The specular reflection of light off light

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It is well known that any classical electromagnetic field obeys the superposition principle—a fact that would seem to preclude the interaction and hence the deflection of light with light in free space. Nevertheless, the permissible phenomenon of *interference* between light waves can be viewed as a type of interaction. It is shown that under certain circumstances it is possible to consistently interpret interference as the specular reflection of two electromagnetic waves off each other in the vacuum.

I. INTRODUCTION

As a direct consequence of the linearity of Maxwell's equations in free space, the classical electromagnetic field in a vacuum obeys the superposition principle.[1] One often thinks of this in terms of two light waves not being able to interact with one another. There is, of course, the possibility (classically) of light–light interaction in a nonlinear medium.[2] Even in a vacuum, the scattering of light by light can occur in quantum electrodynamics as a low probability second-order process represented by a Feynman diagram of the box type.[3] This QED scattering cross section is, however, very small and difficult to observe.[4] These considerations aside, within the context of classical electromagnetic theory no such interaction between light waves is thought to occur outside of a medium. Of course coherently prepared light waves can *interfere* with each other—might not interference be thought of as a type of interaction? In this work we shall extend this idea of "interaction" as far as we can. In particular, we argue that the interference of two coherent beams of light in a vacuum can be consistently interpreted as a process of specular reflection in which the effect of the interference fringes is indistinguishable from that of a mirror.

In the interest of clarity and simplicity we shall consider the gedanken experiment illustrated in Fig. 1. Two, classical, monochromatic beams of light with equal intensity I, frequency ω, polarization axis z, and diameter d are made to intersect orthogonally in free space. In addition we require the two beams to be 180° out of phase with each other, as indicated in the figure. Without loss of generality we take the beams to have a square cross section of area d^2 and furthermore we presuppose $d=\lambda$, the wavelength of the light being used. (These restrictive conditions are rather unrealizable physically, but they simplify the calculations and bring out the essence of our argument.) With respect to the coordinate system of Fig. 1 we may write down plane-wave solutions to Maxwell's equations in free space as

$$\mathbf{E}_h(\mathbf{r},t) = -E_0 e^{i(kx-\omega t)}\hat{\mathbf{z}} \quad (|y|\leqslant d/2,\ |z|\leqslant d/2), \tag{1a}$$

$$\mathbf{B}_h(\mathbf{r},t) = +E_0 e^{i(kx-\omega t)}\hat{\mathbf{y}} \quad (|y|\leqslant d/2, |z|\leqslant d/2), \tag{1b}$$

$$\mathbf{E}_v(\mathbf{r},t) = E_0 e^{i(ky-\omega t)}\hat{\mathbf{z}} \quad (|x|\leqslant d/2,\ |z|\leqslant d/2), \tag{1c}$$

$$\mathbf{B}_v(\mathbf{r},t) = E_0 e^{i(ky-\omega t)}\hat{\mathbf{x}} \quad (|x|\leqslant d/2,\ |z|\leqslant d/2). \tag{1d}$$

The subscripts h and v refer to the horizontal and vertical beams, respectively. We are using Gaussian units and hence $|\mathbf{E}|^2 = |\mathbf{B}|^2$ for a plane wave. A factor of $e^{i\pi} = -1$ has been included in the horizontal equations to give the desired 180° phase difference between beams at intersection. The fields are assumed to be zero outside of the beams. (A more physical model could be obtained by making the beams circular in cross section with a Gaussian intensity profile—but the present configuration is sufficient for our purposes here.) We assume, without loss of generality, that E_0 is real; the hatted vectors $\hat{\mathbf{x}}$ etc. are unit normals in their respective directions according to the coordinate system implied in Fig. 1.

The time-averaged expressions for the electromagnetic energy density $u(\mathbf{r},t)$ and Poynting vector $\mathbf{S}(\mathbf{r},t)$ may be written, in general, for harmonically varying fields as

$$\langle u(\mathbf{r},t)\rangle = (1/16\pi)(\mathbf{E}\cdot\mathbf{E}^* + \mathbf{B}\cdot\mathbf{B}^*), \tag{2a}$$

$$\langle \mathbf{S}(\mathbf{r},t)\rangle = (c/8\pi)\mathrm{Re}(\mathbf{E}\times\mathbf{B}^*), \tag{2b}$$

where the angular brackets denote the time averaging over the rapid harmonic oscillations at frequency ω. We shall define an *interference fringe* to be the locus of points (x,y,z) that minimize the energy density (2a) in the cube of intersection between the two beams. For the setup in Fig. 1, we define the total fields $\mathbf{E}$ and $\mathbf{B}$ within the cube as $\mathbf{E} = \mathbf{E}_h + \mathbf{E}_v$ and $\mathbf{B} = \mathbf{B}_h + \mathbf{B}_v$, which can be written immediately as

$$\mathbf{E} = E_0(e^{i\eta} - e^{i\xi})\hat{\mathbf{z}}, \tag{3a}$$

$$\mathbf{B} = E_0(e^{i\eta}\,\hat{\mathbf{x}} + e^{i\xi}\,\hat{\mathbf{y}}), \tag{3b}$$

where we have defined $\xi = kx - \omega t$ and $\eta = ky - \omega t$. We now use Eqs. (1)–(3) to compute the energy density in the horizontal beam, the vertical beam, and the cube of intersection. These are given respectively by

$$\langle u_h\rangle = (1/8\pi)E_0^2, \tag{4a}$$

$$\langle u_v\rangle = (1/8\pi)E_0^2, \tag{4b}$$

$$\langle u\rangle = (1/8\pi)E_0^2\{2 - \cos[k(x-y)]\}. \tag{4c}$$

The cosine term in (4c) is purely an interference term. For two incoherent beams it vanishes and we would have $\langle u\rangle = \langle u_h\rangle + \langle u_v\rangle$. The cosine determines the locus of the fringes; by inspection we see that (4c) is minimum only if

$$y = x - n\lambda, \quad n = 0, \pm 1, \pm 2, \ldots . \tag{5}$$

using $k = 2\pi/\lambda$. There are an infinitude of fringes, which are planes formed by taking in the xy plane straight lines of slope $m=1$ and y intercept $b = -n\lambda$, and translating

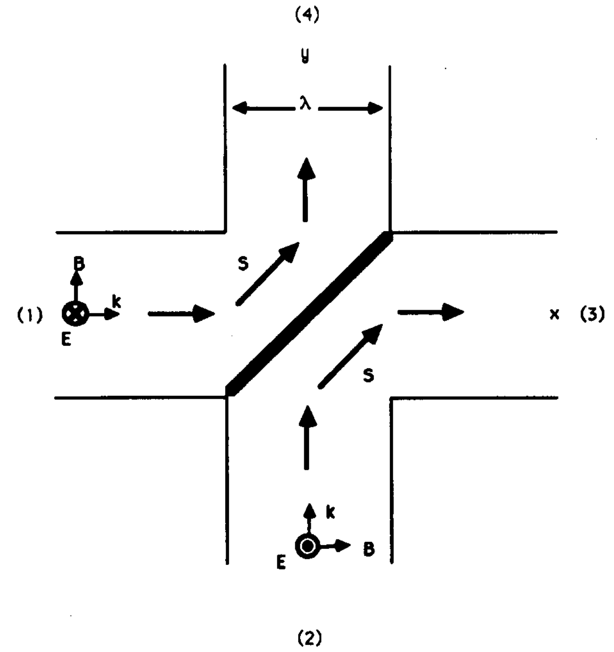

Fig. 1. Two light beams of square rectangular cross section of area d^2 are made to intersect orthogonally in free space. We assume that the beams are of equal intensity, frequency, and polarization orientation, but 180° out of phase. If the beam width is chosen to be $d = \lambda$, the wavelength, then a single interference fringe runs through the cube of intersection in the plane $x = y$. Both the Poynting vector **S** and the energy density u vanish in this plane, and hence there is no power flow through it. The Poynting vector shows that the power of incident beam (1) emerges in upper beam (4) and that of (2) in (3). In fact either the (1)→(4) or the (2)→(3) branches are indistinguishable from what one would see if the fringe plane were replaced with a perfectly conducting plane and one supposed that specular reflection were taking place at this plane.

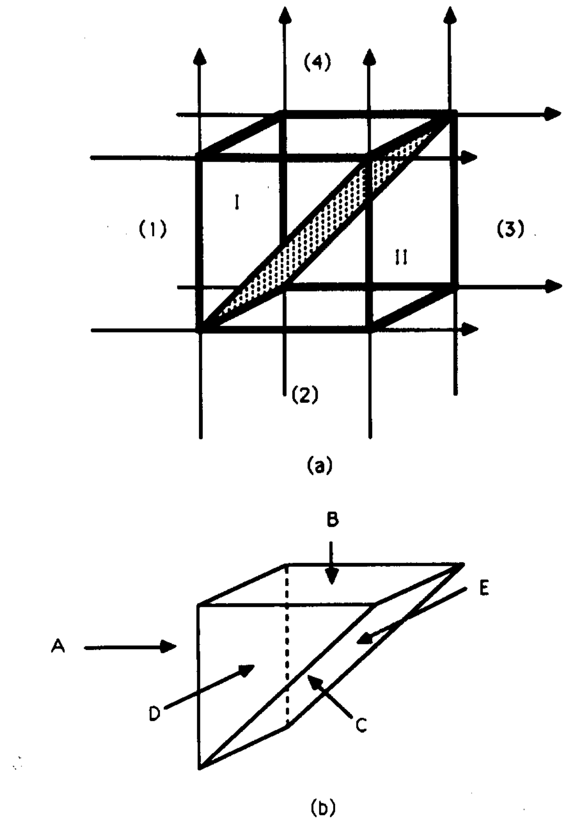

Fig. 2. The fringe plane of Fig. 1 divides the cube of intersection into two prisms I and II, depicted in (a). To track the power flow through the cube one may integrate the Poynting vector over the surface of each prism. For the initial symmetric case we call case zero, there is no power flow through the fringe, labeled C in (b), and all the incident power through surface A, for example, escapes upwards through surface B.

them in the z direction. We notice now the utility of choosing $d = \lambda$, as this gives us only *one* fringe plane, crossing through the middle of the cube diagonally. This plane naturally divides the cube of intersection into two prisms which we label I and II as in Fig. 2(a). We evaluate the fields of Eq. (3) on the fringe plane to get

$$\mathbf{E}|_{x=y} = \mathbf{0}, \tag{6a}$$

$$\mathbf{B}|_{x=y} = E_0 e^{i\xi}(\hat{\mathbf{x}}+\hat{\mathbf{y}}) \equiv E_0 e^{i\eta}(\hat{\mathbf{x}}+\hat{\mathbf{y}}). \tag{6b}$$

The electric field vanishes but there remains a magnetic field that is transverse to the fringe plane. If smoke or dust particles are blown through the cube, at the fringe they will appear dark since the dipole scattering of light will depend on **E** and not **B**. If we now evaluate the density (4c) here also, we see that

$$\langle u \rangle|_{x=y} = (1/4\pi)E_0^2, \tag{7}$$

which arises from the **B** field alone. This energy is constant throughout the cube and is not subject to interference effects since $\mathbf{B}_h$ and $\mathbf{B}_v$ are orthogonal and hence cannot interfere. The vanishing of **E** alone defines the fringe. We now investigate the Poynting vector (2b) in the cube. We have for the h beam alone, the v beam alone, and the cube, respectively,

$$\langle \mathbf{S}_h \rangle = (c/8\pi)E_0^2\hat{\mathbf{x}}, \tag{8a}$$

$$\langle \mathbf{S}_v \rangle = (c/8\pi)E_0^2\hat{\mathbf{y}}, \tag{8b}$$

$$\langle \mathbf{S} \rangle = (c/8\pi)E_0^2\{1 - \cos[k(x-y)]\}(\hat{\mathbf{x}}+\hat{\mathbf{y}}). \tag{8c}$$

It is easy to see that the first term in the braces of (8c) is the incoherent sum of (8a) and (8b), while the second term is the interference term. We see that power flow in the cube is parallel to the fringe at an angle of + 45°. If we calculate (8c) on the fringe plane itself, we get

$$\langle \mathbf{S} \rangle|_{x=y} \equiv \mathbf{0}, \tag{9}$$

which is consistent with the fact (6a) that **E** vanishes there, since $\langle \mathbf{S} \rangle \propto \mathbf{E} \times \mathbf{B}^*$. The total power flow through the fringe plane is consequently zero. Classically, one would have to wonder how the energy in the two beams *does* get across this plane. The simplest explanation is that, in fact, it does not.

On the fringe plane we see that, from Eqs. (6), we have $\mathbf{E}|_{x=y} \equiv \mathbf{0}$ and $\mathbf{B}|_{x=y} \equiv \mathbf{B}_{\parallel}$ (The symbols $\parallel$ and $\perp$ indicate transverse and normal directions, respectively, to the plane.) Hence it follows trivially that

$$\mathbf{E}_{\parallel}|_{x=y} \equiv \mathbf{0}, \tag{10a}$$

$$\mathbf{B}_{\perp}|_{x=y} \equiv \mathbf{0}, \tag{10b}$$

which we recall are precisely the boundary conditions for an electromagnetic field at a perfectly conducting surface. Since the fields cannot distinguish the plane from a perfect conductor, a reasonable interpretation—complementary to that of the more usual superposition principle—is that the beams respond to the fringe as if it *were* a perfect mirror. Hence, we propose an alternative to the usual viewpoint: Namely that no energy is passing through the fringe plane and that the two beams are specularly reflecting off it.

Let us label the various legs of the beams (1)–(4) as shown in Fig. 1. If we isolate, say, the (1) and (4) branches, the resultant figure is indistinguishable from a reflection process off a mirror: Angle of incidence equals that of reflection, there is the correct 180° phase change, and the Poynting vector's behavior is identical to that one would see for such a beam reflection. (We must assume that the "mirror" is edgeless to preclude diffraction effects.) An interesting question arises: If a process, usually explained in the context of superposition, is physically indistinguishable from specular reflection under all observing conditions—would we not have to conclude that both viewpoints offer equally valid descriptions of this phenomenon?

An immediate counterargument to this specular reflection scenario presents itself: Suppose one were to "label" in some fashion one of the incident beams, say (1). Since clearly by the superposition principle the label must emerge unscathed on the opposite side in branch (3) and not up in branch (4)—the mirror interpretation seems at first to be inconsistent with superposition. This is because, naively, one would think that the specular reflection interpretation would predict that labels on an incident beam would end up in a reflected beam and not in a transmitted beam as superposition would seem to require. Compelling as this simple argument may seem for rejecting our novel, alternative viewpoint, we note a subtlety, namely, that the properties of the fringe depend upon the properties of the light generating it. In order to label the light we must change it, and in turn we alter the properties of the mirror. In the remainder of this paper we shall show that no matter how we label beam (1) the mirror always compensates in such a way that the label attached to beam (1) emerges in (3) and not in (4)—in a fashion always consistent with the specular reflection scenario as well as the superposition principle.

Before we begin, let us first realize that, classically, light waves have only four distinct parameters that we can alter to use as a label for an incident beam: amplitude, polarization, phase, and frequency. In what follows we shall analyze the response of the beams and the fringe plane to a small change in each one of these parameters in turn. When we are through, we will find the reflection interpretation of the process still standing on equal footing with that of superposition. We begin by making a small change in the amplitude of an incident beam.

II. AMPLITUDE MODULATION

Suppose in the original symmetric configuration of Fig. 1 (a configuration that we shall henceforth refer to as case zero) we increase the amplitude of incident beam (1) by $E_0 \rightarrow E_0 + \Delta E_0$. How does the increase in amplitude get through the fringe and over into branch (3)—as superposition requires—without contradicting the reflection scenario? The answer, we shall see, is that an amplitude mismatch between beams causes the fringe mirror to become partially transmitting.

The horizontal beam fields (1a) and (1b) can be written as

$$\mathbf{E}_h = -(E_0 + \Delta E_0)e^{i\xi}\,\hat{\mathbf{z}}, \tag{11a}$$

$$\mathbf{B}_h = (E_0 + \Delta E_0)e^{i\xi}\,\hat{\mathbf{y}}, \tag{11b}$$

with the vertical beam fields (1c) and (1d) unchanged. The sum of the fields in the cube are then

$$\mathbf{E} = E_0\,[e^{i\eta} - (1+\epsilon)e^{i\xi}]\,\hat{\mathbf{z}}, \tag{12a}$$

$$\mathbf{B} = E_0\,[e^{i\eta}\,\hat{\mathbf{x}} + (1+\epsilon)e^{i\xi}\,\hat{\mathbf{y}}], \tag{12b}$$

where we have defined $\epsilon = \Delta E_0/E_0$. The respective energy densities for the h beam, the v beam, and the cube are

$$\langle u_h \rangle = (1/8\pi)E_0^2(1+\epsilon)^2, \tag{13a}$$

$$\langle u_v \rangle = (1/8\pi)E_0^2, \tag{13b}$$

$$\langle u \rangle = (1/8\pi)E_0^2[2 - \cos\Phi + \epsilon(\epsilon + 2 - \cos\Phi)], \tag{13c}$$

where we have defined the phase factor $\Phi(x,y) = k(x-y)$. It is clear that as $\epsilon \rightarrow 0$ we obtain the previous expressions (4). For the Poynting vectors we now have

$$\langle \mathbf{S}_h \rangle = (c/8\pi)E_0^2(1+\epsilon)^2\,\hat{\mathbf{x}}, \tag{14a}$$

$$\langle \mathbf{S}_v \rangle = (c/8\pi)E_0^2\,\hat{\mathbf{y}}, \tag{14b}$$

$$\langle \mathbf{S} \rangle = (c/8\pi)E_0^2[(1-\cos\Phi)(\hat{\mathbf{x}}+\hat{\mathbf{y}}) + \epsilon(\epsilon+2-\cos\Phi)\hat{\mathbf{x}} - \epsilon\cos\Phi\,\hat{\mathbf{y}}]. \tag{14c}$$

The expression (13c) is still minimized by the fringe condition (5), and so the fringe plane has not moved. If we evaluate $\langle u \rangle$ and $\langle \mathbf{S} \rangle$ here we get

$$\langle u \rangle|_{x=y} = (1/8\pi)E_0^2(1+\epsilon+\epsilon^2), \tag{15a}$$

$$\langle \mathbf{S} \rangle|_{x=y} = (c/8\pi)E_0^2[(\epsilon+\epsilon^2)\hat{\mathbf{x}} - \epsilon\hat{\mathbf{y}}]. \tag{15b}$$

The energy density is now greater than the background magnetic energy (7) and the Poynting vector is no longer zero. Apparently the mirror is "leaking." To see clearly what is going on we shall study the energy flowing through prisms I and II, as labeled in Fig. 2(a), which border on the fringe plane and divide the cube in half. The five surfaces of prism I are labeled A–E as in Fig. 2(b), with branch (1) coming in A, branch (4) going out B, and the fringe plane on C. Clearly there is no power flow through D and E since they are parallel to $\langle \mathbf{S} \rangle$, given by (14c). If $d\sigma$ is a differential of area then we have the following surface integrals

$$I_{\rm IN} = \int_A \langle \mathbf{S}_h \rangle\cdot\hat{\mathbf{x}}\,d\sigma \equiv \int_A \langle \mathbf{S} \rangle\cdot\hat{\mathbf{x}}\,d\sigma = \frac{c}{8\pi}E_0^2(1+\epsilon)^2 d^2, \tag{16a}$$

$$I_R = \int_B \langle \mathbf{S}_v \rangle\cdot\hat{\mathbf{y}}\,d\sigma \equiv \int_B \langle \mathbf{S} \rangle\cdot\hat{\mathbf{y}}\,d\sigma = \frac{c}{8\pi}E_0^2 d^2, \tag{16b}$$

$$I_T = \int_C \langle \mathbf{S} \rangle\cdot\hat{\mathbf{n}}\,d\sigma = \frac{c}{8\pi}E_0^2\epsilon(2+\epsilon)d^2, \tag{16c}$$

$$I_Z = \int_D \langle \mathbf{S} \rangle\cdot\hat{\mathbf{z}}\,d\sigma \equiv \int_E \langle \mathbf{S} \rangle\cdot\hat{\mathbf{z}}\,d\sigma \equiv 0, \tag{16d}$$

where the subscripts IN, R, T, and Z stand for incident, reflected, transmitted, and z direction, respectively. The vector $\hat{\mathbf{n}}$ is normal to C and is defined as

$$\hat{\mathbf{n}} = (\hat{\mathbf{x}} - \hat{\mathbf{y}})/\sqrt{2}. \tag{17}$$

It is clear then that we have

$$I_{\rm IN} \equiv I_R + I_T. \tag{18}$$

In other words, the extra energy in beam (1) associated with the increase in amplitude I_T is leaking through the fringe, leaving the reflected branch (4) depleted in energy I_R so that it exactly matches branch (2) in power. A similar calculation with prism II quickly verifies that this leaking power is added to incident beam (2) upon its reflection and emerges in beam (3) to match the intensity of (1) as predicted by superposition.

To understand this in terms of amplitudes and not just intensities, consider the expressions **E** and **B** for the fields in the cube, given by (12), when evaluated on the fringe plane:

$$\mathbf{E}|_{x=y} = -\Delta E_0 e^{i\xi}\,\hat{\mathbf{z}}, \tag{19a}$$

$$\mathbf{B}|_{x=y} = E_0 e^{i\xi}(\hat{\mathbf{x}}+\hat{\mathbf{y}}) + \Delta E_0 e^{i\xi}\,\hat{\mathbf{y}}, \tag{19b}$$

where $\xi \equiv \eta$ since $x=y$. Comparing (19) to Eqs. (6) in case zero, we may identify the leaking fields throughout the system as

$$\Delta\mathbf{E} = -\Delta E_0 e^{i\xi}\,\hat{\mathbf{z}}, \tag{20a}$$

$$\Delta\mathbf{B} = +\Delta E_0 e^{i\xi}\,\hat{\mathbf{y}}, \tag{20b}$$

which are precisely the amplitude increases made to beam (1) originally. Hence, we have shown that the 100% reflective mirror we first had is now partially transmitting. The mirror has adjusted itself in just the right way to insure that the outcome is the same as one gets by superposition. We conjecture that this always is the case. We turn now to polarization labeling.

III. POLARIZATION EFFECTS

Let us now rotate the polarization axis of incident beam (1) of Fig. 1 in a right-handed sense by a small angle $\Delta\theta$ without changing the intensity. We shall see that this problem differs from the previous example, case one, in that the fringe plane leaks polarization information without losing its 100% reflectivity. This might be suspected since all four legs have identical intensities. To see how the mirror accomplishes this feat we proceed in the same fashion as before.

Upon rotation, the fields in the horizontal beam, (1a) and (1b), can now be written as

$$\mathbf{E}_h = E_0 e^{i\xi}(\sin\Delta\theta\,\hat{\mathbf{y}} - \cos\Delta\theta\,\hat{\mathbf{z}}), \tag{21a}$$

$$\mathbf{B}_h = E_0 e^{i\xi}(\cos\Delta\theta\,\hat{\mathbf{y}} + \sin\Delta\theta\,\hat{\mathbf{z}}), \tag{21b}$$

whereas the vertical equations (1c) and (1d) are unchanged. The fields in the cube can be then written as

$$\mathbf{E} = E_0\{e^{i\xi}[\sin\Delta\theta\,\hat{\mathbf{y}} + (1-\cos\Delta\theta)\hat{\mathbf{z}}] + (e^{i\eta} - e^{i\xi})\hat{\mathbf{z}}\}, \tag{22a}$$

$$\mathbf{B} = E_0\{(e^{i\eta}\,\hat{\mathbf{x}} + e^{i\xi}\,\hat{\mathbf{y}}) + e^{i\xi}[(\cos\Delta\theta - 1)\hat{\mathbf{y}} + \sin\Delta\theta\,\hat{\mathbf{z}}]\}, \tag{22b}$$

where $\xi = kx - \omega t$, $\eta = ky - \omega t$ as before. The energy densities can be computed now, and they are

$$\langle u_h\rangle = (1/8\pi)E_0^2, \tag{23a}$$

$$\langle u_v\rangle = (1/8\pi)E_0^2, \tag{23b}$$

$$\langle u\rangle = (1/8\pi)E_0^2(2 - \cos\Phi\cos\Delta\theta), \tag{23c}$$

where $\Phi = \xi - \eta \equiv k(x-y)$, as before. Notice that (23a) and (23b) are the same as the similar equations (4a) and (4b) from case zero. This is a result of the fact that only the polarization of beam (1) is changed and not the intensity. The Poynting vectors follow immediately also. They are

$$\langle \mathbf{S}_h\rangle = (c/8\pi)E_0^2\hat{\mathbf{x}}, \tag{24a}$$

$$\langle \mathbf{S}_v\rangle = (c/8\pi)E_0^2\hat{\mathbf{y}}, \tag{24b}$$

$$\langle \mathbf{S}\rangle = (c/8\pi)E_0^2[(1-\cos\Phi\cos\Delta\theta)(\hat{\mathbf{x}}+\hat{\mathbf{y}}) - \cos\Phi\sin\Delta\theta\,\hat{\mathbf{z}}], \tag{24c}$$

where again for the h and v beams there is no change.

Once again $\langle u\rangle$ is minimum at $x=y$ or $\Phi=0$, fixing the mirror's location on the diagonal plane as before. On this plane we have

$$\langle u\rangle|_{x=y} = (1/8\pi)E_0^2(2-\cos\Delta\theta), \tag{25a}$$

$$\langle \mathbf{S}\rangle|_{x=y} = (c/8\pi)E_0^2[(1-\cos\Delta\theta)(\hat{\mathbf{x}}+\hat{\mathbf{y}}) - \sin\Delta\theta\,\hat{\mathbf{z}}]. \tag{25b}$$

We see that $\langle u\rangle$ is greater on the fringe than in case zero and $\langle \mathbf{S}\rangle$ is also non-null here, and transverse to the fringe plane. (The component of $\langle \mathbf{S}\rangle$ in the z direction might seem disturbing at first glance—but it is a transient interference effect that vanishes upon integration over the surface of the cube.) Referring to Fig. 2 we can carry out the required surface integrals on prism I to track the energy flow. The results are

$$I_{\rm IN} = \int_A \langle \mathbf{S}_h\rangle\cdot\hat{\mathbf{x}}\,d\sigma \equiv \int_A \langle \mathbf{S}\rangle\cdot\hat{\mathbf{x}}\,d\sigma = \frac{c}{8\pi}E_0^2 d^2, \tag{26a}$$

$$I_R = \int_B \langle \mathbf{S}_v\rangle\cdot\hat{\mathbf{y}}\,d\sigma \equiv \int_B \langle \mathbf{S}\rangle\cdot\hat{\mathbf{y}}\,d\sigma = \frac{c}{8\pi}E_0^2 d^2, \tag{26b}$$

$$I_T = \int_C \langle \mathbf{S}\rangle\cdot\hat{\mathbf{n}}\,d\sigma \equiv 0, \tag{26c}$$

$$I_Z = I_D \equiv I_E \equiv \int_{D,E} \langle \mathbf{S}\rangle\cdot\hat{\mathbf{z}}\,d\sigma \equiv 0, \tag{26d}$$

with $\hat{\mathbf{n}}$ given by Eq. (17) as before. As promised, there is no net energy flow I_Z in the z direction, as is clear from (26d). There is also no net *energy* transmission through the fringe plane, by (26c), hence $I_{\rm IN} = I_R$, and it appears that all the energy of incident beam (1) is reflected up into beam (4). We shall now consider field amplitudes at the fringe to see just how polarization information gets through. Equations (22) at $x=y$ or $\xi\equiv\eta$ can easily be calculated and compared to the case zero results of Eq. (6). From this we identify the contributions

$$\Delta\mathbf{E} = E_0 e^{i\xi}[\sin\Delta\theta\,\hat{\mathbf{y}} + (1-\cos\Delta\theta)\,\hat{\mathbf{z}}], \tag{27a}$$

$$\Delta\mathbf{B} = E_0 e^{i\xi}[(\cos\Delta\theta - 1)\hat{\mathbf{y}} + \sin\Delta\theta\,\hat{\mathbf{z}}], \tag{27b}$$

which survive on the fringe plane as field contributions that are not present in case zero. The fields of Eq. (27) do not give rise to energy flow through the fringe plane–Eq. (26c) shows this is zero—but instead act as information flow pertaining to the beam polarization. The mirror has acquired the properties of a lossless polarization rotator. The tagged incident beam (1) is rotated back to the polarization of beam (2) and reflected upwards into (4), while the reverse happens to incident beam (2) as it reflects into (3). Everything once again conspires to give an end result that would be expected by the direct application of the superposition principle. Our attempt to refute the mirror interpretation by polarization labeling has failed; we consider next the phase.

IV. PHASE LABELING

The situation considered in the introduction requires a strict 180° phase difference between the horizontal and vertical beams as, say, measured with respect to some third reference beam. Suppose we change the phase in incident beam (1) by a small amount $\Delta\phi$, how does this change emerge oppositely in beam (3), as required by superposition, in a manner reconcilable with the mirror idea?

The phase change results in horizontal fields (1a) and (1b) now given by

$$\mathbf{E}_h = -E_0 e^{i(\xi + \Delta\phi)}\hat{\mathbf{z}}, \quad (28a)$$

$$\mathbf{B}_h = E_0 e^{i(\xi + \Delta\phi)}\hat{\mathbf{y}}, \quad (28b)$$

with the vertical fields (1c) and (1d) unaltered. Total fields in the cube of intersection are then

$$\mathbf{E} = E_0 [-e^{i(\xi + \Delta\phi)} + e^{i\eta}]\hat{\mathbf{z}}, \quad (29a)$$

$$\mathbf{B} = E_0 [e^{i\eta}\,\hat{\mathbf{x}} + e^{i(\xi + \Delta\phi)}\hat{\mathbf{y}}], \quad (29b)$$

giving for the energy density

$$\langle u \rangle = (1/8\pi)E_0^2[2 - \cos(\Phi + \Delta\phi)], \quad (30)$$

where we have defined $\Phi = k(x - y)$ as before. This expression is not minimized by Eq. (5), but rather by a new fringe plane condition, namely,

$$y = x + (\Delta\phi/2\pi - n)\lambda. \quad (31)$$

This straight line equation has solutions inside the cube of sides λ for $n = 0,1$. These correspond to planes of slope $m = 1$ as before, but with y intercepts at $b = \Delta\phi\lambda/2\pi$ and $b = \Delta\phi\lambda/2\pi - \lambda$. The first fringe has moved up off the diagonal and a second fringe has come into the cube (see Fig. 3). We note that the energy density at the two fringes,

$$\langle u \rangle|_{\text{fringe}} = (1/8\pi)E_0^2 \quad (32)$$

is the same as in case zero, Eq. (7). We suspect the fringes then to be perfectly reflecting. The Poynting vector behavior confirms this; we have

$$\langle \mathbf{S} \rangle = (c/8\pi)E_0^2[1 - \cos(\Phi + \Delta\phi)](\hat{\mathbf{x}} + \hat{\mathbf{y}}), \quad (33a)$$

$$\langle \mathbf{S} \rangle|_{\text{fringe}} = \mathbf{0}. \quad (33b)$$

Energy flow throughout the cube is parallel to the surface of the two fringe planes at an angle of $+45°$, and it is zero through and on the planes. Classically, one would have to interpret that the fringes are acting like guides for the beam energy.

In case zero the location of the fringe on the diagonal insured that branches (1)→(4) and (2)→(3) traversed equal optical paths through the cube. If we treat the two fringes now as perfect, edgeless mirrors, their asymmetric locations will produce optical path differences for the reflecting beams. Since such changes are equivalent to a change in phase, we see the mechanism that will remove the additional phase from beam (1) and put it in beam (3). Consider Fig. 3(a) for $\Delta\phi = \pi/2$. One can easily see that, before reflection, the ray tracing the center of beam (1) travels a distance $\Delta\lambda \equiv \Delta\phi\lambda/2\pi$ less than before—removing its extra phase $\Delta\phi$ with respect to the imagined reference beam. Incident beam (2), on the other hand, travels the same distance *more* than before, giving it the required extra shift. To see that this holds for all parts of the beam, independent of a particular choice of $\Delta\phi$, consider Fig. 3(b). Since each beam is uniformly constant in cross section, and since optical path differences matter only to within modulo λ, we can translate the bottom fraction of the horizontal beam of width $\Delta\lambda$ with its fringe bit in the lower right corner of the cube–upwards by an amount λ. This process maintains the overall optical path length–mod λ—and results in a diagram seemingly identical to Fig. 1. However, now it is obvious that all parts of beam (1) travel an optical path length of $\Delta\lambda$ less, and those of beam (2) the same amount more. Hence, the phase label $\Delta\phi$ is efficiently transmitted across the fringes. This idea of phase tagging, then, cannot constitute a contradiction for the mirror point of view. We have one parameter left to investigate—frequency. In the next section we shall detune one of the incident beams.

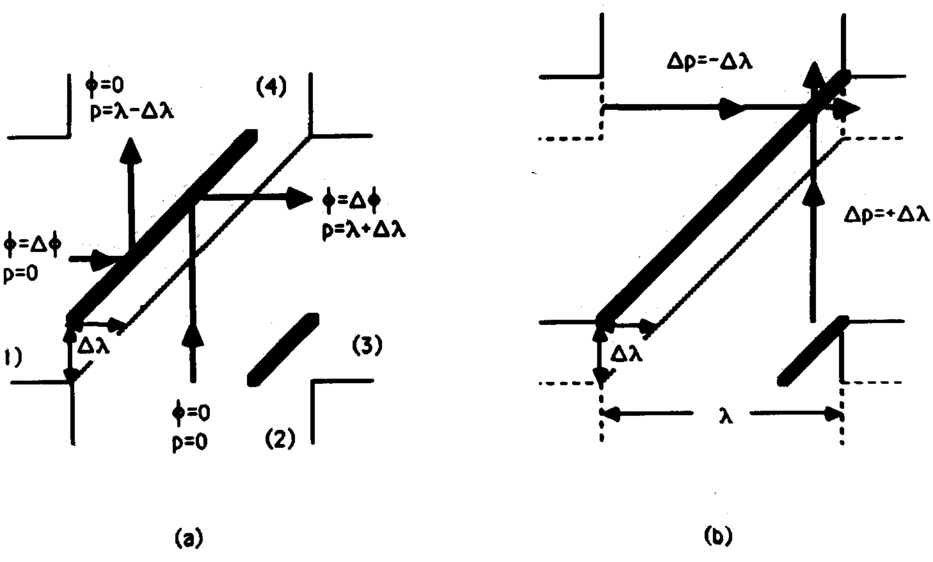

Fig. 3. If one alters the phase of incident beam (1) by $\Delta\phi$, the original fringe plane moves upward and a second fringe enters the cube. The y intercepts of the two planes are at $\Delta\lambda$ and $\Delta\lambda - \lambda$, where $\Delta\lambda \equiv \Delta\phi\lambda/2\pi$. In (a) we trace a centered ray in each incident beam. Beam (1)→(4) travels a distance $\Delta\lambda$ less than before—losing its additional phase. The optical path distance traveled we denote as p. The reverse occurs for (2)→(3) which gains $\Delta\phi$ in phase. Hence, the mirror diagonal acts to transmit the phase change horizontally through the cube. In (b) we translate the bottom $\Delta\lambda$-fraction of the horizontal beam upward by λ, preserving the overall optical path length modulo λ. The two fringes may be treated as a single fringe then, and one sees that *all* parts of the (1)→(4) branch lose $\Delta\phi$ while all of (2)→(3) gain it. (We supress the 180° phase change on reflection for clarity.)

V. DETUNING

Reconciling the specular reflection interpretation with that of superposition is quite challenging in this fourth case where we consider the detuning of an incident beam. We will see that we have to invoke the special theory of relativity before a satisfactory account emerges. Essentially what happens is as follows: The detuning of a beam causes beating between the two beams. This beating results in the motion of the fringe mirror at a velocity dependent on the detuning. The incident beam (1) at the higher frequency strikes a receding mirror and is Doppler-shifted down upon reflection. The reverse happens to the incoming beam (2) at the lower frequency. It will turn out that the specular reflection idea and detuning are indeed compatible with each other and consistent with the predictions of the superposition principle.

We begin by detuning beam (1) in Fig. 1 by an amount $\Delta\omega$. The equations (1a) and (1b) are now

$$\mathbf{E}_h = -E_0 e^{i(\xi + \Delta\xi)}\hat{\mathbf{z}}, \quad (34a)$$

$$\mathbf{B}_h = E_0 e^{i(\xi + \Delta\xi)}\hat{\mathbf{y}}, \quad (34b)$$

while the vertical equations (1c) and (1d) are as before. We have defined $\xi = kx - \omega t$, $\eta = ky - \omega t$ as before, and the new quantity $\Delta\xi = \Delta kx - \Delta\omega t$, where, recognizing that $ck = \omega$, we have $\Delta k = \Delta\omega/c$. We may compute the fields $\mathbf{E} = \mathbf{E}_h + \mathbf{E}_v$ and $\mathbf{B} = \mathbf{B}_h + B_v$ in the cube, as before, and from these the energy density, namely,

$$\langle u \rangle = (1/8\pi)E_0^2\{2 - \cos[\Phi + \Delta k(x - ct)], \quad (35)$$

where $\Phi = k(x - y)$ as usual. The loci of points minimizing (35) turn out to obey the equation

$$y = (1 + \zeta)x - \zeta ct - n\lambda, \quad n = 0, \pm 1, \pm 2, \ldots \quad (36)$$

instead of Eq. (5) from case zero. We have defined the detuning parameter $\zeta = \Delta\omega/\omega \equiv \Delta k/k$. Choosing $n = 0$, this Eq. (36) has slope m, y intercept b, and x intercept a, given by

$$m = 1 + \zeta, \quad (37a)$$

$$b = -\zeta ct, \quad (37b)$$

$$a = [(1 + \zeta)/\zeta]ct. \quad (37c)$$

The fringe is no longer at an angle of $+45°$ and in addition it is moving more or less downward and rightward [see Fig. 4(a)]. The velocity is easily seen to be $\mathbf{v} = \dot{a}\hat{\mathbf{x}} + \dot{b}\hat{\mathbf{y}}$, and is in a direction $\hat{\mathbf{v}}$ that is not even normal to the fringe plane. One may easily compute the Poynting vector in the cube as

$$\langle \mathbf{S} \rangle = (c/8\pi)E_0^2[1 - \cos(\Phi + \Delta\Phi)](\hat{\mathbf{x}} + \hat{\mathbf{y}}), \quad (38)$$

with $\Delta\Phi = \Delta k(x - ct)$. Evaluating (35) and (38) on the fringe we get

$$\langle u \rangle|_{\text{fringe}} = (1/8\pi)E_0^2, \quad (39a)$$

$$\langle \mathbf{S} \rangle|_{\text{fringe}} = \mathbf{0}, \quad (39b)$$

which illustrates that the plane, although in motion, is still perfectly reflecting. These developments at first seem rather pathological; in particular the change in angle associated with (37a) would seem to violate the "angle of incidence equals the angle of reflection" law and hence trash the whole specular reflection business. However, due to the relativistic Doppler shift this law does not hold in a reference frame where the mirror is in motion, except in the trivial case where it is translating transversely in its own plane.[5] The general law of reflection from a moving mirror is derived by Lorentz boosting into a frame where the mirror is at rest, applying the usual reflection law there, and then boosting back. We shall do something of the sort here by boosting into a frame S' which makes the reflection process easier to analyze. Consider Fig. 4(a), which we call frame S (not to be confused with the Poynting vector $\mathbf{S}$). Since the horizontal beam is at a higher frequency than the vertical beam, one might suppose that because of the Doppler effect there exists a frame S' translating in the x direction at some constant velocity V in which the horizon-

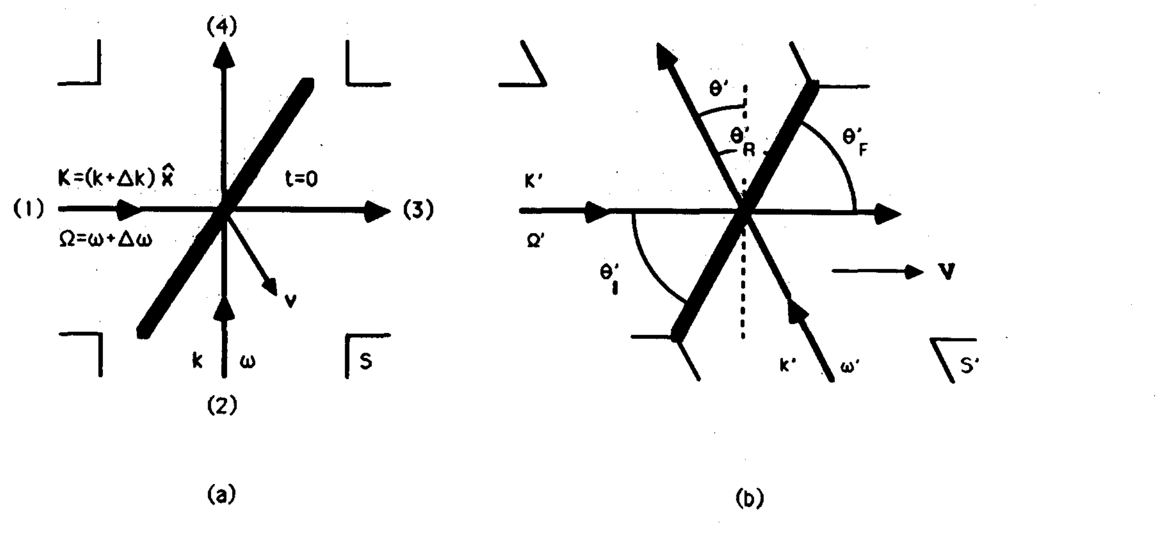

Fig. 4. If incident beam (1) is detuned by an amount $+\Delta\omega$, beating occurs. The fringe plane tilts counterclockwise and moves at constant velocity $\mathbf{v}$ as indicated. The law of reflection, "angle of incidence θ_I equals angle of reflection θ_R," does not hold for moving mirrors in general. The reflection process (1) → (4) takes place off a receding mirror—Doppler shifting the beam down by an amount $\Delta\omega$. The reflection (2) → (3) is off an oncoming mirror and is Doppler-shifted up by $\Delta\omega$. The result is then the same as one would expect from superposition. This is best seen in (b) where we have boosted to a frame S' moving at velocity $\mathbf{V}$ parallel to $\mathbf{x}$ in which the frequency of the two beams, Ω' and ω', are equal. In this frame ordinary specular reflection can be interpreted as occurring and the fringe is stationary with $\theta'_I = \theta'_R$; indicating that the Law of Reflection now holds.

tal and vertical beams have the same frequency. Such a frame indeed exists. Let us recall the Lorentz transformations, on our coordinates, for a general four vector $A^{\mu} = [A_0, \mathbf{A}]$

$$A'_x = \gamma(A_x - \beta A_0), \tag{40a}$$

$$A'_0 = \gamma(A_0 - \beta A_x), \tag{40b}$$

$$A'_y = A_y, \quad A'_z = A_z, \tag{40c}$$

where we employ the standard notation $\beta = v/c$ and $\gamma = (1-\beta^2)^{-1/2}$. Let us denote $\omega + \Delta\omega = \Omega$ and $\mathbf{k} + \Delta\mathbf{k} \equiv \mathbf{K} = K\hat{\mathbf{x}}$ for the h beam and leave ω and $\mathbf{k} = k\hat{\mathbf{y}}$ for the v beam. Then we seek a velocity $\mathbf{V} = V\hat{\mathbf{x}}$ for S' in which $\Omega' \equiv \omega'$, i.e., so that the h and v beams are equal in frequencies. The solution, employing Eqs. (40), is

$$\mathbf{V} = [\zeta/(1+\zeta)]c\hat{\mathbf{x}}. \tag{41}$$

Using this definition of $\mathbf{V}$ in β and γ we may now transform the fringe plane Eq. (36) into the S' frame using (40) again. The result is

$$y' = \sqrt{1+2\zeta}\,x' - n\lambda', \tag{42}$$

where $\lambda' = 2\pi c/\omega' = 2\pi c/\Omega'$. As should be obvious from the requirement that $\Omega' = \omega'$, the beating has stopped in S' and the mirror is at rest. The fact that the slope $m' = \sqrt{1+2\zeta} \neq 1$ is not a problem here, because viewed in this frame the "vertical beam" is no longer vertical. This is made clear by computing the transformed four vectors $K'^{\mu} \equiv [\Omega'/c, \mathbf{K}']$ and $k'^{\mu} \equiv [\omega'/c, \mathbf{k}']$. In components we have:

$$K'_x = \gamma(\omega/c), \quad k'_x = -(\omega/c)\beta\gamma, \tag{43a}$$

$$K'_y = 0, \quad k'_y = (\omega/c), \tag{43b}$$

$$K'_0 = \gamma(\omega/c), \quad k'_0 = \gamma(\omega/c). \tag{43c}$$

The fact that $k'_x < 0$ tells us that the "vertical" beam has tilted counterclockwise by an angle θ' given by

$$\tan\theta' = \left|\frac{k_x}{k_y}\right| = \beta\gamma \tag{44}$$

as shown in Fig. 4(b). As a final proof that ordinary reflection is taking place in S', we verify that the usual law of reflection holds for beam (1) reflecting into beam (4) in the S' frame. Referring to Fig. 4(b), we see that we must prove that $\theta'_R = \theta'_I$. From the geometry we can write down the following relationships, referring to Fig. 4(b),

$$\tan\theta'_I = \tan\theta'_F = \sqrt{1+2\zeta}, \tag{45a}$$

$$\theta'_R = \pi/2 - \theta'_I + \theta', \tag{45b}$$

where θ' is given by Eq. (44) above. Using elementary trigonometry and algebra one arrives sooner or later at

$$\tan(\theta'_I - \theta'_R) = 0, \tag{46}$$

which gives only $\theta_I = \theta_R$ under physically sensible conditions. The law of reflection does indeed hold, and hence we see that a frequency detuning is understandable within the specular reflection interpretation—a fact that is easiest to see in this Lorentz frame where the fringe is at rest. Detuning was the last parameter to check. Hence, we have refuted the claim that the specular reflection interpretation of this interference process is not consistent with some type of incident beam labeling—a claim that would have invalidated the mirror viewpoint as contradictory to ordinary superposition.

VI. SUMMARY AND CONCLUSIONS

We began by considering a gedanken experiment in Sec. I concerning two orthogonally intersecting coherent beams of light 180° out of phase with each other but equal in all other respects. Analysis of the Poynting vector and the electromagnetic field inside the region of intersection gave rise to the conjecture that the interference fringe was indistinguishable from a perfect edgeless mirror and that the two incident beams could in fact be viewed as reflecting specularly off the fringe. In order to reconcile this idea with the superposition principle, we analyzed the mirror response to each of the four possible ways one can classically label or tag a light beam. We find that such a labeling of an incident beam always alters the fringe in such a way that the label is transmitted across the fringe in a manner consistent with either the mirror hypothesis or superposition. We claim then, at least, for this simple configuration, that we have proved the indistinguishability between the two interpretations.

Why do we want to interpret an ordinary interference effect that has been understood for years as a process of specular reflection? Well, in some sense the reflection idea gives us a more satisfactory account of what the energy in the field is really doing during such an interference process. Certainly, having a totally new way of looking at an old phenomenon cannot hurt and could perhaps lead to new insights in optics. The specular reflection interpretation perhaps could open new vistas when considering various interference effects; one can treat "interference" to some extent as an "interaction."

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