formalism of the double solution by the intervention of a quantum potential. The coherence of all these conceptions is thus clearly shown.

It is now appropriate to reason as follows. Relativistic dynamics teaches us that Lagrange's function of a free particle of rest mass $M_0$ in motion with a velocity $\beta c$ is $L = -M_0c^2(1 - \beta^2)^{1/2}$ and that the action integral is

$$\int L\,dt = -\int M_0c^2(1 - \beta^2)^{1/2}\,dt = -\int M_0c^2\,ds \tag{13}$$

$ds$ being the proper time of the particle. It is therefore tempting to establish a relation between the two fundamental relativistic invariants, action and entropy. But in order to be able to do so, we have to give a well-determined value to the action integral by choosing properly the time interval over which the integration extends. From our point of view, it seems natural to choose as integration interval the period $T$ of the particle of normal proper mass $m_0$ in the reference system where it is moving with a velocity $\beta c$. Since $1/T = m_0c^2(1 - \beta^2)^{1/2}$, we thus define the cyclical action integral as

$$A = -\int_0^T M_0c^2(1 - \beta^2)^{1/2}\,dt \tag{14}$$

Since the period $T$ is very small, it is natural to suppose that $M_0$ and $\beta$ remain essentially constant over the time of the integration, which allows us to write

$$A = -M_0c^2/m_0c^2 \tag{15}$$

and, in order to define the entropy $S$ of the state of the particle, one is led to write

$$S/k = A/h \tag{16}$$

where $k$ and $h$ are respectively the Boltzmann and Planck constants. Since $Q_0 = M_0c^2$, we deduce from (16) the formula

$$\delta S = -k\,\delta Q_0/m_0c^2 \tag{17}$$

We have thus succeeded in attributing a given entropy to the movement of the particle in its wave, and eventually a certain probability $P$ defined by the famous Boltzmann formula $P = e^{S/k}$. We shall explain later the origin of the minus sign on the right-hand side of the relation (17).

I felt one could draw from these considerations two conclusions that seemed important for the interpretation of quantum physics:

1. The principle of least action is but a particular case of the second law of thermodynamics.

2. The privileged role, whose paradoxical character has been underlined by Schrödinger, that present quantum mechanics attributes to plane monochromatic waves and to stationary states of quantified systems can be explained by the fact that they correspond to entropy maxima, not because the other states are nonexistent, but only because they are of a lesser probability.

As far as the second of these conclusions is concerned, I refer to the demonstration outlined in one of my works.[9] But, in view of the great interest which attaches to the identification of the principle of least action with the second law of thermodynamics, I will summarize the demonstration previously offered.

Hamilton's principle of least action tells us that if a particle, in its natural motion according to classical dynamics, leaves a point $A$ at the instant $t_0$ to arrive at a point $B$ at $t_1$, then the action integral taken along this motion is a minimum compared with the same integral taken along all other possible motions that would lead the particle from the point $A$ at time $t_0$ to point $B$ at instant $t_1$. We are thus led to write

$$\int_{t_0}^{t_1} [\delta L]_{M_0}\, dt = 0; \qquad \int_{t_0}^{t} [\delta^2 L]_{M_0}\, dt > 0 \tag{18}$$

both variations being taken while maintaining the rest mass $M_0$ constant and equal to its normal value $m_0$.

I have introduced here a hypothesis which to me seems to have a very interesting significance The curve $ACB$ (Fig. 1) represents the natural trajectory. But I have supposed that the alternate trajectories, such as $AC'B$, do not correspond, as is usually assumed, to fictitious motions imagined by the theorist, but to movements that can really occur when the rest mass $M_0$ of the particle undergoes a succession of fluctuations between $t_0$ and $t_1$, drawing it away momentarily from its normal value $m_0$. Thus, the alternate trajectory $AC'B$ must, according to Hamilton's principle, be determined by the equation

$$\int_{t_0}^{t_1} \delta(L + \delta L)\, dt = \int_{t_0}^{t_1} (\delta L + \delta^2 L)\, dt = 0 \tag{19}$$

But, as the rest mass is not supposed to be constant any longer, one must write

$$\delta L = [\delta L]_{M_0} + \delta_{M_0} L; \qquad \delta^2 L = [\delta^2 L]_{M_0} + \delta^2_{M_0} L \tag{20}$$

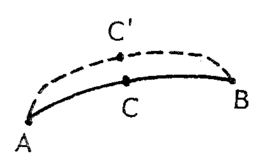


Fig. 1

where $\delta^2_{M_0} L$ represents all of the terms in $\delta^2 L$ which depend on the variation of $M_0$. Therefore, we have on $AC'B$

$$\int_{t_0}^{t_1} \{[\delta L]_{M_0} + \delta_{M_0} L + [\delta^2 L]_{M_0} + \delta^2_{M_0} L\}\, dt = 0 \tag{21}$$

But the integral of the first term is zero by virtue of Hamilton's principle, and it is easy to verify that the fourth is negligible compared to the others. Finally, there remains:

$$-\int_{t_0}^{t_1} \delta_{M_0} L = -(t_1 - t_0)\, \overline{\delta_{M_0} L} = \int_{t_0}^{t_1} [\delta^2 L]_{M_0}\, dt > 0 \tag{22}$$

being the time average of $\overline{\delta_{M_0} L}$ between $t_0$ and $t_1$. The formulas previously accepted lead us to suppose that $-\delta_{M_0} L$ represents the heat received by the particle, and formula (22) shows that the temporal mean of the quantity of heat is zero on the natural trajectory, while it is positive on the "hypothetical" trajectory. Thus, when the minus sign in (17) is taken into account, the formula (17) shows that the entropy $S$ diminishes on the average when one goes from $ACB$ to $AC'B$. On the natural trajectory the entropy is therefore maximal relative to the fluctuations subject to the conditions of Hamiltonian variation. The natural trajectory is therefore more likely than the other trajectories. Thus, in the framework of our concepts, there appears to be a very curious link between the principle of least action and the second law of thermodynamics.[1]

We arrive now at another very important point. The thermodynamic conception of the particle just outlined leads us to think that even when it seems to us that a particle is isolated from all macroscopic bodies capable of exchanging heat with the particle, it is constantly in thermal contact with a kind of thermostat hidden in what we call the vacuum. When a particle, or a set or particles, is in contact with a thermostat of temperature $T$, we know from the work of Boltzmann and Gibbs that the probability of its energy having value $E$ is $P_0 e^{-E/kT}$. In this expression, $P_0$ is often called the "*a priori* probability," and we say that it is the probability of the state of the particle or particles under consideration in the absence of all contact with a macroscopic thermostat. It seems to me that we must identify this *a priori* probability with the one defined above, since, even though it seems isolated, every particle is in contact with a hidden thermostat.

Any attempt to establish the exact nature of this hidden thermostat seems premature, but it appears related to the "subquantum level" proposed by Bohm and Vigier fifteen years ago,[(13)] or at least to a part of this subquantum level.

During its guidance movement, the mass $M_0$ of the particle generally varies. We must interpret this phenomenon by saying that it exchanges heat with the hidden thermostat. The heat exchanges are linked to the variations of the quantum potential, that is, to the variations of the wave amplitude at the point where the particle is found;

[1] On the question of kinetic focus, see ref. 12.

one sees that the wave acts as an intermediary between the particle and the hidden thermostat.

It is normal to suppose that a particle is a very simple system, and because of this simplicity it is preferable not to attribute to it a proper temperature and entropy. The hidden thermostat, on the contrary, whatever its real nature, must be a very complex system which permits us to attribute an entropy and an apparent temperature to the particle. The entropy appearing in formula (17), which determines the probability of the state of the particle, is therefore the entropy of the thermostat, while the quantity $\delta Q_0 = \delta M_0 c^2$ is (in the proper system of the particle) the quantity of heat it receives from the thermostat when $M_0$ grows or that which it yields to it when $M_0$ decreases. With this understanding, one can therefore write in the reference system where the particle has a velocity $\beta c$,

$$\delta S = -\frac{\delta Q_0(1-\beta^2)^{1/2}}{T_0(1-\beta^2)^{1/2}} = -\frac{\delta Q}{T} = -k\frac{\delta Q_0}{m_0 c^2} \tag{23}$$

provided we recall the relativistic formula for the temperature transition

$$T = T_0(1-\beta^2)^{1/2}$$

and suppose that $T_0 = m_0 c^2/k$. Thus, we find formula (17) again, and the presence of the minus sign on the right-hand side is now explained.

It might seem strange that the apparent temperature $T$ of the thermostat for the particle depends on the proper mass $m_0$ of the particle and differs according to the nature of the latter. But as remarked above, it is by the intervention of its wave that the particle is in thermal contact with the hidden thermostat. This remark seems to give meaning to the fact that, for each particle, in each point of its trajectory, the apparent temperature of the thermostat could, perhaps by means of some resonance effect, depend on the local frequency, which is itself a function of the rest mass. A more detailed description of the hidden thermostat might some day permit further clarification of this point.

But a very important point has still to be examined. The great progress accomplished in thermodynamics, when the molecular structure of matter and statistical mechanics were introduced, suggested that when a body is in a stable thermodynamic state, it is nonetheless constantly subject to small fluctuations of zero average around this state. This made it possible to develop the theory of fluctuations and of Brownian motion. We must expect to encounter some analogous circumstances in the description of the particle motion in terms of our hidden thermodynamics. Without attempting to study the question in depth, we shall limit ourselves to two aspects of it.

First, we have seen that, in the regular guidance movement, some heat exchanges occur between the particles and the hidden thermostat through the quantum potential that can be defined by the formula $q = M_0 c^2 - m_0 c^2$. But the fluctuation theory leads us to assume that the wave amplitude also must undergo constant fluctuations, giving rise to a fluctuating quantum potential $q_f$ of zero average. We must then write

$$M_0 c^2 = m_0 c^2 + q + q_f \tag{24}$$

and, as $\overline{q_f}$ is zero, we have, on the average,

$$\overline{M}_0 c^2 = m_0 c^2 + q \tag{25}$$

If the potential $q$ is zero, this simply gives us

$$\overline{M}_0 = m_0 \tag{26}$$

Therefore, we see that, because of the absence of a nonfluctuating quantum potential, the normal proper mass $m_0$ of the particle can then be considered as the mean value of the fluctuating proper mass.

Here is another important point. I have shown in my previous publications that, in order to justify the well-established fact that the expression $|\psi(x, y, z, t)|^2\, d\tau$ gives, at least with Schrödinger's equation, the probability for the presence of the particle in the element of volume $d\tau$ at the instant $t$, it is necessary that the particle jump continually from one guidance trajectory to another, as a result of the continual perturbation coming from the subquantal milieu. The guidance trajectories would really be followed only if the particle were not undergoing continual perturbations due to its random heat exchanges with the hidden thermostat. In other words, a Brownian motion is superposed on the guidance movement. A simple comparison will make this clearer. A granule placed on the surface of a liquid is caught by the general movement of the latter. If the granule is heavy enough not to feel the action of individual shocks received from the invisible molecules of the fluid, it will follow one of the hydrodynamic streamlines. If the granule is a particle, the assembly of the molecules of the fluid is comparable to the hidden thermostat of our theory, and the streamline described by the particle is its guiding trajectory. But if the granule is sufficiently light, its movement will be continually perturbed by the individual random impacts of the molecules of the fluid. Thus, the granule will have, besides a regular movement along one of the streamlines of the global flow of the fluid, a Brownian movement which will make it pass from one streamline to another. One can represent Brownian movement approximately by a diffusion equation of the form $\partial\rho/\partial t = D\,\Delta\rho$, and it is interesting to seek, as various authors have done recently, the value of the coefficient $D$ in the case of the Schrödinger equation corresponding to the Brownian movement.

I have recently studied[14] the same question starting from the idea that, even during the periods of random perturbations, the internal phase of the particle remains equal to that of the wave. I have found the value $D = (2\pi/3)\hbar/m$, which differs only by a numerical coefficient from the one found by other authors.

This concludes the account of my present ideas on the reinterpretation of wave mechanics with the help of images which had guided me in my early work. My collaborators and I are working actively to develop these ideas in various directions. Today, I am convinced that the conceptions developed in the present article, when suitably developed and corrected at certain points, may in the future provide a real physical interpretation of present quantum mechanics.

## REFERENCES

1. L. de Broglie, *Compt. Rend. Acad. Sci. Paris* **177**, 506, 548, 630 (1923).
2. L. de Broglie, *J. Physique* (serie 6) **VIII** (5), 225 (1927). Thèse de Doctorat (Masson, Paris, 1924).
3. L. de Broglie, *Une interprétation causale et non linéaire de la Mécanique ondulatoire: la théorie de la double solution* (Gauthier-Villars, Paris, 1956) [English translation: Elsevier, Amsterdam, 1960].
4. L. de Broglie, *La théorie de la Mesure en Mécanique ondulatoire* (Gauthier-Villars, Paris, 1957).
5. L. de Broglie, *J. Physique* **20**, 963 (1959).
6. L. de Broglie, *Etude critique des bases de l'interprétation actuelle de la Mécanique ondulatoire* (Gauthier-Villars, Paris, 1963) [English translation: Elsevier, Amsterdam, 1964].
7. L. de Broglie, *Ondes électromagnétiques et Photons* (Gauthier-Villars, Paris, 1968).
8. Mumm Thiounn, *Cahiers Phys.* **174**, 53 (1965); *Compt. Rend. Acad. Sci. Paris* **B262**, 657 (1966); *Port. Phys.* 657 (1966).
9. L. de Broglie, *La Thermodynamique de la particule isolée (ou Thermodynamique cachée des particules)* (Gauthier-Villars, Paris, 1964).
10. L. de Broglie, La Thermodynamique cachée des particules, *Ann. Inst. Henri Poincaré* I (1), 1–19 (1964).
11. L. de Broglie, Thermodynamique relativiste et Mécanique ondulatoire, *Ann. Inst. Henri Poincaré* IX (2), 89–108 (1968).
12. L. de Broglie, *Compt. Rend. Acad. Sci. Paris* **257**, 1822 (1963).
13. D. Bohm and J. P. Vigier, *Phys. Rev.* **96**, 208 (1954).
14. L. de Broglie, *Compt. Rend. Acad. Sci. Paris* **264**, 1041 (1967).